Dark Energy and the Big RIP

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Abstract

Dark Energy is an enigmatic phenomenon in astrophysics, resulting in the acceleration of universe expansion. Whether its density will remain constant, *i.e.* whether $w = -1$, remains a heated debate topic.[1] The essay will contain:

- Introduction to the **Cosmic Distance Scale**
- Explanation of how **Supernova Ia** lead to the discovery of Dark Energy
- Properties of **Dark Energy**
- A detour to **Einstein's Field Equation (EFE)**
- A brief introduction to the future of the universe and what the **Big Rip** is
- A mathematical estimation of the **Timescale** of its occurrence using **cosmological parameter results from the final full-mission Planck measurements** of the cosmic microwave background (CMB) anisotropies (Aghanim et al., 2020)

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1 Discovering Dark Energy

1.1 Introduction to Supernova Ia

1.1.1 Cosmic Distance Scale

- To crack the expansion rate of the universe we need to set up a **Cosmic Distance Scale**
- Cosmic Distances are particularly difficult to crack. Parallax, Cepheid Variables, **Type Ia supernovae** (Ryden, 2003) allow us to estimate to a reasonable good standard
- For more please check Figure (1)

1.1.2 Standard Candles

- Standard Candles are Astrophysical objects with known luminosity L (Normand, n.d.), those include and are not limited to Cepheid Variables and **Type Ia supernovae**. With measurable flux, one can estimate the required distance by the **Inverse-Square Law**

$$F = \frac{L}{4\pi D^2} \tag{1}$$

1.1.3 White dwarfs

- White dwarfs have a particularly interesting Radius and Mass relationship (*The white dwarf mass-radius relationship*, n.d.):

$$R \sim M^{-\frac{1}{3}} \tag{2}$$

- With decreasing radius density increases and electron speed becomes relativistic, resulting in a **Chandrasekhar Mass Limit**. Below [Figure (2)] is a picture of White Dwarfs.

[1]We should note that while the density is a constant, the volume of the universe increases, so dark energy is being "produced" continuously

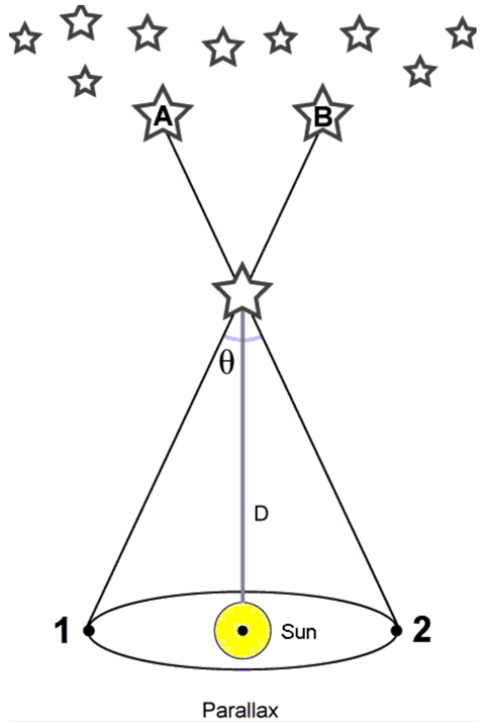

Figure 1: Image is obtained here: (*https://physics.weber.edu/carroll/expand/parallax.htm*, n.d.) There are various assumptions behind using parallaxes. As size of angular motion is inversely proportional to distance of the star (This is left as an exercise for the readers), we treat the "background stars" as fixed. I would like you to pay attention to the Hipparcos Project. Launched in August 1989, Hipparcos successfully observed the celestial sphere for 3.5 years before operations ceased in March 1993. Calculations from observations by the main instrument generated the Hipparcos Catalogue of 118,218 stars charted with the highest precision. An auxiliary star mapper pinpointed many more stars with lesser but still unprecedented accuracy, in the Tycho Catalogue of 1,058,332 stars!(*THE HIPPARCOS SPACE ASTROMETRY MISSION*, n.d.)

