Definition of Inverse Functions

(In linear algebra).

1. Definition

Let $f: A \to B$ be a surjective and injective function.
The inverse of f is another function $f^{-1}: B \to A$ defined by

$$\forall x \in A,\ \forall y \in B,\quad x = f^{-1}(y) \iff y = f(x).$$

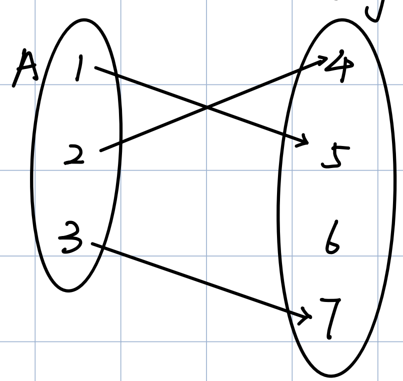
(assuming this is a function.)

sufficient and necessary condition for having an inverse function.

2. Surjective and Injective.

Let $f: A \to B$ be a function

1) Surjective (onto): when range $f = B$

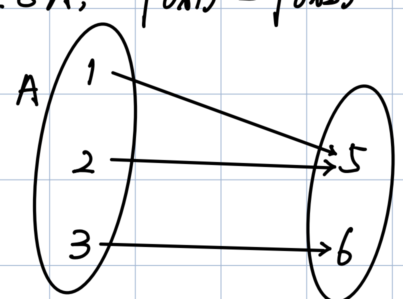


there is an element in the codomin that is not in the range.

2) Injective (one-to-one):

$$\forall x_1, x_2 \in A,\quad x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$ or equivalent

$$\forall x_1, x_2 \in A,\quad f(x_1) = f(x_2) \Rightarrow x_1 = x_2.$$



there are two different inputs that get mapped to the same output.

$g(x)=\sqrt{x}$ is the inverse function of $f(x)=x^2$ when $x\in[0,+\infty)$.

3) f has an inverse $\Leftrightarrow \begin{cases} f \text{ is injective} \\ f \text{ is surjective} \end{cases}$

In calculus, we automatically shrinking the codomain in order to fit the range; we forget the codomin entirely and work only with the range.; do it without saying it explicitly. $\longrightarrow$ never use 'surjective' or 'codomain'.

① Let f be a one-to-one function.

Let domain $f = A$, range $f = C$.

The inverse f^{-1} is another function satisfying domain $f^{-1} = C$, range $f^{-1} = A$, and defined by

$\forall x \in A$, $\forall x \in C$, $x = f^{-1}(y) \Leftrightarrow y = f(x)$.

3. Property (other way of def).

1) $\forall x \in A$, $f^{-1}(f(x)) = x$.

$\forall y \in C$, $f(f^{-1}(y)) = y$.