

# CSC236 Problem Set 1

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# 1 Question 1

- (a) According to the definition of P:

$$\forall g_1 \in G_1, \exists t_1 \in T_1, t_1 \text{ tiles } g_1 \implies \forall g_2 \in G_2, \exists t_2 \in T_2, t_2 \text{ tiles } g_2$$

- (b) Firstly, assume

$$\forall g_1 \in G_1, \exists t_1 \in T_1, t_1 \text{ tiles } g, \text{ which is the antecedent.}$$

Secondly, I will do the consequent part, which:

*Let  $g_2$  be an arbitrary element from  $G_2$*

Then, I want to prove that

$$\exists t_2 \in T_2, t_2 \text{ tiles } g_2$$

by selecting a satisfying element  $t_2$  from  $T_2$  and prove the element  $t_2$  satisfies  $t_2 \text{ tiles } g_2$ .

- (c) The diagram above illustrates one instance of  $G_2$  grids, which being tiled by triominoes.

Firstly, we already know that for P(1), the statement  $\forall g_1 \in G_1, \exists t_1 \in T_1, t_1 \text{ tiles } g$  is true which is the antecedent of this direct proof.

Secondly, the above diagram is an element of the set of all  $2^2 \times 2^2$  grid with one square removed, which is an element of  $G_2$ . By visualising those colorful triominoes, we see a combination triominoes,  $t_2$ , which is an element of the set of all tilings of elements of  $G_2$  using triominoes, belonging to  $T_2$ , exists and tiles  $g_2$ .

Therefore, the diagram above illustrates an instance of that direct proof.

- (d) Given the statement to prove:  $\forall n \in \mathbb{N}, P(n)$ , which for each natural n you can tile any  $2^n \times 2^n$  grid with one cell missing using only triominoes.

**Proof:** We prove this by Simple Induction on n.

**Base Case:** Let  $n = 1$ .

Since  $G_1$  is the set of all  $2^1 \times 2^1$  grids with one cell removed, which by definition is a single triominoe.

Therefore,  $\forall g_1 \in G_1, \exists t_1 \in T_1, t_1 \text{ tiles } g_1$  is true, which P(1) is true.

**Induction Step:** Let  $k \in \mathbb{N}$ .

**Induction Hypothesis:** Assume that  $P(k)$  is true.

By Induction Hypothesis, we know that  $P(k)$  is true, which  $\forall g_k \in G_k, \exists t_k \in T_k, t_k \text{ tiles } g_k$  is true. I will take 3 different  $g_k$ s, the first with right bottom corner square missing, the second with right top corner square missing, and the third with left top corner square missing. I will make the missing corners in these 3  $g_k$ s face inwards and add a triomino which will result in getting a 'L' shape. The remaining  $\frac{1}{4}$  place is missing a cell to form a  $g_{k+1}$ , which can actually be an arbitrary element from  $G_k$ . By Induction Hypothesis, since  $\forall g_k \in G_k, \exists t_k \in T_k, t_k \text{ tiles } g_k$  is true, the remaining  $G_k$  place can be covered by trimonoies, proving the  $P(k + 1)$  is true.

Therefore, we've proved  $\forall n \in \mathbb{N}, P(n)$  is true.

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## Question 2