1.1.4 Supernovae Explosion

- Supernova Ia (Introduced in **1.1.1**) occurs in a binary (or double) star system, in which at least one of the two stars must be a white dwarf(Introduced in **1.1.3**). The white dwarf gets more and more massive as it pulls in material from its companion star through accretion. If the white dwarf grows to over 1.44 $M_\odot$, the electron degeneracy pressure is no longer strong enough to prevent the star from collapsing [2]. At this point, the star explodes as a Type Ia supernova.(*Type Ia Supernovae*, 2022)
- Type Ia supernovae are one of the brightest events in the Universe. They are many times brighter than other kinds of supernovae. Type Ia supernovae are always the same brightness (luminosity). This is because the explosion always takes place when the white dwarf reaches a set mass.(*Type Ia Supernovae*, 2022) And this is where you can use your knowledge obtained from **1.1.2**)
- Below I have a picture of supernova Ia (*Supernova in the Spiral Galaxy NGC 2525 ESA/Hubble*, 2020)

1.2 Nobel Prize Discovery

In 1998, cosmology was shaken at its foundations as two research teams, The Supernova Cosmology Project and The High-z Supernova Search Team presented their findings. The two research teams found over 50 distant supernovae whose light was weaker than expected – this was a sign that the expansion of the Universe was accelerating. The force driving this is dark energy. (*The nobel prize in physics 2011*, 2011) For $z << 1$ we have (Heymans & Lawrence, 2022)

$$D_L \approx \frac{c(1+z)(z - \frac{1+q}{2}z^2)}{H_0} \tag{3}$$

Where $q = -\left(1 + \frac{\dot{H}}{H^2}\right)$ and $\dot{H}$ is the rate of change of the Hubble parameter [3]. We find that $q < 0$, and

[2] $M_\odot$ refers to Mass of the Sun
[3] The Hubble Constant is not a Constant - We can, rather surprisingly, deduce it on our own. Do search it online if you are interested!

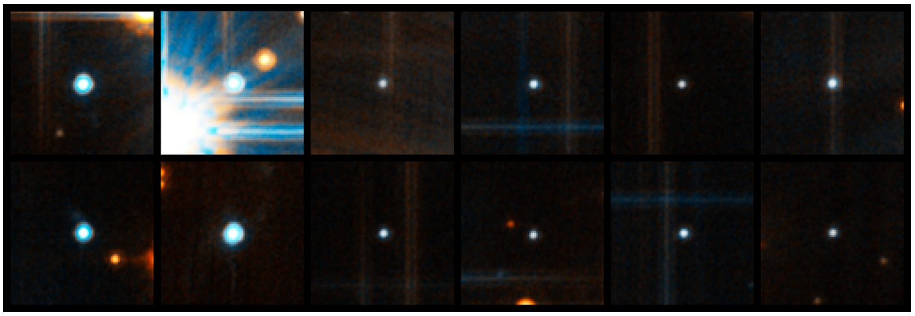

Figure 2: Source: (*White Dwarfs in NGC 6397 (Hubble) (Image from ESA/Hubble website)*, 2007) White Dwarfs in NGC 6397. It is only 7,800 light-years from the Earth. Side Note: In measuring the distance from earth, Parallex (as introduced before) was used and the wobbles of the stars represented only 1/100th of a pixel on the telescope's camera, and Hubble measured them to within 1/3,000th of a pixel. STScI representatives framed this result as "the equivalent to measuring the size of an automobile tire on the moon to a precision of 1 inch. (Urrutia, 2018)

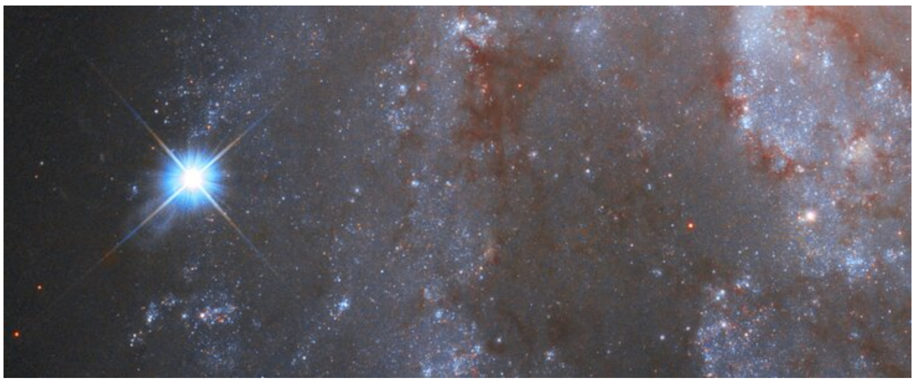

Figure 3: Source: (*Supernova in the Spiral Galaxy NGC 2525 ESA/Hubble*, 2020). The supernova, formally known as SN2018gv, was first spotted in mid-January 2018.

hence the expansion of the universe is increasing.

