Linear Independent

dependent：有别的解.

1. Definition

1) Algebric: A set of vectors $\{\vec{v_1}, \vec{v_2}, \dots, \vec{v_p}\}$ in $\mathbb{R}^n$ is lin inde if the vector equation $x_1\vec{v_1} + x_2\vec{v_2} + x_3\vec{v_3} + \dots + x_p\vec{v_p} = \vec{0}$ only has trivial solution. ($x_1 = x_2 = x_3 = \dots = x_p = 0$ 是唯一解).

e.g. $\left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\4\end{bmatrix}\right\}$

$x_1\begin{bmatrix}1\\2\end{bmatrix} + x_2\begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\begin{cases} x_1 + 3x_2 = 0 \\ 2x_1 + 4x_2 = 0 \end{cases} \longrightarrow \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 2 & 4 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]$

$\therefore$ lin inde

2) Geometric: A set of vectors $\{v_1, v_2, \dots, v_p\}$ is lin dep if for at least one i, $v_i \in \text{span}\{v_1, v_2, \dots, v_{i-1}, v_{i+1}, \dots, v_p\}$

3) Any set $\{v_1, v_2, \dots, v_p\}$ in $\mathbb{R}^n$ is lin dep if $p > n$.

e.g. $\left\{\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}6\\5\\4\end{bmatrix}, \begin{bmatrix}3\\5\\7\end{bmatrix}\right\}$

$p = 4$. $n = 3$. $p > n$. $\therefore$ lin dep.

4) If a set contains a zero vector, then the set must be lin dep.

e.g. $\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}\right\}$.

$0\begin{bmatrix}1\\2\\3\end{bmatrix} + 0\begin{bmatrix}4\\5\\6\end{bmatrix} + n\begin{bmatrix}0\\0\\0\end{bmatrix} = \vec{0}$, $\forall n \in \mathbb{R}$.