Uniform Probability on Finite Spaces.

be careful whether it's finite and equally likely

1. Uniform Probability: Suppose $S = \{s_1, s_2, \dots, s_n\}$ is some finite ① sample space, of finite size $|S| = n$ and each element is equally likely ②.

1) Discret Uniform Distribution:

$$P(s_1) = P(s_2) = \cdots = P(s_n) = \frac{1}{n}.$$

event is a subset of sample space

2). For any event $A = \{a_1, a_2, \dots, a_k\}$, by additivity, we have.

$$P(A) = P(a_1) + P(a_2) + \cdots + P(a_k) = \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n} = \frac{k}{n} = \frac{|A|}{|S|}.$$

(The number of elements in A divided by the number of elements in S).

the subsets can be the same e.g. 3 fair dices.

2. Multiplication Principle (To calculate the number of elements in S): If S is made up by choosing one element of each of the subsets $S_1, S_2, \dots, S_k$. i.e. if $S = S_1 \times S_2 \times \cdots \times S_k$ (Cartesian Product), then $|S| = |S_1| \times |S_2| \times \cdots \times |S_k|$.

e.g. Rolled 3 fair six-sided dice. What's $P(\text{sum} \geq 17)$.

$|S| = |S_1| \times |S_1| \times |S_1| = 6^3 = 216.$

When sum ≥ 17. we only have $\{666, 566, 656, 665\}$. which.

$$P(\text{sum} \geq 17) = \frac{|A|}{|S|} = \frac{4}{216} = \frac{1}{54}.$$

3. Frequent Questions.

1). At least

e.g. $P(\text{At least one six when rolling a fair six-sided dice 4 times})$.

$|S| = |S_1| \cdot |S_2| \cdot |S_3| \cdot |S_4| = 6^4 = 1296.$

$A^c = \{\text{no six when rolling 4 times}\} = \{1, 2, 3, 4, 5\}^4$, which.

$|A^c| = 5^4 = 625$, so $P(A^c) = \frac{|A^c|}{|S|} = \frac{625}{1296} \doteq 0.482.$

$P(A) = 1 - P(A^c) \doteq 1 - 0.482 = 0.518.$

2) Pick 'in order' or 'ignoring order'.

① Permutation: the number of ways picking k distinct items, in order, out

40 people have same bir.:
$|S| = 365^{40}$.
$|A^c| = {}^{365}P_{40}$. (affect).
$= \cdots\cdots$
$\cdots\cdots$

use to cal. $|S|$ or $|A|$.

of n items total: ${}^{n}P_k$ (don't use the sign) $= \frac{n!}{(n-k)!}$

$\longrightarrow$ after pick one will affect the next pick.

② Combinations: the number of ways picking k distinct items out of n items total, ignoring order: $\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$

$\longrightarrow$ just care about the result.

e.g. Suppose there are 10 people in the party, and you randomly pick 3 of the people. What is the P of choice to be 3 richest?

① In order (1-2-3).

S: choose 3 people from 10 in order:

$|S| = {}^{10}P_3 = \frac{10!}{(10-3)!} = 10 \times 9 \times 8 = 720$.

A: $\{1-2-3\}$; $|A| = 1$.

$P(A) = \frac{|A|}{|S|} = \frac{1}{720}$

(after picking the richest out of ten, we pick the richest in the left 9, has affect for next pick).

② In any order

S: choose 3 people from 10 in any order.

$|S| = \binom{10}{3} = \frac{10!}{3!(10-3)!} = 120$

(Only care about the result).

A: $\{1, 2, 3\}$; $|A| = 1$.

$P(A) = \frac{|A|}{|S|} = \frac{1}{120}$.

3). Picking deck cards.

① P(club or 7).

From I.E. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. where

when see a 'or' think about a union which I.E.

A: card is a club; B: card is a 7.

$P(A) = \frac{1}{4}$; $P(B) = \frac{4}{52} = \frac{1}{13}$; $P(A \cap B) = \frac{1}{52}$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{4}{13}$

② Draw a pair of distinct cards. P(both are J/Q/K).

S: take 2 cards from the deck. (not mention in order).

$|S| = \binom{52}{2} = \frac{52!}{50!\,2!} = 1326$.

A: distinct pair of face card

$|A| = \binom{12}{2} = \frac{12!}{10!\,2!} = 66.$

both are J/Q/K means we need to pick 2 from face card not from all.

$P(A) = \frac{|A|}{|S|} = \frac{11}{221}$

4). Exactly

e.g. Filp 10 fair coins. P(exactly 6 heads).

S: H or T filp coins.

$|S| = |S_1| \cdot |S_2| \cdots |S_{10}| = 2^{10}$ (don't choose any thing so just apply multiplication rule).

A: choose 6 coins from 10 to be H (don't mention order).

$|A| = \binom{10}{6} = \frac{10!}{6!\,4!} = 210.$

$P(A) = \frac{|A|}{|S|} = \frac{210}{2^{10}} \doteq 0.205$

① In general. flip n coins. P(exactly k heads) $= \binom{n}{k} \cdot \frac{1}{2^n}$

(actually B.D as P is firm).