

①  $\exists x (Fx \vee \sim Gx) \cdot \forall y (Fy \rightarrow Ay) \cdot \forall z Gz \cdot \therefore \sim \forall x \sim Ax$ . (BR)

1. ~~show~~  $\sim \forall x \sim Ax$ .

|    |                       |          |
|----|-----------------------|----------|
| 2. | [ $\forall x \sim Ax$ | assid.   |
| 3. | $F_i \vee \sim G_i$   | Pr1 EI/i |
| 4. | $F_i \rightarrow A_i$ | Pr2 UI/i |
| 5. | $\sim A_i$            | 2 UI/i   |
| 6. | $\sim F_i$            | 4 5 MT.  |
| 7. | $\sim G_i$            | 3 6 MTP. |
| 8. | $G_i$                 | Pr3 UI/i |
| 9. | [                     | 7 8 id.  |

②  $\exists y \sim Fy \cdot \forall x (Gx \rightarrow Fx) \cdot \forall x \sim (Hx \wedge \sim Gx) \cdot \forall y (Ly \vee Hy) \cdot \therefore \exists z (Lz \vee Jz)$ . (BR)

1. ~~show~~  $\exists z (Lz \vee Jz)$ .

|     |                              |          |
|-----|------------------------------|----------|
| 2.  | [ $\sim F_i$                 | Pr1 EI/i |
| 3.  | $G_i \rightarrow F_i$        | Pr2 UI/i |
| 4.  | $\sim G_i$                   | 2 3 MT.  |
| 5.  | $\sim (H_i \wedge \sim G_i)$ | Pr3 UI/i |
| 6.  | $L_i \vee H_i$               | Pr4 UI/i |
| 7.  | <del>show</del> $L_i$        |          |
| 8.  | [ $\sim L_i$                 | assid.   |
| 9.  | $H_i$                        | 6 7 MTP. |
| 10. | $H_i \wedge \sim G_i$        | 4 9 ADJ. |
| 11. | [                            | 5 10 id. |
| 12. | $L_i \vee J_i$               | 7 ADD.   |

13.  $\exists z (Lz \vee Jz).$  12. EG.

14. 13 DD

③  $\exists x Fx \rightarrow \forall y (Jy \vee Hy). \sim \exists x Hx. \forall x (Jx \rightarrow Gx). \therefore \forall x (Fx \rightarrow Gx)$  (BR)

1. show  $\forall x (Fx \rightarrow Gx).$

2.  $\boxed{\text{show } Fx \rightarrow Gx}$

3.  $\boxed{Fx.}$  asscd.

4.  $\boxed{\text{show } Gx.}$

5.  $\boxed{\sim Gx}$  ass id.

6.  $\boxed{\exists x Fx}$  3 EG.

7.  $\boxed{\forall y (Jy \vee Hy)}$

P-1 6 MP.  $\rightarrow$  don't use UI hurriedly.

8.  $\boxed{Jx \rightarrow Gx.}$

P-3 UI

9.  $\boxed{\sim Jx}$

5 8 MT

10.  $\boxed{Jx \vee Hx.}$

7 UI/x

11.  $\boxed{Hx.}$

9 10 MTP.

} EG can be same variable.

12.  $\boxed{\exists x Hx}$

11 EG.

13.  $\boxed{\quad}$  P-2 12 id.

14.  $\boxed{\quad}$  4cd.

15.  $\boxed{\quad}$  2 ucl.

④  $\sim \forall x (Dx \vee Ex). \exists x (Fx \leftrightarrow \sim Ex) \rightarrow \forall z Dz. \therefore \exists x \sim Fx.$  (AR).

1. show  $\exists x \sim Fx.$

2.  $\boxed{\sim \exists x \sim Fx}$  ass id.

3.  $\boxed{\forall x Fx.}$  2 QN DN

|     |   |                 |
|-----|---|-----------------|
| 4.  | $\exists x (\sim D_x \vee E_x)$                   | Pr 1 QN         |
| 5.  | $\exists x (\sim D_x \wedge \sim E_x)$            | 4 DM.           |
| 6.  | $\sim D_i \wedge \sim E_i$                        | 5 EI/i          |
| 7.  | <del>show</del> $\sim \forall z D_z$              |                 |
| 8.  | [ $\forall z D_z$                                 | assid.          |
| 9.  | [ $D_i$   | 8 UI/i          |
| 10. | [ $\sim D_i$                                      | 6 $\neg$ S      |
| 11. | [ ]   | 9 10 id.        |
| 12. | $\sim \exists x (F_x \leftrightarrow \sim E_x)$   | Pr 2 $\neg$ MT. |
| 13. | $\forall x (\sim (F_x \leftrightarrow \sim E_x))$ | 12. QN          |
| 14. | $F_i \leftrightarrow \sim \sim E_i$               | 13 DM UI/i      |
| 15. | $F_i \rightarrow \sim \sim E_i$                   | 14. BC.         |
| 16. | $F_i$   | 3 UI/i          |
| 17. | $E_i$   | 15 16 MP DN     |
| 18. | $\sim E_i$  | 6 S.            |
| 19. | [ ]   | 17 18 id.       |

