Multiplace Derivatives

1. Alphabetic Variance (AV) (derived rule).

$$\frac{\forall\alpha\phi\alpha}{\therefore \forall\beta\phi\beta} \qquad \frac{\exists\alpha\phi\alpha}{\therefore \exists\beta\phi\beta.}$$

restriction: β can't appear free or bounded within ϕ.

1) Usage 1: scope conflict (the most immediate scope wins).

$\exists x(Fx \wedge \forall x(Gx \rightarrow Hx)) \equiv \exists y(Fy \wedge \forall x(Gx \rightarrow Hx))$.

↘ often don't write.

2) Usage 2: In UD.

Show $\forall\alpha\phi\alpha$	Restriction: α can't appear unbound in any previous available line, or in a premise used in an available line.
Show $\phi\alpha$.	
CD/ID/DD	
$\forall\alpha\phi\alpha$ UD.	

↓

Show $\forall\beta\phi\beta$	
show $\phi\beta$.	
CD/ID/DD	
$\forall\beta\phi\beta$ UD	
$\forall\alpha\phi\beta$ AV.	

2. Rules for Identity

1) LL

$$\frac{\begin{array}{l} t = t^{\star} \\ \vdots \\ \delta(t) \end{array}}{\delta(t^{\star})} \qquad \frac{\begin{array}{l} \sim\delta(t^{\star}) \\ \vdots \\ \delta t \end{array}}{t \neq t^{\star}}$$

2) SM (symmetry)

$$\frac{t = t^{\star}}{t^{\star} = t}$$

3) SID (reflexivity)

$$\frac{\vdots}{t = t}$$

e.g. Argument: $\forall x(x=a)$. $\therefore \exists y Fy \leftrightarrow Fa$.

1. show $\exists y Fy \leftrightarrow Fa$.
2. show $\exists y Fy \rightarrow Fa$.
3. $\exists y Fy$ assed.
4. show Fa.
5. $\sim Fa$. assid.
6. Fi 3 ei/i
7. $i=a$ Pr| ui/i
8. Fa. 6 7 LL.
9. 5 8 id.
10. 4 cd.
11. show $Fa \rightarrow \exists y Fy$
12. Fa asscd.
13. $\exists y Fy$. 12 eg.
14. 13 cd.
15. $\exists y Fy \leftrightarrow Fa$. 2 11 cb
16. 15 dd