2 Properties of Dark Energy

2.1 What is Dark Energy?

I think it is common sense that Dark merely reflects that Dark Energy can't be detected directly(As I expect you to know that black implies absense [4]). While there are many speculations, two proposed forms of dark energy are the **Cosmological Constant** and **Scalar Fields** (such as **Quintessence**[5]). Below I will have a brief (actually rather long) introduction to the cosmological constant. I hope meanwhile you will have a grasp of what Astrophysics is - Behind the seemingly stunning pictures, the mathematics is almost menacing.

2.1.1 Very Slight Detour - Einstein's Field Equation

While Einstein's $E = mc^2$ is the most famous, I argue that this equation, compared to other Einstein's equations, is most important for Astrophysicists. The equation describes gravity as a result of spacetime being curved by mass and energy. The solutions to these equations are the components of the metric tensor $g_{\mu\nu}$, which specifies the spacetime geometry.(*Department of Physics*, n.d.) Some symbols you won't know without taking university courses, for example, $R_{\mu v}$ is the **Ricci curvature tensor**.

$$R_{\mu v} - \frac{1}{2} R g_{\mu v} + \Lambda g_{\mu v} = \frac{8\pi G}{c^4} T_{\mu v} \tag{4}$$

[4]Your Photoreceptors aren't. Anyways, that is left for biologists

[5]It is being treated as the fifth fundamental force (Bender, 2022)

Below I have a crazier form (Note that all three equations below are from (Hirvonen, n.d.)). Note the use of **Einstein Summation Convention**. *e.g.* $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = g_{11}dx^1dx^1 + g_{12}dx^1dx^2 + ... + g_{33}dx^3dx^3$:

$$\begin{gathered}\frac{1}{2}g^{\alpha\beta}\partial_\alpha\partial_\mu g_{\beta\nu} + \frac{1}{2}g^{\alpha\beta}\partial_\alpha\partial_\nu g_{\mu\beta} - \frac{1}{2}g^{\alpha\beta}\partial_\alpha\partial_\beta g_{\mu\nu} - \frac{3}{2}g^{\alpha\beta}\partial_\mu\partial_\nu g_{\alpha\beta} \\ -\frac{1}{2}g^{\beta\lambda}g^{\alpha\rho}\partial_\alpha g_{\rho\lambda}\partial_\mu g_{\beta\nu} - \frac{1}{2}g^{\beta\lambda}g^{\alpha\rho}\partial_\alpha g_{\rho\lambda}\partial_\nu g_{\mu\beta} + \frac{1}{4}g^{\beta\lambda}g^{\alpha\rho}\partial_\nu g_{\alpha\lambda}\partial_\mu g_{\rho\beta} + \\ \frac{1}{4|g|}g^{\alpha\beta}\partial_\beta|g|\partial_\nu g_{\mu\alpha} - \frac{1}{4|g|}g^{\alpha\beta}\partial_\beta|g|\partial_\alpha g_{\mu\nu} - \frac{1}{4|g|}g^{\alpha\beta}\partial_\beta|g|\partial_\mu g_{\alpha\nu} \\ +\Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}\end{gathered} \tag{5}$$

