MAT246 Online Quiz 4

1. $10^0 \equiv 1 \pmod{13}$; $10^1 \equiv 10 \pmod{13}$; $10^2 \equiv 9 \pmod{13}$;

$10^3 = 10 \times 10^2 \equiv 10 \times 9 \equiv 12 \pmod{13}$;

$10^4 = 10 \times 10^3 \equiv 10 \times 12 \equiv 3 \pmod{13}$;

$10^5 = 10 \times 10^4 \equiv 10 \times 3 \equiv 4 \pmod{13}$;

$10^6 = 10 \times 10^5 \equiv 10 \times 4 \equiv 1 \pmod{13}$;

$10^7 = 10 \times 10^6 \equiv 10 \times 1 \equiv 10 \pmod{13}$; Thus, we find the overlap session.

$10^8 = 10 \times 10^7 \equiv 10 \times 10 \equiv 9 \pmod{13}$; $10^9 = 10 \times 10^8 = 10 \times 9 \equiv 12 \pmod{13}$;

$10^{10} = 10 \times 10^9 \equiv 10 \times 12 \equiv 3 \pmod{13}$; $10^{11} = 10 \times 10^{10} \equiv 10 \times 3 \equiv 4 \pmod{13}$;

$10^{12} = 10 \times 10^{11} \equiv 10 \times 4 \equiv 1 \pmod{13}$.

2. To calculate the reminder of $n = 9238476153683$ divided by 13, have,

$n = 9 \times 10^{12} + 2 \times 10^{11} + 3 \times 10^{10} + 8 \times 10^9 + 4 \times 10^8 + 7 \times 10^7 + 6 \times 10^6 + 1 \times 10^5 + 5 \times 10^4 + 3 \times 10^3 + 6 \times 10^2 + 8 \times 10^1 + 3 \times 10^0$

$\Rightarrow n \equiv 9 \times 1 + 2 \times 4 + 3 \times 3 + 8 \times 12 + 4 \times 9 + 7 \times 10 + 6 \times 1 + 1 \times 4 + 5 \times 3 + 3 \times 12 + 6 \times 9 + 8 \times 10 + 3 \pmod{13}$

$\Rightarrow n \equiv 9 + 8 + 9 + 96 + 36 + 70 + 6 + 4 + 15 + 36 + 54 + 80 + 3 \pmod{13}$

$\Rightarrow n \equiv 26 + 96 + 36 + 80 + 15 + 170 + 3 \pmod{13}$

$\Rightarrow n \equiv 426 \pmod{13}$

$\Rightarrow n \equiv 4 \times 10^2 + 2 \times 10^1 + 6 \times 10^0 \pmod{13}$.

$\Rightarrow n \equiv 4 \times 9 + 20 + 6 \pmod{13}$

$\Rightarrow n \equiv 62 \pmod{13}$

$\Rightarrow n \equiv 10 \pmod{13}$.

Therefore, the reminder of 9238476153683 when divided by 13 is 10.

3. From theorem 3.1.6, since If $a \equiv b \pmod{m}$, then for every $n \in \mathbb{N}$, $a^n \equiv b^n \pmod{m}$, gives

$9238476153683^{485249} \equiv 10^{485249} \pmod{13}$. (from the result of question 2)

Since in question 1, I've found that $10^n = 10^6 \times 10^{n-6} \equiv 1 \times 10^{n-6} \pmod{13}$, I need to find the reminder of 485249 when divided by 6:

$10^0 \equiv 1 \pmod{6}$; $10^1 \equiv -2 \pmod{6}$; $10^2 \equiv 10 \times 10 \equiv (-2)^2 \equiv 4 \equiv -2 \pmod{6}$.

$10^3 \equiv 10^1 \times 10^2 \equiv (-2)^2 \equiv 4 \equiv -2 \pmod{6}$; $10^4 \equiv 10 \times 10^3 \equiv (-2)^2 \equiv 4 \equiv -2 \pmod{6}$.

$10^5 \equiv 10^1 \times 10^4 \equiv (-2)^2 \equiv 4 \equiv -2 \pmod{6}$, which.

$485249 = 4 \times 10^5 + 8 \times 10^4 + 5 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 9 \times 10^0$

$\Rightarrow 485249 \equiv 4 \times (-2) + 8 \times (-2) + 5 \times (-2) + 2 \times (-2) + 4 \times (-2) + 9 \pmod{6}$.

$\Rightarrow 485249 \equiv -8 - 16 - 10 - 4 - 8 + 9 \pmod{6}$

$\Rightarrow 485249 \equiv -37 \pmod{6}$

$\Rightarrow 485249 \equiv -1 \pmod{6}$

$\Rightarrow 485249 \equiv 5 \pmod{6}$.

Thus $10^{485249} \equiv 10^{485249-6} \equiv 10^{485249-6\times 2} \equiv 10^{485249-6\times 3} \equiv \cdots\cdots \equiv 10^5 \equiv 4 \pmod{13}$