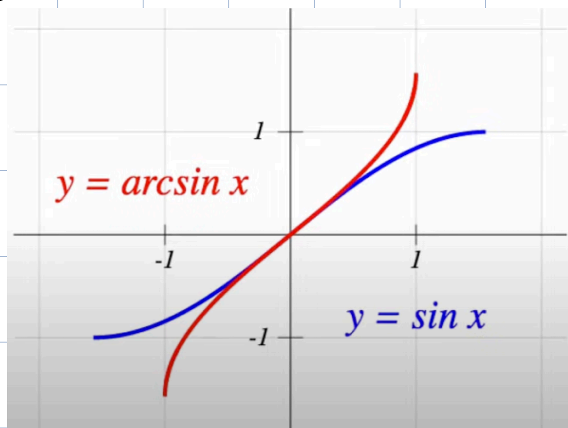
Arcsin, Arccos, and Arctan.

arcsin isn't the inverse function of sin
→ not one-to-one

1. Arcsin: the inverse function of the restriction of sin to $[-\frac{\pi}{2}, \frac{\pi}{2}]$



$x = \arcsin y \iff y = \sin x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, $-1 \le y \le 1$

1) composition of sin and arcsin

$\longrightarrow \arcsin(\sin x) = x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\longrightarrow \sin(\arcsin y) = y$ for $-1 \le y \le 1$.

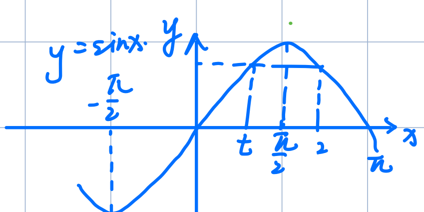
Not true for other nums.

$\longrightarrow \sin(\arcsin 2)$ is not defined.

when arcsin y is out of it domain. then its not defined.

$\longrightarrow \arcsin(\sin 2) = \pi - 2$.

$\arcsin(\sin 2) = t \iff \begin{cases} \sin t = \sin 2. \\ -\frac{\pi}{2} \le t \le \frac{\pi}{2}. \end{cases}$



$t = \pi - 2$.

when sinx is out of its domain we need to solve. the value t which satisfies the domain.

2) derivative $(-1 \le x \le 1)$.

$$\frac{d}{dx} \sin(\arcsin x) = \frac{d}{dx}[x].$$

$$\Rightarrow \cos(\arcsin x) \cdot \frac{d}{dx}(\arcsin x) = 1$$

$$\Rightarrow \qquad \frac{d}{dx} \arcsin x = \frac{1}{\cos(\arcsin x)}$$

Let $\theta = \arcsin x$. $\quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

$\cos\theta = \pm\sqrt{1-\sin^2\theta}$ since $\cos\theta > 0$ when $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

$\cos\theta = \sqrt{1-x^2}$.

$$\therefore \frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

2. Arccos x: the inverse function of the restriction of cos to $[0, \pi]$; domain: $[-1, 1]$

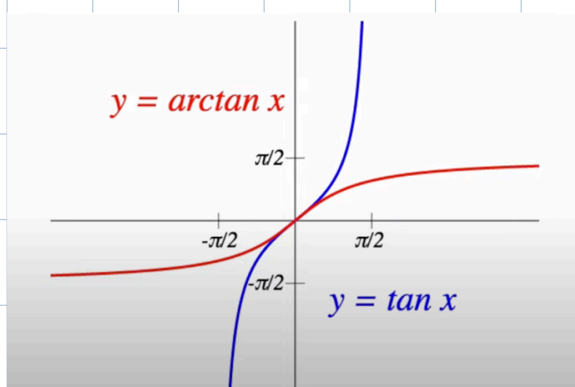
1) derivative $(-1 \le x \le 1)$.

$$\frac{d}{dx}(\arccos x) = \frac{1}{(\cos y)'} \qquad x = \cos y.$$

$$= \frac{1}{-\sin y}$$

$$= \frac{-1}{\sqrt{1-x^2}}.$$

3. Arctan x: domain: $[-\frac{\pi}{2}, \frac{\pi}{2}]$; range: $(-\infty, +\infty)$.



1) derivative:

$$\frac{d}{dx}(\arctan x) = \frac{1}{(\tan y)'}$$

$x = \tan y$. $\qquad = \frac{1}{\sec^2 y}$

$\sec^2 y = 1 + \tan^2 y$. $\qquad = \frac{1}{1+x^2}$.