A Sneak Peak

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Abstract

This set of notes is made with reference to the Astrophysics Coursebook by The University of Edinburgh (Introductory Astrophysics, Course PHYS08050) . I have gained permission from Professor Catherine Heymans to put my modified notes in my website. It is also available in my personal website.

This is a special version of the notes (A Sneak Peak!) If you want the original copy, please email henry36c@gmail.com or ask Henry in person (Well he's quite a nice guy)

1 Some Black Holes

- If a small mass Δm starts from a distance and falls onto a large mass M, it gains kinetic energy:

$$E = \frac{GM\Delta m}{R} \tag{1}$$

- For gases however, friction and collisions randomise the energy and turn it into heat
- For a black hole:

$$R_{\text{Event Horizon}} = \frac{2GM}{c^2} \tag{2}$$

- By substitution,

$$E = \frac{GM\Delta m}{\frac{2GM}{c^2}} \tag{3}$$

$$= \frac{1}{2}\Delta mc^2 \tag{4}$$

- The efficiency, μ, is 0.5, which is much higher than $\mu_{\text{nuc}} = 0.007$
- This is obviously, a fallacy
- **Incorrect Assumptions**
  - Radial free fall is unlikely
  - At larger distances, the accreting material will always have some angular momentum, and end up forming a rotating disc around the black hole
  - Friction between neighbouring radial annuli then allows the material to slowly spiral inwards, forming a gradually heated accretion disc
  - Efficiency μ is reduced by the following 2 effects:
    * Effective radius is **not** the Event Horizon. For non-rotating black-holes, the ISCO is located at:

$$r_{\text{ms}} = 6\frac{GM}{c^2} \tag{5}$$

$$= 3R_{\text{EH}} \tag{6}$$

    * I.e. There is no stable orbit $< 3R_{\text{EH}}$

* And therefore the thermal energy gained by the gas decreases
* Some energy (half) is converted to rotational energy, as:

$$\Delta K = \frac{dK}{dR}\Delta R \tag{7}$$
$$= -\frac{GM\Delta m}{2R^2}\Delta R \tag{8}$$

$$\Delta U = \frac{dU}{dR}\Delta R \tag{9}$$
$$= -\frac{GM\Delta m}{R^2}\Delta R \tag{10}$$

$$\frac{\Delta K}{\Delta U} = \frac{-\frac{GM\Delta m}{2R^2}\Delta R}{-\frac{GM\Delta m}{R^2}\Delta R} \tag{11}$$
$$= \frac{1}{2} \tag{12}$$

Combining with other effects, we have $\mu \approx 0.1$

1.1 Eddington Luminosity

- A photon has energy $E = \frac{hc}{\lambda}$ and momentum $p = \frac{h}{\lambda} = \frac{E}{c}$. So you can visualize momentum flux being $\frac{S}{c}$, where S is the radiation flux
- Most of that flux may pass straight through, but some of it will scatter on the electrons inside atoms. That scattering produces a force on the electrons, which drag the atoms with them. The scattering process has a cross-section $\sigma_e = 6.6510^{-29}m^2$
- The **rate** [1] of momentum transfer (or in other words, radiation force on each atom) is therefore:

$$\frac{dp}{dt} = \frac{S\sigma_e}{c} \tag{13}$$

- Combining with $L = \frac{S}{4\pi R^2}$ we obtain:

$$F_{\text{radiation}} = \frac{L\sigma_e}{4\pi R^2 c} \tag{14}$$

- Assuming all the gas are hydrogen and each hydrogen atom composes of 1 proton, which has mass of m_p. As $m_e << m_p$, we have:

$$F_{\text{gravitation}} = \frac{GMm_p}{R^2} \tag{15}$$

- At the limiting luminosity, the outward radiation force is equal to the inwards gravitational force, *i.e.* $F_{\text{radiation}} = F_{\text{gravitation}}$. Combining everything gives us:

$$L_{\text{Max}} = \frac{4\pi G m_p c}{\sigma_e} \times M \tag{16}$$

2 Distance-redshift relation

- This section resulted in a Nobel Prize in 2011. They used distant supernova as standard candles to probe the Universe

[1] This implies differentiation

- We have the formula $D = \frac{cz}{H_0}$. There is an extension that works for more distant galaxies:

$$D_L \approx \frac{c(1+z)(z - \frac{1+q}{2}z^2)}{H_0} \tag{17}$$

- Where $q = -\left(1 + \frac{\dot{H}}{H^2}\right)$ and $\dot{H}$ is the rate of change of the Hubble parameter

3 Concordant cosmology

- Baryonic content $\Omega_b \approx 0.05$
- The total dark and baryonic matter content $\Omega_m \approx 0.3$
- Dark energy content $\Omega_\Lambda \approx 0.7$
- The university is flat and has critical density: $\Omega_b + \Omega_m + \Omega_\Lambda = 1$