Polynomials.

1. A poly. of degree n has at most n roots; if 'multi-plicities' are counted it has exactly n roots.

2. Fundamental Theorem of Algebra: Every non-constant polynomial with complex coefficient has a complex root.

3. Thm. 9.3.4.: If r is a complex number and $p(z)$ is a non-constant polynomial with complex coefficients, then there exists a polynomial $q(z)$ and a constant c. s.t. $p(z)=(z-r)q(z)+c$.

   1). r is root of $p(z)$. $\Leftrightarrow c=0$.

   2). Thm. 9.3.5: (Divisibility relation of poly.): If $\exists q(z)$ s.t. $p(z)=f(z)\cdot q(z)$. then $f(z)$ is a factor of $p(z)$

4. Thm. 9.3.6. (factor theorem): The complex num. r is a root of a poly. $p(z)$ iff. $z-r$ is a factor of $p(z)$ ($p(z)=(z-r)q(z)=f(z)\cdot q(z)$).

5. Solve the equation / Find the root.

   1). $f(x)=3x^3+x^2+x-2$. (find the root).

   ① Get rational roots. by R.R.T.

   By R.R.T. $m|-2$, $n|3$. which. $m: \pm 1, \pm 2$; $n: \pm 1, \pm 3$.

   Possible $\frac{m}{n}$: $\pm 1, \pm 2, \pm 3, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{3}{2}$.

   Substitute gives $x=\frac{2}{3}$ is a root.

   ② Use the root to factorize $f(x)$

   $f(x)=(3x-2)(x^2+x+1)$ by Long division.

   ③ Calculate x.

   $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{3}i}{2}.$$

If its not 2-D. then solve it as 1D does.

2). $z^4+z^2+1=0$.

Let $x=z^2$, gives. $x^2+x+1=0$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{3}i}{2}.$$

Let $z=r(\cos\theta+i\sin\theta)$. gives. $z^2=r^2(\cos 2\theta+i\sin 2\theta)$.

① $r^2(\cos 2\theta+i\sin 2\theta)=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$.

$$r=\sqrt{a^2+b^2}=\sqrt{\frac{1}{4}+\frac{3}{4}}=1.$$

$$\begin{cases}\cos 2\theta=-\frac{1}{2}\\ \sin 2\theta=\frac{\sqrt{3}}{2}\end{cases}\Rightarrow 2\theta=\frac{2}{3}\pi \text{ or } \frac{8}{3}\pi.\Rightarrow \theta=\frac{\pi}{3} \text{ or } \frac{4}{3}\pi.$$

$z_1=\cos(\frac{\pi}{3})+i\sin(\frac{\pi}{3})$. $z_2=\cos(\frac{4}{3}\pi)+i\sin(\frac{4}{3}\pi)$.

② $r^2(\cos 2\theta+i\sin 2\theta)=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$.

⋮