PL Derivative

1. Core Idea: Substitution

1) All instances of α are replaced with a singular term β. (replace a bounded variable with a free one).

i.e. Symbolic Sentence $\longrightarrow$ Symbolic Formula.

$\forall\alpha\phi\alpha \rightarrow \phi\beta$.

$\exists\alpha\phi\alpha \rightarrow \phi\beta$.

2. Basic Rules.

1) Universal Instantiation (UI). ($\forall$ elimination).

$$\frac{\forall\alpha\phi\alpha.}{\phi\beta}$$

restriction: β can't be a bound variable within $\phi\alpha$.

e.g. $\forall x(Fx \rightarrow Gx)$.

$Fx \rightarrow Gx$ Pr1 UI (✓: substitute x for x).

$Fi \rightarrow Gi$ Pr1 UI/i (✓: substitute i for x).

$Fy \rightarrow Gx$. Pr1 UI/y (X: should substitute all)

$Fab \rightarrow Gab$ Pr1 UI/ab (X: not a singular term).

e.g. $\forall x(Fx \rightarrow \exists y(Gx \wedge Hy \leftrightarrow \sim Fy \vee Hx))$.

contradict the restriction, can not instantiate a letter that bounded to a quantifier within the scope.

$Fy \rightarrow \exists y(Gy \wedge Hy \leftrightarrow \sim Fy \vee Hx)$ Pr2 UI/y. (X).

2) Existential Instantiation (EI) ($\exists$ elimination).

$$\frac{\exists\alpha\phi\alpha}{\phi\beta}$$

restriction: β must be an arbitrary term (i–z) that doesn't occur in any previous.

line or premises.

2) Existential Generalization (EG) ($\exists$ Introduction)

$$\frac{\phi\beta.}{\exists\alpha\phi\alpha}$$

restriction 1: α cannot be a bound variable within $\phi\beta$.

restriction 2: if α is different from β, then α can't be free with $\phi\beta$.

e.g. $Fa \wedge Ga$.

① Correct

$\longrightarrow \exists x(Fx \wedge Gx)$; $\exists y(Fy \wedge Gy)$

$\longrightarrow \exists x(Fa \wedge Gx)$; $\exists x(Fx \wedge Ga)$.

$\longrightarrow \exists x(Fa \wedge Ga)$.

② Wrong.

$\longrightarrow Fa \wedge \forall x(Ha \rightarrow Gx) \longrightarrow \exists x(Fx \wedge \forall x(Hx \rightarrow Gx))$ (x).

contradicts R1.

$\longrightarrow Fx \wedge Gy \longrightarrow \exists y(Fy \wedge Gy)$.

contradicts R2. (different from x, but Gy).

3. Universal Derivation (UD).

1) Why no UG: since this is the deductive logic, we can't make it inductively.

e.g. All students are happy. All happy people are nice.

$\therefore$ All students are nice (inductive) ($\uparrow$)

2) Arbitrary Reasoning: take an arbitrary member in

the UD.

① Let x be an arbitrary student

② Knowing x is happy.

③ Knowing x is nice.

④ Since x can be any student, then we know that all students are nice

3) Structure

Show $\forall\alpha\,\phi\alpha$

Show $\phi\alpha$.

CD / ID / DD

$\forall\alpha\,\phi\alpha$ UD.

Restriction: α can't appear unbound in any previous available line, or in a premise used in an available line.

because. Restrict. we need to used UD quickly or check it carefully. and we don't do the simplification when we can.

4). Look at most recent 'show' line.

① $\phi \to \psi$: start CD.

② $\forall\alpha\,\phi\alpha$: start UD.

③ else: start ID.

4. Golden Rules.

1). Use EI / UD first, then use UI / EG.

(Invoke arbitrary terms) (matching free terms).

match it to sth. useful.

exception: buried existential

2) Don't do simplification although we can. → check less to make sure it's arbitrary. (EI & UD need to be arbitrary).

3). bounded: $\forall\alpha\phi\alpha$; $\exists\alpha\phi\alpha$ bounded and can be changed

free: β, $\phi\beta$ free and fixed.

5. Quantifier Negation (QN).

$$\frac{\sim\forall\alpha\phi}{\exists\alpha\sim\phi} \qquad \frac{\forall\alpha\sim\phi}{\sim\exists\alpha\phi} \qquad \frac{\sim\exists\alpha\phi}{\forall\alpha\sim\phi} \qquad \frac{\exists\alpha\sim\phi}{\sim\forall\alpha\phi}$$

quantifer or a negation of quantifier must be main operator.