Conditional Statements.

1. Proposition: $P(x)$ has a true value.

1) negation: not $P(x) = \neg P(x)$; whenever $P(x)$ is true, $\neg P(x)$ is not true.

2. 'If, then' statement:

1) 'If $P(x)$ then $Q(x)$' is true ~~whenever~~ if $P(x)$ is true then $Q(x)$ is true; when $P(x)$ is false, we don't care about $Q(x)$ part.

e.g. Assume $x \in A \Rightarrow x > 0$. conclude?

- $x \notin A$ No conclusion. (no $P(x)$; don't care)
- $x > 0$ No conclusion.
- $x \leq 0$ $\Rightarrow$ $x \notin A$. (contrapositive)

e.g. $0 > 1 \Rightarrow 103574289$ is prime.; T/F?

True! when $P(x)$ is false, don't care about $Q(x)$

$P(x)$	$Q(x)$	$P(x) \Rightarrow Q(x)$.
T	T	T
T	F	F
F	T	T
F	F	T

$P(x) \Rightarrow Q(x)$

$\neq Q(x) \Rightarrow P(x)$

$= \neg Q(x) \Rightarrow \neg P(x)$

e.g. If it's sunny, I wear my sun-glasses. If $P(x)$ is false, $Q(x)$ is true, this is saying that it's not sunny,

but I wore my sunglasses anyway.
(This doesn't invalid the original statement)
Not being sunny and me not wearing my sunglasses. (doesn't invalid)

2) $P(x) \Rightarrow Q(x)$
① = 'If $P(x)$ then $Q(x)$'.
② converse: $Q(x) \Rightarrow P(x)$

→ not logically equivalent.
③ contrapositive: $\neg Q(x) \Rightarrow \neg P(x)$
→ equivalent
→ considering using the contrapositive. when it's hard to proof the original statement.

e.g. Let x be a positive integer. If $x \neq 1$, then $x \neq x^2$.

Solution:

When showing the question in contrapositive gives:
'Let ...... If $x = x^2$, then $x = 1$.'
Of course, this follows:

$x^2 = x.$

$\Leftrightarrow x^2 - x = 0.$

$\Leftrightarrow x(x-1) = 0.$

$\Leftrightarrow x = 0$ or $x = 1.$

and since we said x is a positive integal, x must be 1.

there's always an implicit quantifier; but it comes to negation, should be explicitly.

3. If and only if:

1) $P(x) \Leftrightarrow Q(x)$

- $P(x) \Rightarrow Q(x)$ and $Q(x) \Rightarrow P(x)$
- P and Q are equivalent.
- P and Q both true / false.

4. Negation of a conditional statement.

e.g. let $A \subseteq \mathbb{R}$.

$(\forall x \in \mathbb{R})$ If $x \in A$, then $x > 0$.

means.

$\forall x \in \mathbb{R}$ $\begin{cases} x \in A \text{ and } x > 0. & \text{or.} \\ x \notin A. \text{ and } x > 0 & \text{or.} \\ x \notin A \text{ and } x \leq 0 \end{cases}$

Negation: $\exists x \in \mathbb{R}.$ s.t. $x \in A.$ and $x \leq 0.$