In - Ex. & Dear.

画 Vienn 图.

total - miss/fix 1个 + ⋯ 2个. - 3个 + ⋯

1. Principle: $\sum_{S\subseteq[m]} (-1)^{|S|} \cdot N(S)$.

1). # of surjections. $f: [n] \to [m]$.

$$S(n,m) = \sum_{k=0}^{m} (-1)^k \binom{m}{n} (m-k)^n.$$

2. Dearangement: without fixed point. $\sigma(i) \neq i$. $\forall i \in [n]$.

1) # of dear. of $[n]$: $d_n = \sum_{k=0}^{n} (-1)^k \binom{n}{k} \cdot (n-k)! = \sum_{k=0}^{n} (-1)^k \frac{n!}{k!}$

2). $\lim_{n\to\infty} \frac{d_n}{n!} = \frac{1}{e}$.

3). $d_n = \begin{cases} nd_{n-1}+1 & \text{odd } n. \\ nd_{n-1}-1 & \text{even } n. \end{cases}$

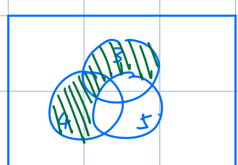
4). $d_n = (n-1)(d_{n-1}+d_{n-2})$.

3. Euler's ϕ: # of $\mathbb{Z}^+$ small or equal to n and gcd $=1$.

1). $\phi(n) = n \cdot \prod_{i=1}^{m} \frac{p_i - 1}{p_i}$ ← every prime that construct n.

4. Exercise.

1). # Int. less than or equal to 2017 are multiples of 3. or 4 but not 5.



$\frac{2017}{3} = 672 \cdots 1$.

$\frac{2017}{4} = 504 \cdots 1$.

$\frac{2017}{5} = 403 \cdots 2$.

$\frac{2017}{12} = 168 \cdots$ __.

$\frac{2017}{15} = 134 \cdots$ __.

$\frac{2017}{20} = 100 \cdots$ __.

$\frac{2017}{60} = 33 \cdots$.

not 3, 4, 5: $2017 - 672 - 504 - 403 +$

$168 + 134 + 100 - 33 = 840$.

$\therefore \# = 2017 - 840 - 403 = 774$.

3). Let $n \in \mathbb{N}$. $n \geq 5$. How many. der. σ of $[n]$ has $\sigma(3) = 5$.

Since. $\sigma(3)=5$, $\underbrace{\{1,2,4,5,\dots,n\}}_{n-1} \to \underbrace{\{1,2,3,4,6,\dots,n\}}_{n-1}$.

Consider the probable fixed point in domain. since $\sigma(3)=5$ given. $\sigma(5)\neq 5$; thus, there're $n-2$ probable fixed pts.

$\therefore$ all probability: $(n-1)! - \binom{n-2}{1}\cdot(n-1-1)! + \binom{n-2}{2}\cdot(n-1-2)! - \cdots$

choose 1 pt. to fix. all # (位置). fixed pt.

$= \sum_{k=0}^{n-2} (-1)^k \cdot \binom{n-2}{k} \cdot (n-1-k)!$

4) Consider bijections $f:[6]\to[6]$. with $f(2)\neq 2$. $f(4)\neq 4$. $f(6)\neq 6$. How many such f. (= surjections.)

All $-$ 1 fixed pt $+$ 2 fixed pts. $-$ 3 fixed pt.

$\Rightarrow 6! - \binom{3}{1}(6-1)! + \binom{3}{2}(6-2)! - \binom{3}{3}(6-3)!$

fixed pt. 只可从这3个中选.

有不等号.

G.F. $\ln$-$\bar{z}x$