# Predicate Logic

## 1. Symbol of Predicate Logic

1) Atomic Sentence Letters: $P - Z$
2) Logical Connectives: $\sim, \rightarrow, \leftrightarrow, \wedge, \vee$
3) Brackets and Parentheses: ( ), [ ]

sentential logic.

4) Quantifiers
   ① Universal Quantifier: $\forall$ (for all)
   ② Existential Quantifier: $\exists$ (some, there exist, at least one).
5) Individual Constants or Names: $a - h$
6) Variables: $i - z$
   e.g. I don't like cats
7) Operation Letters: $a - h$; also $a^0, a^1, a^2, \ldots$

subjects.

multiplace PL

8) Predicate Letters: $A - O$; also $F^0, F^1, F^2, \ldots$
9) Identity Sign: $=$

Predicates

## 2. Atomic & Molecular & Quantified Formula.

1) Atomic Formula:
   ① $P - Z$
   ② (Predicate Letter)(Name Letter)
   ③ (Predicate Letter)(Variable Letter)
2) Molecular Formula: Using $\sim, \rightarrow, \leftrightarrow, \wedge, \vee$ and parentheses on atomic and molecular formulas.
   e.g. a: Louis C.K. b: Tina Fey.

A: {1} is amazing. H: {1} is hilarious

Tina Fey and Louis C.K. are hilarious

$Ha \wedge Hb$

3) Quantified Formula

① (Quantifier)(Variable)(Atomic or Molecular)

4) Well-formed Formula.

① Atomic Formula. ② Molecular Formula.

③ Quantified Formula.

e.g. C': a is material D': a is phenomenal.

Everything is material or phenomenal.

$\forall x(Cx \vee Dx)$

when we say everything tastes so good, it even includes computer

3. Universe of Discourse.: when we say '$\forall$' (everything), it's literally everything in the universe. (we have to restrict the domin).

1) Canonical Form of Universally Quantified Sentence. : we always combine universal quantifiers with conditional)

$\forall\alpha(\phi\alpha \to \psi\alpha)$ : $\forall\alpha$(Group $\alpha \to$ Property $\alpha$)

predicates.

→ in same bracket, the variable is the same.

e.g. D': a is a dog. C': a is cute.

All dogs are cute.

$\cancel{\forall x(Dx \wedge Cx)}$ (x)

$\forall x(Dx \to Cx)$

① Stylistic Variants

→ All dogs are cute

→ Dogs are cute

→ $\boxed{\text{The}}$ dog is cute. → a dog is cute

→ Any dog is cute

→ If you're a dog, then you're cute

② Complicated Restricted Clauses.

(group part) the restricted clause can change between '∧' and '→' but predicate clause can't (property part).

e.g. D': a is a dog; C': a is cute; F': a has a cube head.

Dogs with cube heads are cute.

$\forall x(Dx \land Fx \to Cx)$ or $\forall x(Fx \land Dx \to Cx)$

$\forall x(Dx \to (Fx \to Cx))$ or $\forall x(Fx \to (Dx \to Cx))$

e.g. D': a is a dog; G': a is a cat; F': a is fluffy.

Cats and Dogs are fluffy.

$\forall x(Dx \lor Cx \to Fx)$ or $\forall x(Cx \lor Dx \to Fx)$.

$\forall x(Dx \to Fx) \land \forall x(Gx \to Fx)$. 可使用相同 variable, 因为在 bracket 内才生效.

③ Complicated Predicate Clauses.

e.g. D': a is a dog; C': a is cute; F': a has a sphere head

H': a is hilarious

Sphere headed dogs are cute and hilarious.

$\forall x(Dx \land Fx \to Cx \land Hx)$ or $\forall x(Fx \land Dx \to Cx \land Hx)$.

$\forall x(Dx \to (Fx \to Cx \land Hx))$ or $\forall x(Fx \to (Dx \to (Cx \land Hx)))$.

2) Canonial Form of the Existentially Quantified Sentence.

The inner is alway an '$\wedge$' when quantifier is an existential.

$\exists \alpha (\phi \alpha \wedge \psi \alpha)$: $\exists \alpha$ (Group $\alpha$ $\wedge$ Property $\alpha$)

(Some $\phi$'s are $\psi$'s; At least one $\phi$ is a $\psi$; There is a $\phi$ that is a $\psi$).

e.g. $F'$: {1} is a rapper; $H'$: {1} is from Toronto;

$K'$: {1} is started from the bottom.

There is a rapper from Toronto who started from the bottom.

$\exists x (Fx \wedge Hx \wedge Kx)$.

3) Tranform

① Not All ...... $(\sim \forall x (Fx \rightarrow Gx))$

= Some ...... is not. $(\exists x (Fx \wedge \sim Gx))$

② None of ......, $(\forall x (Fx \rightarrow \sim Gx))$

= Not something is ...... $(\sim \exists x (Fx \wedge Gx))$

e.g. $F'$: a is a restaurant; $G'$: a is good.

Not all restaurants are good.

$\sim \forall x (Fx \rightarrow Gx)$ or $\exists x (Fx \wedge \sim Gx)$.

e.g. a: Joe; $A'$: a is sad; $F'$: a is a store; $G'$: a is open; $J'$: a is good; $K'$: a is weather; $M'$: a is a person; $N'$: a is on the street; $O'$: a is nice.

Assuming that none of the stores are open, Joe will be sad unless there is good weather and the people on the street are nice.

$\forall x (Fx \rightarrow \sim Gx) \rightarrow (Aa \vee (\exists x (Jx \wedge Kx) \wedge \forall x (Mx \wedge Nx \rightarrow Ox)))$