System of First Order Linear Equations.

1. System of FOLE.

$$\begin{cases} x_1' = F_1(x_1, \dots, x_n) = p_{11}(t)x_1 + \cdots + p_{1n}(t)x_n + g_1(t). \\ \vdots \\ x_n' = F_n(x_1, \dots, x_n) = p_{n1}(t)x_1 + \cdots + p_{nn}(t)x_n + g_n(t) \end{cases}$$

1) sol^{n} to the system.

$$\begin{cases} x_1 = \phi_1(t). \\ x_2 = \phi_2(t). \end{cases}$$

2). Initial Value.

$$\begin{array}{ccc} x_1(t_0) = x_1^0 & \longleftarrow & x_1 = \phi_1(t) \\ \vdots & & \vdots \\ x_n(t_0) = x_n^0 & \longleftarrow & x_n = \phi_n(t) \end{array}$$

2. 题型

1). Transform given equation into system

①最高导了几次就设几次，然后写成 system.

e.g. $u'' + 0.5u' + 2u = 0$.

i). Let $x_1 = u$, $x_1'' + 0.5x_1' + 2x_1 = 0$.

ii) Let $x_2 = x_1'$, $x_2' + \underbrace{0.5x_2 + 2x_1}_{} = 0$. ← $F(x_1, x_2)$ 是可以的.

iii) $\begin{cases} x_1' = x_2. \\ x_2' = -2x_1 - 0.5x_2. \end{cases}$

e.g. $u^{(4)} - u = 0$.

i). Let $x_1 = u$, $x_1^{(4)} - x_1 = 0$.

ii) Let $x_2 = x_1'$, $x_2^{(3)} - x_1 = 0$.

iii) Let $x_3 = x_2'$, $x_3^{(2)} - x_1 = 0$

iv) Let $x_4 = x_3'$, $x_4' - x_1 = 0$.

v). $\begin{cases} x_1' = x_2 \\ x_2' = x_3 . \\ x_3' = x_4 . \\ x_4' = x_1 . \end{cases}$

2) Transform the system into equation.

① 通过 $x_1' = F_1(x_1, x_2)$ 得到 $x_2 = G_1(x_1, x_1')$.

② 通过 $x_2 = G_1(x_1, x_1')$ 得到 $x_2' = G_1'(x_1, x_1')$ 另一个关于 x_2' 的式子.

③ $G_1'(x_1, x_1') = F_2(x_1, x_2) = F_2(x_1, G_1(x_1, x_1'))$ 回代即可.

e.g. $\begin{cases} x_1' = 3x_1 - 2x_2 , & x_1(0) = 3 \\ x_2' = 2x_1 - 2x_2 , & x_2(0) = \frac{1}{2} \end{cases}$

(a) change into equation.

(b) solve x_1, x_2.

(a) i) $2x_2 = 3x_1 - x_1' \Rightarrow x_2 = \frac{3}{2}x_1 - \frac{1}{2}x_1'$ ①

ii) $\Rightarrow x_2' = \frac{3}{2}x_1' - \frac{1}{2}x_1''$ ②.

iii). $\frac{3}{2}x_1' - \frac{1}{2}x_1'' = 2x_1 - 2\cdot(\frac{3}{2}x_1 - \frac{1}{2}x_1')$.

$\Rightarrow \frac{3}{2}x_1' - \frac{1}{2}x_1'' = 2x_1 - 3x_1 + x_1'$

$\Rightarrow 0 = \frac{1}{2}x_1'' - \frac{1}{2}x_1' - x_1$

$\Rightarrow x_1'' - x_1' - 2x_1 = 0$.

(b) $r^2 - r - 2 = 0$.

$\Rightarrow (r-2)(r+1) = 0$. $r_1 = 2$. $r_2 = -1$.

$x_1(t) = C_1 \cdot e^{2t} + C_2 \cdot e^{-t}$.

$x_1'(t) = 2C_1 \cdot e^{2t} - C_2 \cdot e^{-t}$

$\therefore x_2(t) = \frac{3}{2}(C_1 \cdot e^{2t} + C_2 \cdot e^{-t}) - \frac{1}{2}(2C_1 \cdot e^{2t} - C_2 \cdot e^{-t})$

$= \frac{3}{2}C_1 \cdot e^{2t} + \frac{3}{2}C_2 \cdot e^{-t} - C_1 \cdot e^{2t} + C_2 \cdot e^{-t}$.

$= \frac{1}{2}C_1 \cdot e^{2t} + \frac{5}{2}C_2 \cdot e^{-t}$.

Since $x_1(0) = 3$, gives, $C_1 + C_2 = 3$.

$x_2(0) = \frac{1}{2}$, gives, $\frac{1}{2}C_1 + \frac{5}{2}C_2 = \frac{1}{2}$.

$\Rightarrow \begin{cases} C_1 = \frac{7}{2} \\ C_2 = -\frac{1}{2} \end{cases}$

$\therefore x_1(t) = \frac{7}{2} \cdot e^{2t} - \frac{1}{2}e^{-t}$

$x_2(t) = \frac{7}{4} \cdot e^{2t} - \frac{5}{4}e^{-t}$.