Determine Validity of Argument.

1. Question:

Consider the following statement:

If the argument is valid, give a derivation.

If the argument is invalid. provide a counter-example.

2. Solution:

$pr1, pr2, pr3 \dots, prn$ are premises:

$$\left.\begin{array}{l} pr1 \\ pr2 \\ pr3 \\ \vdots \\ prn \end{array}\right\} \text{premises}$$
$$\therefore q \qquad \text{conclusion.}$$

This is a valid argument if:

$$(pr1 \wedge pr2 \wedge pr3 \wedge \cdots \wedge prn) \longrightarrow q \equiv T$$

3. Examples.

Determine whether the following is valid:

1) $p \to r$
$q \to r$
$q \vee \sim r$
$\therefore \sim p$

If the argument is valid:

$$(p \to r) \wedge (q \to r) \wedge (q \vee \sim r) \longrightarrow \sim p \equiv T. \text{ (is tautology).}$$

We try to provide a counter-example. where conc is false & antecedent is true.

$$\therefore \sim p \equiv F \to p \equiv T.$$

$$((p \to r) \wedge (q \to r) \wedge (q \vee \sim r)) \equiv T$$

$$\Rightarrow ((T \to r) \wedge (q \to r) \wedge (q \vee \sim r)) \equiv T.$$

where $(T \to r)$, $(q \to r)$, $(q \vee \sim r)$ are all True.

$\therefore r \equiv T$. $\therefore \sim r \equiv F$.

where $(q \to T)$, $(q \vee F)$ are all True

$\therefore q \equiv T$.

$\therefore p \equiv T$, $q \equiv T$, $r \equiv T$.

$((p \to r) \wedge (q \to r) \wedge (q \vee \sim r)) \to \sim p$

$\equiv ((T \to T) \wedge (T \to T) \wedge (T \vee F)) \to F$

$\equiv (T \wedge T \wedge T) \to F$

$\equiv T \to F$

Therefore, if p, q, r are true, this argument is invalid.

2) $p \to q$

$q \to r$

$p \to r$

When seeing the conclusion is a single sentence. start an invalid analysis. Otherwise start a prove analysis.

1. $((p \to q) \wedge (q \to r)) \to (p \to r)$
2. $\sim((p \to q) \wedge (q \to r)) \vee (p \to r)$ 1 CDJ.
3. $(\sim(p \to q) \vee \sim(q \to r)) \vee (p \to r)$ 2 DM.
4. $(p \wedge \sim q) \vee (q \wedge \sim r) \vee (p \to r)$ 3. NC.
5. $(p \wedge \sim q) \vee (q \wedge \sim r) \vee (\sim p \vee r)$ 4 CDJ.
6. $((p \wedge \sim q) \vee \sim p) \vee ((q \wedge \sim r) \vee r))$. ☆ distribution.
7. $(\underbrace{(p \vee \sim p)}_{T} \wedge (\sim q \vee \sim p)) \vee ((q \vee r) \wedge \underbrace{(\sim r \vee r)}_{T})$.
8. $(T \wedge (\sim q \vee \sim p)) \vee ((q \vee r) \wedge T))$
9. $(\sim q \vee \sim p) \vee (q \vee r)$
10. $(\sim q \vee q) \vee \sim p \vee r$
11. $T \vee \sim p \vee r$.
12. T.

$\therefore$ Valid.

aim: turn everything into '$\vee$' or '$\wedge$' to see wether it's always True.

Q1: $((p \to r) \wedge (q \to r) \wedge (q \vee \sim r)) \to \sim p$.

$\equiv \sim((p \to r) \wedge (q \to r) \wedge (q \vee \sim r)) \vee \sim p$.

$\equiv (\sim(p \to r) \vee \sim(q \to r) \vee \sim(q \vee \sim r)) \vee \sim p$.

$\equiv (p \wedge \sim r) \vee (q \wedge \sim r) \vee (q \wedge r) \vee \sim p$

$\equiv ((p \vee \sim p) \wedge (\sim p \vee \sim r)) \vee (q \wedge \sim r) \vee (q \wedge r).$

$\equiv (T \wedge (\sim p \vee \sim r)) \vee (q \wedge \sim r) \vee (q \wedge r).$

can't go further. Thus. start and invalid analyse.