And so if we write it out in terms of summation as learnt in high school:

$$\begin{aligned}&\frac{1}{2}\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}g^{\alpha\beta}\partial_\alpha\partial_\mu g_{\beta\nu} + \frac{1}{2}\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}g^{\alpha\beta}\partial_\alpha\partial_\nu g_{\mu\beta} - \frac{1}{2}\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}g^{\alpha\beta}\partial_\alpha\partial_\beta g_{\mu\nu} \\ &- \frac{3}{2}\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}g^{\alpha\beta}\partial_\mu\partial_\nu g_{\alpha\beta} - \frac{1}{2}\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}\sum_{\rho=0}^{3}\sum_{\lambda=0}^{3}g^{\beta\lambda}g^{\alpha\rho}\partial_\alpha g_{\rho\lambda}\partial_\mu g_{\beta\nu} \\ &- \frac{1}{2}\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}\sum_{\rho=0}^{3}\sum_{\lambda=0}^{3}g^{\beta\lambda}g^{\alpha\rho}\partial_\alpha g_{\rho\lambda}\partial_\nu g_{\mu\beta} + \frac{1}{4}\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}\sum_{\rho=0}^{3}\sum_{\lambda=0}^{3}g^{\beta\lambda}g^{\alpha\rho}\partial_\nu g_{\alpha\beta}\partial_\mu g_{\rho\beta} \\ &+ \frac{1}{4|g|}\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}g^{\alpha\beta}\partial_\beta|g|\partial_\nu g_{\mu\alpha} - \frac{1}{4|g|}\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}g^{\alpha\beta}\partial_\beta|g|\partial_\alpha g_{\mu\nu} \\ &- \frac{1}{4|g|}\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}g^{\alpha\beta}\partial_\beta|g|\partial_\mu g_{\alpha\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}\end{aligned} \tag{6}$$

To show exactly how complicated this is, below I'll show the **First Term** being completely expanded out:

$$\begin{gathered}\frac{1}{2}\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}g^{\alpha\beta}\partial_\alpha\partial_\mu g_{\beta\nu} = \frac{1}{2}g^{00}\partial_0\partial_\mu g_{0\nu} + \frac{1}{2}g^{01}\partial_0\partial_\mu g_{1\nu} + \frac{1}{2}g^{02}\partial_0\partial_\mu g_{2\nu} + \frac{1}{2}g^{03}\partial_0\partial_\mu g_{3\nu} \\ +\frac{1}{2}g^{10}\partial_1\partial_\mu g_{0\nu} + \frac{1}{2}g^{11}\partial_1\partial_\mu g_{1\nu} + \frac{1}{2}g^{12}\partial_1\partial_\mu g_{2\nu} + \frac{1}{2}g^{13}\partial_1\partial_\mu g_{3\nu} + \frac{1}{2}g^{20}\partial_2\partial_\mu g_{0\nu} + \frac{1}{2}g^{21}\partial_2\partial_\mu g_{1\nu} \\ +\frac{1}{2}g^{22}\partial_2\partial_\mu g_{2\nu} + \frac{1}{2}g^{23}\partial_2\partial_\mu g_{3\nu} + \frac{1}{2}g^{30}\partial_3\partial_\mu g_{0\nu} + \frac{1}{2}g^{31}\partial_3\partial_\mu g_{1\nu} + \frac{1}{2}g^{32}\partial_3\partial_\mu g_{2\nu} + \frac{1}{2}g^{33}\partial_3\partial_\mu g_{3\nu}\end{gathered} \tag{7}$$

Well that's Astrophysics... Behind all the beautiful pictures, you need maths.

2.1.2 Back to topic - Λ

After all this fuss, you should pay attention to the Λ symbol, which is depicting **dark energy**. The value is taken as:

$$\Lambda = 3\left(\frac{H_0}{c}\right)^2\Omega_\Lambda \tag{8}$$

Using results in 2018, *i.e.* $\Omega_\Lambda = 0.6889 \pm 0.0056$ and $H_0 = 67.66 \pm 0.42$ (Aghanim et al., 2020), Λ is close to $2.888 \times 10^{-122}{l_p}^2$ where l_p is the Planck Length [6]

2.2 Prevalence

The concordant cosmology model of our universe shows that dark energy is a major constituent of our universe. Below I will show the components in the universe for comparison. (*Dark Energy, dark matter*, n.d.)

- Baryonic content $\Omega_b \approx 0.05$
- Dark energy content $\Omega_\Lambda \approx 0.68$
- Dark matter content $\Omega_c \approx 0.27$

2.3 Why Great Effect?

Its prevalence in the universe allows it to exert a great effect, while gravity diminishes over distance by, again, the **inverse-square law**. In case the reader forgets, it is:

$$F = G\frac{Mm}{r^2} \tag{9}$$

[6] The Planck Length is defined as: $l_p = \sqrt{\frac{\hbar G}{c^3}}$ (*Planck length*, n.d.)

