Math Induction

CSC236:

$0 \in \mathbb{N}$

1. Simple Induction

1) Principle: $(P(0) \wedge \forall n, P(n) \Rightarrow P(n+1)) \Rightarrow \forall n, P(n)$.

($(P(0) \wedge \forall n, P(n) \Rightarrow P(n+1))$: prove. $\forall n, P(n)$: conclude.)

2) Structure:

① Given the statement to prove: $\forall n \in \mathbb{N}, P(n)$, (describe what's $P(n)$ and don't include $\forall n \in \mathbb{N}$ in it).

② Let $n \in \mathbb{N}$.

③ Base Case: $n = 0$ (or $n = 1$ or ..... or $n = e$)

④ Induction Step: Let $n \in \mathbb{N}$. 此处若 B.C 不唯一也无须说明 $n \geq e$.

⑤ Induction Hypothesis: Assume $P(n)$, i.e., ...

WTS: $P(n+1)$

2. Complete Induction.

1) Principle: $(\forall n, (\forall k < n, P(k)) \Rightarrow P(n)) \Rightarrow \forall n, P(n)$

($(\forall n, (\forall k < n, P(k)) \Rightarrow P(n))$: prove. $\forall n, P(n)$: conclude.)

2) Structure:

① Given the statement to prove: $\forall n \in \mathbb{N}, (n \geq n_0,)\ P(n)$, (describe what's $P(n)$ and don't include $\forall n \in \mathbb{N}, (n \geq n_0,)$ in it).

② Let $n \in \mathbb{N}$.

③ Base Case: $n = 0\ (n = n_0)$ or $n = 1\ (n = n_0 + 1)$ or ..... $n = e\ (n = n_0 + e)$

④ Induction Step. Let $n \in \mathbb{N}$. $n > e$. $(n > n_0 + e)$. ← 若 B.C 不唯一须说明.

⑤ Induction Hypothesis. Assume $\forall k$. $0/n_0 \leq k < n$, $P(k)$.

WTS: $P(n)$.

(Mention when using I.H).