MAT246 Online Quiz 3.

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1. I'll apply induction on n,

WTS: $\forall n \in \mathbb{N}.\ P(n)$, where $P(n): 5 \mid (6^n - 1)$

Let $n \in \mathbb{N}$.

Base Case: $n=1$.

When $n=1$. $6^n - 1 = 6 - 1 = 5$.

Since $\exists k \in \mathbb{Z}.$ s.t. $k=1$. $5k = 5$. gives $5 \mid 5$.

We've proved the base case is True.

Induction Step: Let $n \in \mathbb{N}$, $n \geq 1$.

Induction Hypothesis: Assume $P(n)$ is True, which $\exists k_1 \in \mathbb{Z}.\ 5k_1 = 6^n - 1$.

Since $6^{n+1} - 1$

$= 6^n \cdot 6 - 1$ (definition of exponention number).

$= 6^n \cdot (5+1) - 1$

$= 6^n \cdot 5 + 6^n - 1$ (distributivity for addition).

$= 6^n \cdot 5 + (6^n - 1)$. (Associativity for addition).

$= 5 \cdot 6^n + 5k_1$ (Induction Hypothesis).

$= 5(6^n + k_1)$

Thus. $\exists k_2 \in \mathbb{N}.$ s.t. $k_2 = 6^n + k_1$. which. $5 \mid 6^{n+1} - 1$.

We've proved $P(n) \Rightarrow P(n+1)$ is True.

Therefore. $\forall n \in \mathbb{N}$, $5 \mid (6^n - 1)$.

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2. I'll apply Induction on n.

WTS. $\forall n \in \mathbb{N}.\ \forall m \in \mathbb{N}.\ P(n)$. which $P(n): m \mid ((m+1)^n - 1)$.

Let $n \in \mathbb{N}$.

Let $m \in \mathbb{N}$. $m > 1$.

Base Case. $n=1$

When $n=1$. $((m+1)-1) = m$. which m/m is True.

I've proved that the base case is True.

Induction Step: Let $n \in \mathbb{N}$. $n \geq 1$.

Induction Hypothesis: Assume $P(n)$ is True. which $\exists p_1 \in \mathbb{Z}$. s.t.

$p_1 . m = (m+1)^n - 1$.

$$(m+1)^{n+1} - 1$$

$$= (m+1)^n \cdot (m+1) - 1. \quad (\text{def}^{\text{n}} \text{ of exponential number}).$$

$$= m \cdot (m+1)^n + (m+1)^n - 1 \quad (\text{distributivity for addition}).$$

$$= m \cdot (m+1)^n + ((m+1)^n - 1) \quad (\text{associativity for addition}).$$

$$= m \cdot (m+1)^n + p_1 m \quad (\text{Induction Hypothesis}).$$

$$= m \cdot ((m+1)^n + p_1).$$

Thus. $\exists p_2 \in \mathbb{Z}$. s.t. $k_2 = (m+1)^n + p_1$ which. $m / ((m+1)^{n+1} - 1)$

We've proved $P(n) \Rightarrow P(n+1)$ is True

Therefore. $\forall n \in \mathbb{N}$. $\forall m \in \mathbb{N}$. $m / ((m+1)^n - 1)$.

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3. a). 1 is not in S, $1 \notin S$.

Assume for contradiction: $1 \in S$.

Since 1 is the smallest natural number, $\forall s' \in S$, $1 \leq s'$. which means 1 is the smallest element in S. contradicts to the assumption of S.

b). Yes, $(k+1) \in S^c$.

Assume $(k+1) \notin S^c$. Since $(k+1) \in \mathbb{N}$, gives. $(k+1) \in S$.

Since $\forall s'' \in S$, $(k+1) \leq s'$ as all the natural numbers that are less or equal to $(k+1)$ are in S^c, which means $(k+1)$ is the smallest element in S, contradicts to the assumption of S.

c). To prove S is an empty set, it's the same as proving all natural numbers are in S^c.

WTP: $\forall n \in \mathbb{N}.\ n \in S^c$.

I'll apply PCMI on n.

Let $n \in \mathbb{N}$.

Base Case: $n=1$.

From 3.a), we've proved that $1 \notin S$. Since $1 \in \mathbb{N}$. thus. $1 \in S^c$.

I've proved that base case is True.

Induction Step. Let $k \in \mathbb{N}$.

Induction Hypothesis: Assume all of the natural number from 1 though k are in S^c, which $1, 2, \ldots\ldots k \in S^c$.

From 3b). we've proved that $(k+1) \in S^c$ as well.

I've proved that $k+1$ is also in S^c.

Therefore. I've proved that all natural numbers are in S^c.

Since $S \subseteq \mathbb{N}$, $S^c = \mathbb{N}$, gives. $S = \varnothing$.

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