Problem Set 3.

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Q3.

proof: Let $t \in \mathbb{N}$.

Let c be a sequence of length l with $\Sigma c > t$.

Let $A = \{\Sigma a : a$ is a sequence of length l with $a \le c$ and $\Sigma a \ge t\}$.

WTS: $\exists c'$ be a sequence of length l. ① $c' \le c$ and ② $\Sigma c' = t$.

Since c is a sequence of length l with ① $c \le c$ and ② $\Sigma c > t \Rightarrow \Sigma c \ge t$.

gives A is not an empty set.

By WOP. gives. $\exists c_{min} \in A$, c_{min} has length l with $c_{min} \le c$ and $\Sigma c_{min} = t_0 \ge t$.

Since $\Sigma c_{min} = t_0 \ge t$, and we want to show $\Sigma c_{min} = t_0 = t$

Assume for contradiction $\Sigma c_{min} = t_0 > t$, which $\Sigma c_{min} \ge t+1$.

Since $t \in \mathbb{N}$ and $\Sigma c_{min} \ge t+1$, gives Σc_{min} is positive and some element of c_{min}, say c_{min_k} is positive and $c_{min_k} \ge 1$.

Take $c_{min}' = c_{min} \setminus \{c_{min_k}\} + \{c_{min_k} - 1\}$, which c_{min}' is a sequence which the element c_{min_k} decreased by 1 and other elements in c_{min} remain the same.

Since the number of elements in c_{min}' is the same as c_{min}, then length of c_{min}' is still l, gives $c_{min}' \le c_{min} \le c$, from the question's definition.

Since c_{min}' is a sequence which the element c_{min_k} decreased by 1 and other elements in c_{min} remain the same, $\Sigma c_{min}' = \Sigma c_{min} - 1 \ge (t+1) - 1 = t$.

Since $c_{min}' \le c$ and $\Sigma c_{min}' \ge t$, gives $c_{min}' \in A$.

However, since $\Sigma c_{min}' = \Sigma c_{min} - 1 < \Sigma c_{min}$, $c_{min}' \le c_{min}$, contradicts to c_{min} is the least element in A.

Therefore, since $\Sigma c_{min} \ge t$ and Σc_{min} not bigger then t, $\Sigma c_{min} = t$, which we've found such $c' = c_{min}$, that $\Sigma c' = t$ ∎