

Q1 知识点梳理

- Prove a function is bijective

1. Injective (1-1)

2. Surjective (Onto)

- Cardinality

↳ Definition:

↳ Definition 10.1.7 - Equal cardinality (same cardinality):

↳ 证明 same cardinality 的方法:

1. 构造一个 bijection

2. 使用传递性

- Definition of Countable

- Finite sequences of elements of set A

Q2 知识点梳理

- RRT

If $\frac{m}{n}$ is a rational root of the polynomial $a_k x^k + a_{k-1} x^{k-1} + \cdots + a_1 x + a_0$, where a_j are integers and m and n are relatively prime, then m divides a_0 and n divides a_k .

- Check 是否是 field 的四个条件:

- 1.
- 2.
- 3.
- 4.

- Tower of fields

A tower of fields is a finite sequence F_0, F_1, \cdots, F_n of subfields of \mathbb{R} such that $F_0 = \mathbb{Q}$ and for each i from 1 to n , there is a positive number r_i in F_{i-1} such that $\sqrt{r_i}$ is not in F_{i-1} and $F_i = F_{i-1}(\sqrt{r_i})$.

- Field Extension:

Q3 知识点梳理

- De Moivre's Theorem

For every natural number n ,

$$(r(\cos \theta + i \sin \theta))^n = r^n(\cos(n\theta) + i \sin(n\theta)).$$

- n^{th} Roots of Unity

- Method of 12.4.12

Solve $z^7 = 1$ without De Moivre's Theorem

- **Axioms of Greek Construction**

- ↳ Axiom 1: any two points can be joined to create a line segment

- ↳ Axiom 2: Any line segment can be extended to a line

- ↳ Axiom 3: given any point and any line segment (length), we can draw a circle that has the point as its center and the length as its radius.

- ↳ Axiom 4: The only way points are born is of the intersection of two lines, two circles, or lines and circles.

- 基础作图 1 - 垂直平分线段

- 基础作图 2 - 角平分线

- 基础作图 3 - 复制一条线段

- 基础作图 4 - 复制一个角度

Q4 知识点梳理

- **Theorem 9.3.4**

If r is a complex number and $p(z)$ is a non-constant polynomial with complex coefficients, then there exists a polynomial $q(z)$ and a constant c such that

$$p(z) = (z - r)q(z) + c$$

- **Theorem 9.3.5.** Divisibility relation for polynomial

- **Theorem 9.3.6** The Factor Theorem

The complex number r is a root of a polynomial $p(z)$ if and only if $z - r$ is a factor of $p(z)$.

- Long division of Polynomial