

# Models.

## 1. Definitions.

- 1) Model: an interpretation designed to show a particular semantic property.
- 2) Countermodel (example): an interpretation designed to show the absence of a particular semantic property.

## 2. Categories.

1) Intensional.

Infinito

Concrete.

2) Extensional.

Finite.

Abstract.

## 3. Abstract Translation (Key Skills). (don't consider UD, Pr first).

1)  $\forall x (Fx \rightarrow Gx)$ : Everything if in F then in G.

= All Fs are Gs

2)  $\forall x (Fx \rightarrow \neg Gx)$ : Everything if in F then not in G.

= all Fs are not Gs

3)  $\forall x (\neg Fx \rightarrow Gx)$ : Everything if not in F then in G.

4)  $\forall x (Bx \leftrightarrow Gx)$ : If in B then in G and if in G then in B

= B and D are identical

5)  $\neg \forall x (Bx \leftrightarrow Dx)$ : Not everything If in B then in G and if in G then in B

= B and D are different

informal.  
equivalence. ←

6)  $\forall x \sim (Fx \leftrightarrow Gx)$ : Everything is if in F then not in G or if not in G then in F.  
 $= \forall x (Fx \leftrightarrow \sim Gx)$ .  
 $=$  Everything is F exclusive or G.

7)  $\forall x (Fx \vee Gx)$ : Everything is F or G.

8)  $\exists x Fx \wedge \exists x Gx$ : There exist sth. in F and there exist sth. in G.

$=$  F and G are not empty.

9)  $\exists x (Fx \wedge Gx)$ : There exist sth. is in both F and G.

10)  $\exists x (Fx \wedge \sim Gx)$ : sth. is in F and not in G

11)  $\exists x (Fx \wedge Gx) \rightarrow \sim \exists y \forall y$ : If sth is in both F and G, then nothing is in D.

4. Informal Equivalence: helping abstract translation

e.g.  $\sim \exists x (Fx \rightarrow \sim Gx)$ .

$= \forall x \sim (Fx \rightarrow \sim Gx)$ .

$= \forall x (Fx \wedge Gx)$ . Everything is in F and G.

5. Finite Abstract Model Tips.

1) Focus on Abstract Translation.

→ use informal equivalence. appropriately.

2) Begin with a UD of 2 elements. (don't close right bracket).

3) Start with constants → existentials.

→ Because universal Q always has a conditional

4) rechecking.

e.g.  $\{ \forall x (Bx \leftrightarrow Dx), \exists x (Ax \wedge \sim Dx), \exists x Dx \}$ . consistent

S1: B and D are identical

$\downarrow$   
all true.

S2: There exists sth. in A and not in D.

S3: D is not empty.

UD:  $\{0, 1\}$ .

B:  $\{0\}$

D:  $\{0\}$

A:  $\{1\}$

e.g.  $Fa \wedge Ma, \forall x (Mx \vee Hx), \sim \forall x (Fx \leftrightarrow Hx) \therefore \forall x (Fx \rightarrow Mx)$  Invalid.

Pr1. a is in F and M.

Pr2. Everything is in M inclusive or H.

Pr3. F and H are different

$\sim C$ : There exists sth. in F and not in M.

UD:  $\{0, 1\}$

a: 0

F:  $\{0, 1\}$

M:  $\{0\}$

H:  $\{1\}$

$$\sim \forall x (Fx \rightarrow Mx) \\ = \exists x (Fx \wedge \sim Mx)$$

e.g.  $\sim \forall x (Fx \rightarrow Gx), \therefore \sim \exists x \forall y (Fy \rightarrow Gx)$ . Invalidity.

Pr1:  $\exists x (Fx \wedge \sim Gx)$ : sth in F and not in G.

$\sim C$ :  $\exists x \forall y (Fy \wedge \sim Gx)$ .

UD =  $\{0, 1\}$ .

$$= \forall y (Fy \wedge \sim G0) \vee \forall y (Fy \wedge \sim G1).$$

$$= [(F_0 \wedge \sim G_0) \wedge (F_1 \wedge \sim G_0)] \vee [(F_0 \wedge \sim G_1) \wedge (F_1 \wedge \sim G_1)].$$

$F_0$ : T

$F_1$ : T

$G_0$ : F

$G_1$ : 不重要.

总结: 一个 variable, 直接翻译.

两个 variable 用 expand. ( $\exists$ : 用  $\vee$ ;

$\forall$ : 用  $\wedge$ ) 先列出  $\forall$  下的所有情况.

e.g.  $\forall x \exists y (F_x \leftrightarrow G_y)$ .  $\therefore \exists y \forall x (F_x \leftrightarrow G_y)$ . Invalidity.

Pr:  $\exists y (F_0 \leftrightarrow G_y) \wedge \exists y (F_1 \leftrightarrow G_y)$ .

$$= [(F_0 \leftrightarrow G_0) \vee (F_0 \leftrightarrow G_1)] \wedge [(F_1 \leftrightarrow G_0) \vee (F_1 \leftrightarrow G_1)]$$

$\sim C$ :  $\forall y \exists x (F_x \leftrightarrow \sim G_y)$ .

$$= \exists x (F_x \leftrightarrow \sim G_0) \wedge \exists x (F_x \leftrightarrow \sim G_1).$$

$$= [(F_0 \leftrightarrow \sim G_0) \vee (F_1 \leftrightarrow \sim G_0)] \wedge [(F_0 \leftrightarrow \sim G_1) \vee (F_1 \leftrightarrow \sim G_1)].$$

$F_0$ : F

$F_1$ : T

$G_0$ : T

$G_1$ : F

$F_0$  或  $F_1$  有一个与  $\sim G_0$  相同即可.