Random Variables

1. Random Variables: any function from S to $\mathbb{R}$, represents some random quantity in the experiment.

e.g. Roll 3 dice. $X(s)$ = number of 5s.; $Y(s)$ = number of 3s; $Z = X - Y$.

Sample Space S: {111, 112, ...}, $s \in S$.

① when $s = 335$. $X(s) = 1$, $Y(s) = 2$, $Z = X - Y = -1$

② when $X(s) = 3$, gives $s = 555$.

1). Indicator Function: $I_A(s) = 1$ if $s \in A$ otherwise $I_A(s) = 0$ (get whether s is in A).

2. Distribution of Random Variables: the collection of all of the P of the variable being in every possible subset of $\mathbb{R}$.

e.g. $S = \{A, B, C\}$. $P(A) = 0.4$. $P(B) = 0.15$. $P(C) = 0.45$.

Let $X(A) = 1$. $X(B) = 2$. $X(C) = 5$.

① $P(X = 1) = P(A) = 0.4$. (when $X = 1$. 's' = A).

② $P(X \leq 3) = P(X = 1) + P(X = 2) = P(A) + P(B) = 0.55$.

③ For any subset $M \subseteq \mathbb{R}$ (M is an event), we have

$P(X \in M) = P(A) \cdot I_M(1) + P(B) I_M(2) + P(C) \cdot I_M(5)$.

$P(X \in M)$
$= P(X^{-1}(M))$
$= P(\{s \in S : X(s) \in M\})$

e.g. M be the event '>4' ($M = \{s \in S : X(s) \in M\}$). which.

Since 1, 2 is not in M. $I_M(1) = 0$. $I_M(2) = 0$.

$P(X > 4) = P(A) \cdot 0 + P(B) \cdot 0 + P(C) \cdot 1 = 0.45$.