3 Why concern us

The behaviour of dark energy will eventually result in the fate of the universe (and our future). That includes but is not limited to: The Big Freeze, Big Crunch, The Big Bounce, and The Big Rip(James, 2021). Among these, The Big Rip is the most interesting and violently spectacular, and will remain our focus.

4 Big Rip

4.1 Phantom Energy

The ratio between dark energy pressure and its energy density, also known as the equation of state, is defined to be w (Mack, 2020). We can measure this through gravitational lensing (*Light twists its way through a lumpy universe*, n.d.). By seeing how the light of background galaxies is bent behind the mass of clusters, we can see how clusters grew with time in the past and hence the strength of dark energy [7]. Below I have a picture of Gravitational Lensing:

Figure 4: Source: (*Gravitational Lensing from Esa/Hubble*, 2011). The galaxy is one of a group of galaxies called Luminous Red Galaxies. Notice the blue ring - *i.e.* the Einstein Ring, it is a product of Gravitational Lensing.

The universe will accelerate ($\ddot{a} > 0$) if $w < -\frac{1}{3}$ (Carroll, Hoffman, & Trodden, 2003). If $w < -1$ the density of dark energy (known as **phantom energy** in this case) **increases** as the universe expands.(Carlisle, 2017) Current estimations conclude w is close to **-1**. There are several ways in which one may achieve $w < -1$, including purely negative kinetic terms, non-minimal kinetic terms, and scalar-tensor theories (Carroll et al., 2003). In 2018, the Planck Collaboration measured a value of $w = -1.03 \pm 0.03$ (Aghanim et al., 2020) , which is consistent, but errors and inaccuracies in these experiments mean we cannot exclude the possibility of Big Rip occuring in the future in general [8]. In Figure (5) you can see very clearly what will happen in the future

As the observable universe expands, the 4 fundamental forces [9] will have a smaller effect than dark energy, and eventually, even atoms will be "ripped apart" as the electromagnetic force is being overcome.(Mack, 2020) The very existence of civilization, and even spacetime itself, will turn obsolete.[10]

[7] As dark energy can be roughly thought as anti-gravity, it discourages the formation of clusters

[8] Even a value slightly smaller than -1 will eventually cause Big Rip. The closer the value is to -1, the longer it takes. For details, please check the formula stated later

[9] That includes: Electromagnetism, Strong Interaction, Weak Interaction, Gravitation

[10] Obviously, assuming humans are still present!

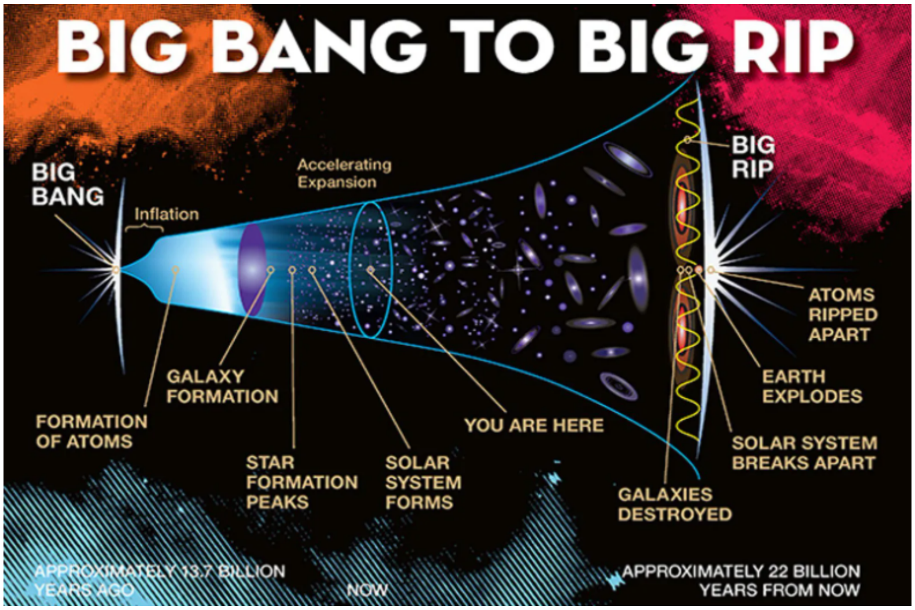

Figure 5: Source: (Teaford, 2015). The Universe from beginning to end.

