

Pigeon Hole.

1. 核心: 找到 鸽子 和 洞 代表什么.

2. Examples:

1) 21 houses for pigeon. 3×7 ; Show always can find 4 vertices of a rec. in the grid all contain pigeon or all contain no pigeon.

each col: $\begin{cases} \geq 2 \text{ filled} - \text{crowded} \\ \geq 2 \text{ empty} - \text{uncrowded} \end{cases}$

crowded + # uncrowded = 7.

WLOG. #crowded ≥ 4 . where crowded only has 3 options.

(1,2); (1,3); (2,3) filled.

\therefore By pigeon hole principle at least 2 options will be the same
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