MAT 137
Tutorial #1– Logic, quantifiers, and definitions
September 20-21, 2022
Due on Thursday, Sept 22 by 11:59pm via GradeScope

- **Submissions are only accepted by Gradescope.** Do not send anything by email. Late submissions are not accepted under any circumstance. Remember you can resubmit anytime before the deadline.
- **Submit your polished solutions using only this template PDF.** You will submit a single PDF with your full written solutions. If your solution is not written using this template PDF (scanned print or digital) then you will receive zero. Do not submit rough work. Organize your work neatly in the space provided.
- **Show your work and justify your steps** on every question, unless otherwise indicated. Put your final answer in the box provided, if necessary.

We recommend you write draft solutions on separate pages and afterwards write your polished solutions here. You must fill out and sign the academic integrity statement below; otherwise, you will receive zero.

Academic integrity statement

I confirm that:

- I have read and followed the policies described in the Policies and FAQ.
- I understand that the collaboration of this worksheet is only among this group. I have not violated this rule while writing this worksheet.
- I understand the consequences of violating the University's academic integrity policies as outlined in the Code of Behaviour on Academic Matters. I have not violated them while writing this assessment.

By signing this document, I agree that the statements above are true.

First Name	Last Name	UofT email	signature	TA Initials
Shujun	Yang	shujun.yang@mail.utoronto.ca	Shujun Yang	
Xuanqi	Wei	henry.wei@mail.utoronto.ca	Henry Wei	
Michelle	Wang	xxuan.wang@mail.utoronto.ca	Michelle Wang	
Wenyi	Li	wenyii.li@mail.utoronto.ca	Wenyi Li	

TA name: Brendon Shutterwhi[illegible] TA signature: BS

1. Write the negation of the following statements without using any negative words ('no', 'not', 'none',etc):

- Every student at University of Toronto has at least one friend whose first name contains a letter comes alphabetically before H.

There exists at least one student at University of Toronto such that all of his friends' name only contain letters that are alphabetically after G.

- If I enjoy learning MAT137, then I have no friends in this course.

I enjoy learning MAT137, and I have at least one friend in this course.

2. Let A be a **non-empty** subset of the real numbers. $A \subseteq \mathbb{R}$.

- We say A is *excited* if $\exists a \in \mathbb{R}$ such that $\forall x \in A, x \geq a$.
  Find an example of excited set. Then prove it is excited by using the definition.

$A = \{0\}$

WTS: $\exists a \in \mathbb{R}$ s.t. $\forall x \in A$, $x \geq a$.

pf. Let $A = \{0\}$. $A \subseteq \mathbb{R}$; $A \neq \emptyset$

Let $x \in A$.

Take $a = -1$, $a \in \mathbb{R}$; Take $x = 0$

Since 0 is the only element of the set A, gives:

$x = 0 > a = -1$

We conclude that when $A = \{0\}$. A is excited as $\exists a \in \mathbb{R}$ s.t. $\forall x \in A$. $x \geq a$. ■

- We say A is *happy* if $\exists a \in A$ such that $\forall x \in A, x \geq a$.
  Find an example of happy set. Then prove it is happy by using the definition.

$A = \{0\}$

WTS: $\exists a \in A$ s.t. $\forall x \in A, x \geq a$.

pf. Let $A = \{0\}$, $A \subseteq \mathbb{R}$. $A \neq \emptyset$.

Take $a \in A$, gives $a = 0$

Since $A \subseteq \mathbb{R}$. $a \in A$, gives. $a \in \mathbb{R}$.

Let $x \in A$; Since 0 is the only element of the set A, take $x = 0 = a$, satisfies $x \geq a$;

We conclude that when $A = \{0\}$. A is excited as $\exists a \in A$. s.t. $\forall x \in A$. $x \geq a$. ■.

- We say A is *normal* if $\forall x \in A$, $\exists a \in \mathbb{R}$ such that $a < x$.
  Find an example of normal set. Then prove it is normal by using the definition.

$A = \{2\}$

WTS: $\forall x \in A$, $\exists a \in \mathbb{R}$. s.t. $a < x$.

pf. Let $A = \{2\}$. $A \subseteq \mathbb{R}$. $A \neq \emptyset$.

Let $x \in A$. since 2 is the only element of A, $x = 2$.

Take $a = 1$, $a \in \mathbb{R}$, statisfies $a = 1 < x = 2$.

Since 2. is the only element of the set A,

To conclude, when $A = \{2\}$. $\forall x \in A$, $\exists a \in \mathbb{R}$ s.t. $a < x$.

Hence, A is normal. ■

- We say A is *good* if $\forall x \in A, \exists a \in A$ such that $a < x$.
  Find an example of good set. Then prove it is good by using the definition.

$A = \mathbb{R}$.

WTS: $\forall x \in A, \exists a \in A$. s.t. $a < x$.

pf. Let $A = \mathbb{R}$; $A \subseteq \mathbb{R}$. $A \neq \emptyset$.

Let $x \in A$.

Take $a = x - 1$. since $A = \mathbb{R}$. gives $x \in \mathbb{R}$. $x - 1 \in \mathbb{R}$,

$a \in \mathbb{R}$;

Therefore, $a \in A$.

Moreover. $a = x - 1 < x$

To conclude. when $A = \mathbb{R}$. $\forall x \in A, \exists a \in A$ s.t. $a < x$.

$\blacksquare$

- We say A is *bad* if $\exists a \in A$ such that $\forall x \in A, a < x$.
  Can you find an example of bad set? Explain why.

No, we can't.

$\forall A \subseteq \mathbb{R}$. the statement is logically equivalent to

There exist a real number $a \in A$ that is smaller than any real number in A.

Apparently. it's false.

Anytime we can obtain an element x from A, s.t. $x = a$ not satisfying. $a < x$. (Since a & x are variables and the question didn't state $a \neq x$).

Note that here A is a non-empty set.
Challenging question: is empty set excited/happy/normal/good/bad? Think about it. You don't need to return your work for this question.

$A = \emptyset$. In $P(x) \Rightarrow Q(x)$ condition. It's equivalent to say.
if $\exists a \in \mathbb{R}$. s.t. $\forall x \in A$. $x \geq a$. then A is excited; If $\exists a \in A$ such that $\forall x \in A$. $x \geq a$. then A is happy......

We can obtain from truth table that

$P(x)$	$Q(x)$	$P(x) \Rightarrow Q(x)$.
T	T	T
T	F	F
F	T	T
F	F	T.

Since $A = \phi$. $P(x)$ is always false. because there isn't any element in it. Therefore. we can conclude when $A = \phi$, it's always excited/happy/normal/good/bad.