Counting Problem.

1. Final Integer solutions: (插板问题).

$x_1 + x_2 + x_3 + x_4 = 30$.

1). $x_i > 0$ for $i = 1, 2, 3, 4$.

$\binom{30-1}{4-1} = \binom{29}{3}$. 29个空插3板

2). $x_i \geq 0$. for $i = 1, 2, 3, 4$.

① 变成正.

$(x_1+1) + (x_2+1) + (x_3+1) + (x_4+1) = 34$. where $x_i + 1 > 0$.

$\therefore \binom{34-1}{4-1} = \binom{33}{3}$.

3). $x_i \geq 0$. for $i = 1, 2, 3, 4$; $2 \leq x_1 \leq 7$.

法一: 固定不同 x_1 的值. 看剩有几种再加和.

$x_1 = 2$. $x_2 + x_3 + x_4 = 28$ $\Rightarrow (x_2+1) + (x_3+1) + (x_4+1) = 31$. $\therefore \binom{31-1}{3-1} = \binom{30}{2}$.

⋮

$x_6 = 7$.

法二: # $x_1 \geq 2$ − # $x_1 \geq 8$.

when $x_1 \geq 2$. $x_1 - 2 \geq 0$. $\Rightarrow (x_1 - 2 + 1) + \cdots = 32$. $\therefore \binom{32-1}{4-1} = \binom{31}{3}$.

when $x_1 \geq 8$. $x_1 - 8 \geq 0$ $\Rightarrow (x_1 - 8 + 1) + \cdots = 26$ $\therefore \binom{26-1}{4-1} = \binom{25}{3}$.

$\therefore \binom{31}{3} - \binom{25}{3}$.

4). $x_1 \geq -5$, $x_2 \geq -1$, $x_3 \geq 1$, $x_4 \geq 2$.

$x_1 + 5 \geq 0$; $x_2 + 1 \geq 0$; $x_3 - 1 \geq 0$; $x_4 - 2 \geq 0$.

$\Rightarrow (x_1 + 5 + 1) + (x_2 + 1 + 1) + (x_3 - 1 + 1) + (x_4 - 2 + 1) = 37$.

$\therefore \binom{37-1}{4-1} = \binom{36}{3}$.

2. Round Table (圆桌问题).

1) n people

① rotation same: $\frac{n!}{n}$.

② R.S. + reflection same: $\frac{n!}{n \cdot 2}$.

eg. 5 couples (5M. 5F). sit round table

1). every man seated between 2 F.

2). every man seated between 2 F one of whom is wife.

3). every man seated with wife.

4) women seated consecutive seats.

1). ① 5M seat on 5 seat table $\frac{5!}{5}$

② 5F seat between. $\frac{5!}{5} \cdot 5!$

2) ① 捆绑: $\frac{5!}{5}$

② 同时交换: $\frac{5!}{5} \cdot 2$

3). ① 捆绑: $\frac{5!}{5}$

② 组内换: $\frac{5!}{5} \cdot 2$

4). ① 捆绑: $\frac{2!}{2}$ (M, F各一组). ; $\frac{6!}{6}$

② 内部排: $\frac{2!}{2} \cdot 5! \cdot 5!$; $\frac{6!}{6} \cdot 5!$