Matrix Operations

1. Sum and Scalar Multiples.

2. Matrix Multiplication.

A row = B col.

$A \cdot B = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & & \\ \vdots & & & \\ a_{n1} & & & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & & & \\ \vdots & & & \\ b_{n1} & & & b_{nn} \end{bmatrix}$

$m \times n \quad n \times k$

must be equal.

will get $m \times k$ matrix.

$= \begin{bmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + \cdots + a_{1n} \cdot b_{1n} & \cdots & a_{11} b_{1n} + a_{12} b_{2n} + \cdots + a_{1n} b_{nn} \\ \vdots & & \vdots \\ a_{n1} \cdot b_{11} + a_{n2} \cdot b_{21} + \cdots + a_{nn} \cdot b_{1n} & \cdots & a_{n1} b_{1n} + a_{n2} b_{2n} + \cdots + a_{nn} \cdot b_{nn} \end{bmatrix}$

1) $A(BC) = (AB) \cdot C$

2) $r(AB) = (rA) \cdot B = A(rB)$.

3) $A(B+C) = AB + AC$.

4) $(B+C)A = BA + CA \longrightarrow I_m A = A = A \cdot I_n$.

(I_m: $m \times m$, A: $m \times n$, I_n: $n \times n$)

5) $AB \neq BA$ (In general).

6) $AB = AC \not\Rightarrow B = C$.

↳ $AB = 0 \not\Rightarrow A = 0$ or $B = 0$

7) Power Matrix.

$A^k = A \cdot \ldots \cdot A$ when $k = 0$. $A^0 = I_n$.

(A^k: $n \times n$)

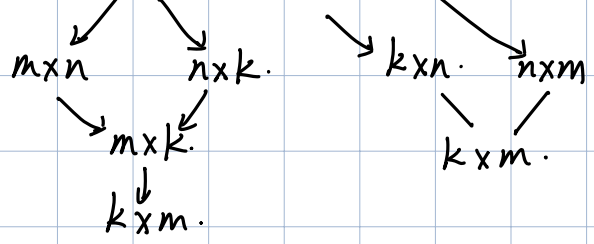
3. Matrix Transpose. (行列调换)

1) $(A^T)^T = A$.

2) $(rA)^T = rA^T \quad \forall r \in \mathbb{R}$.

3) $(A+B)^T = A^T + B^T$

4) $(AB)^T = B^T \cdot A^T$.


m x n
n x k.
m x k.
k x m.
k x n.
n x m
k x m.