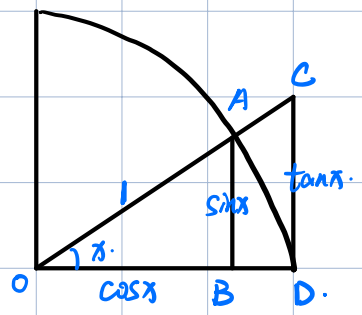
An Important Limit

1. $\lim_{x\to 0} \frac{\sin x}{x} = 1$.

1) Geometric Proof.



$S_{\triangle OAB} \le S_{OAD} \le S_{\triangle COD}$.

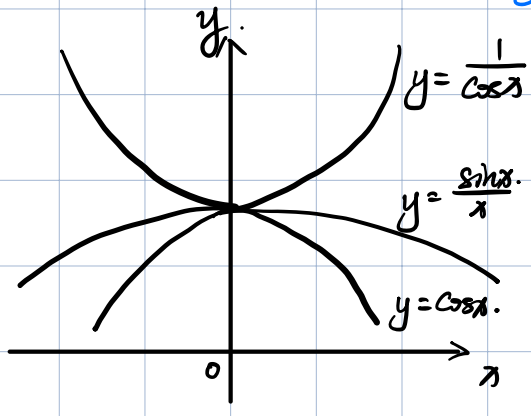
$\Rightarrow \frac{1}{2}\sin x\cos x \le \frac{1}{2}\cdot 1^2 \cdot x \le \frac{1}{2}\tan x$.

$\Rightarrow \sin x\cos x \le x \le \frac{\sin x}{\cos x}$.

$\Rightarrow \frac{\sin x}{x} \le \frac{1}{\cos x}$; $\cos x \le \frac{\sin x}{x}$.

$\Rightarrow \cos x \le \frac{\sin x}{x} \le \frac{1}{\cos x}$ for $0 < |x| < \frac{\pi}{2}$

$\lim_{x\to 0} \cos x = 1$; $\lim_{x\to 0} \frac{1}{\cos x} = 1$



By Squeeze Theorem,

$\lim_{x\to 0} \frac{\sin x}{x} = 1$