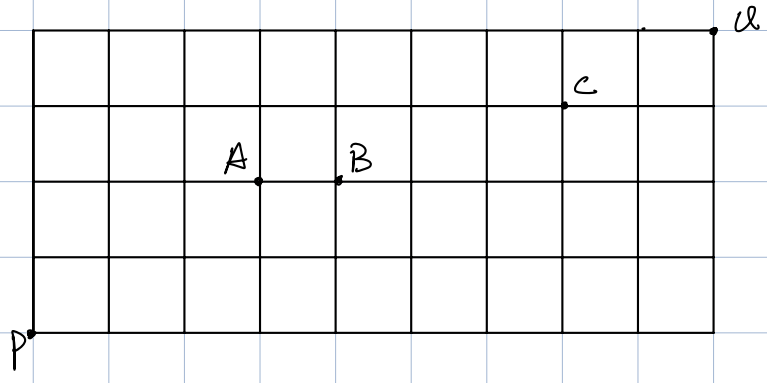
Lattice Path

1. # of L.P. from (m,n) to (p,q) is $\binom{(p-m)+(q-n)}{p-m}$.
↖ 确定一个方向即可.

2. Catalan number: # of L.P. from $(0,0)$ to (n,n) which never goes above the diagonal $y=x$; $C(n) = \frac{1}{n+1}\binom{2n}{n}$.
① ② ③
may use in C.P.
e.g. sequence of len. $2n$, $\forall k$, $\sum_{i=1}^{k} x_i \geq 0$, $x_i \in \{\pm 1\}$, $\sum_{i=1}^{2n} x_i = 0$.

3. Example:
e.g. # of shortest P-Q routes.



注意 n 是右和上之和(X).

1) Pass A 2) Pass A-B 3) Pass A & C 4) delete A-B.

1) P-A-Q: $\binom{5}{3}\binom{7}{2}$

2) P-AB-Q: $\binom{5}{3}\binom{6}{2}$.

3). P-A-C-Q: $\binom{5}{3}\binom{4}{1}\binom{3}{1}$.

4). (P-Q)-(P-AB-Q): $\binom{12}{4} - \binom{5}{3}\cdot\binom{6}{2}$.