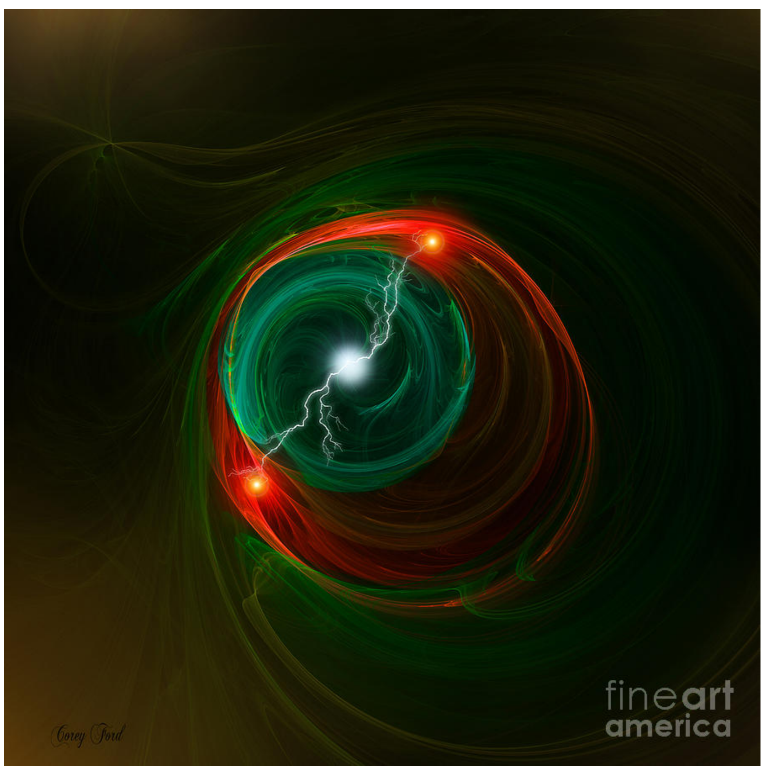

Figure 6: Source: (Ford, n.d.) This is a very beautiful picture with a title seemingly related to dark energy. Obviously Dark Energy is invisible, but it is definitely as mysterious and mighty as this picture seems!

5 Duration Until Big RIP

We can estimate the duration to be (Ruggiero, 2020):

$$t_{\text{rip}} = -\frac{2}{3(1+w)H_0\sqrt{1-\Omega_{m,0}}} \tag{10}$$

With a vertical asymptote at $w = -1$, as one would expect. If we take $w = -1.03$ as from the measured value from Planck Collaboration(Aghanim et al., 2020), we have:

$$t_{\text{rip}} = 7.59 \pm 0.05 \times 10^9 \text{ years} \tag{11}$$

And that's **a lot** , considering the time when sun will consume earth(Schröder & Connon Smith, 2008): [11]

$$t_{\text{earth consumption}} = 7.59 \times 10^9 \text{ years} \tag{12}$$

However, a paper published in 2015 suggested Big Rip can occur 22×10^9 years later (Temperton, 2015). Some scientists even suggest 2.8×10^9 years (Aron, 2016). This depends on the actual value of w, but as no one has come up with a concrete number, this causes a huge disparity in the estimation. [12]

6 Conclusion

Dark Energy remains one of the biggest mysteries in Astrophysics (Cho, 2012). Its density results in drastically different universe models, including the Big RIP. Even though on paper it sounds cool while dangerous, as it won't happen for another few billion years (or more), we need not worry for now (unless we find out we've missed some details!).

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[11] Considering error propagation, 168.8×10^9years for $w = -1.03 - 0.03 = -1.06$, still much larger than 7.59×10^9 years

[12] A slight deviation near the vertical asymptote can produce a drastically different answer!

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