c) Read the proof of Theorem 12.4.12, and explain

i) Why does the expression z_0 have to satisfy $(z_0^6 + z_0^5 + \ldots z_0 + 1) = 0$, and why must the expression $\frac{1}{z_0}$ be defined?

Since by calculation. $z=1$. is a solution to $z^7=1$. and $z^7=1 \Rightarrow z^7-1=0$.
By long division $z^7-1 = (z-1)(z_0^6+z_0^5+\dots+z_0+1)=0$, which $(z_0^6+z_0^5+\dots+z_0+1)=0$.
Since $z=a+bi$. $\bar{z}=a-bi$. gives. $|z| = z\cdot\bar{z} = (a+bi)(a-bi) = a^2+b^2=1$. which.
$\bar{z}=\frac{1}{z}$, as $\bar{z}$ is defined. $\frac{1}{z}$ need to be defined and will be used to express.
$x_0 = z+\bar{z} = (a+bi)+(a-bi) = 2a = z+\frac{1}{z} = 2Re(z)$.

ii) Why could we make the assumption $\frac{1}{z_0} = \overline{z}_0$?

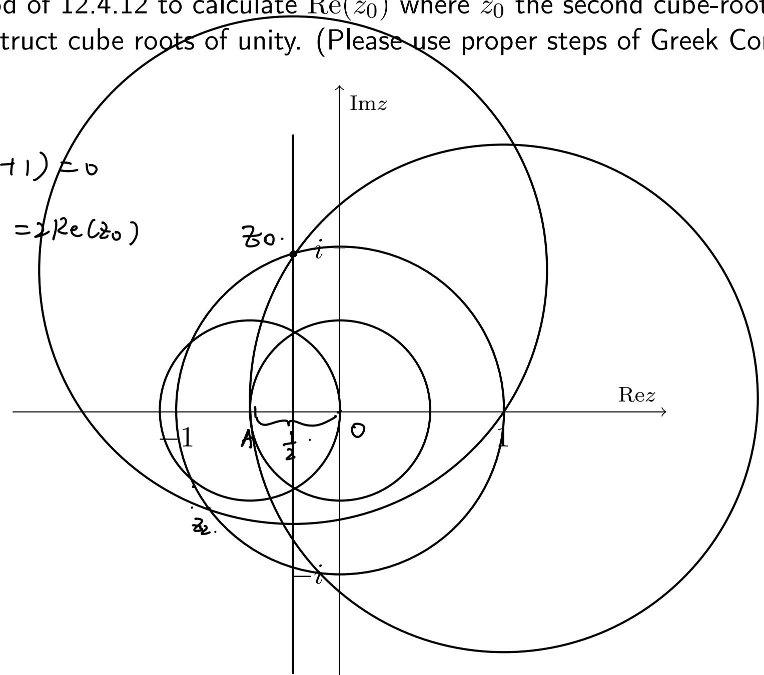
Since $z=a+bi$. $\bar{z}=a-bi$. gives. $|z| = z\cdot\bar{z} = (a+bi)(a-bi) = a^2+b^2=1$. which.
$\bar{z}=\frac{1}{z}$

iii) How did we conclude that $z_0 + \frac{1}{z_0} = 2\mathrm{Re}(z_0)$?

Since $z_0=a+bi$. $\bar{z}_0=a-bi$. gives. $|z| = z\cdot\bar{z}_0 = (a+bi)(a-bi) = a^2+b^2=1$. which.
$\bar{z}_0=\frac{1}{z_0}$, as $\bar{z}_0$ is defined. $\frac{1}{z}$ need to be defined and will be used to express.
$x_0 = z_0+\bar{z}_0 = (a+bi)+(a-bi) = 2a = 2Re(z_0)$.
Since. $x_0 = z_0+\bar{z}_0 = z_0+\frac{1}{z_0}$, gives $z_0+\frac{1}{z_0} = 2Re(z_0)$

d) For $n = 3$, use the method of 12.4.12 to calculate $\mathrm{Re}(z_0)$ where z_0 the second cube-root of unity. Use this information to draw/construct cube roots of unity. (Please use proper steps of Greek Construction)

$z^3=1$.
$\Rightarrow z^3-1 = (z-1)(z^2+z+1)=0$
Similarly. $x_0 = z_0+\frac{1}{z_0} = 2Re(z_0)$
$z^2+z+1=0$.
$\Rightarrow$ $z+1+\frac{1}{z}=0$.
$\Rightarrow$ $(z+\frac{1}{z})+1=0$
$\Rightarrow$ $x_0+1=0$
$\Rightarrow$ $x_0=-1$
$2Re(z_0)=-1$.
$\Rightarrow$ $Re(z_0) = -\frac{1}{2}$.



I used the steps, I described in (b).