Local Extreme Value Theorem.

① extremum
= maximum or minimum
= global extremum

② plural forms:
extremum : extrema
maximum: maxima.
minimum: minima.

③ local extremum can be extremum as well.

1. Definitions:

Let f be a function with domain I.

Let $c \in I$.

1> Maximum: f has a maximum at c when:

$$\forall x \in I,\ f(x) \leq f(c).$$

2> Minimum : f has a minimum at c when:

$$\forall x \in I,\ f(x) \geq f(c).$$

3> Local Maximum: f has a local maximum at c when:

$$\exists \delta > 0,\ \text{s.t. } |x-c| < \delta \Rightarrow f(x) \leq f(c).$$

4> Local Minimum: f has a local minimum at c when:

$$\exists \delta > 0,\ \text{s.t. } |x-c| < \delta \Rightarrow f(x) \geq f(c).$$

end points doesn't count as L max or L min (In some analysis books, it may include.

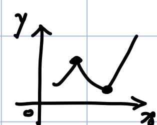
2. The Local Extreme Value Theorem.

Let f be a function with domain an interval I. Let $c \in I$.

If. ① f has a local extremum at c

② c is an interior point to I. (not an end-point).

Then. $f'(c) = 0$ or DNE.



1> Proof.

Assume f has a local extremum at c.

WTS: $f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ is 0 or DNE

Assume f is differentiable at c.

$x \to c^+$

$x - c > 0$

$f(x) - f(c) \leq 0$

$\frac{f(x) - f(c)}{x - c} \leq 0$

$\lim_{x \to c^+} \frac{f(x) - f(c)}{x - c} \leq 0$

$x \to c^-$

$x - c < 0$

$f(x) - f(c) \leq 0$

$\frac{f(x) - f(c)}{x - c} \geq 0$

$\lim_{x \to c^-} \frac{f(x) - f(c)}{x - c} \geq 0$

$\because \lim_{x \to c^+} \frac{f(x) - f(c)}{x - c} = \lim_{x \to c^-} \frac{f(x) - f(c)}{x - c}$ $\quad \therefore f'(c) = 0$.

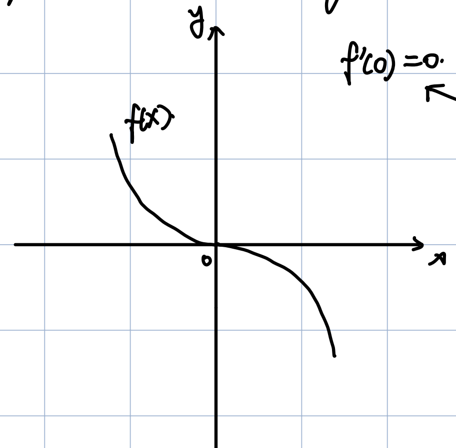
3. Critical Point

c is a critical point of f when

① c is an interior point of the domain of f, and

② $f'(c) = 0$ or DNE.

1) The local extremum point is always a critical point but the critical point is not always a local extremum point.



$f'(0) = 0$. when $c = 0$, it's a critical point but not a local extremum point.

4. Find the extrema.

From EVT, since f is continuous in $[-4, 4]$, x has max & min in $[-4, 4]$.

e.g. Find the extrema of $f(x) = x^3 - 3x^2 - 9x + 35$. when $x \in [-4, 4]$

① Find endpoints & critical points ($f'(c) = 0$ or DNE).

$x = 4$ $f(x) = 15$. $x = -4$. $f(x) = -41$.

$f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x-3)(x+1)$.

$x = 3$ $f(x) = 8$. $x = -1$ $f(x) = 40$.

② Compare.