Problem Set 5

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Q3: $T(n) = \begin{cases} 1 & n=0 \text{ or } n=1. \\ T(\lfloor \frac{n+2}{3} \rfloor) + T(\lceil \frac{2n-2}{3} \rceil) + 1 & n \geq 2. \end{cases}$

Case 1: goes to branch line 1.

① $n = e - b = 0$.

② $T(n) = 1$ as only 1 step.

Case 2: goes to branch line 2-4.

① $n = e - b = (b+1) - b = 1$.

② $T(n) = 1$ as it terminates on line 3 or 4 which are both 1 step

Case 3: goes to 'else' branch.

① $n \geq 2$.

② Line 5 takes 1 step to calculate c, where $c = \lfloor \frac{2b+e+2}{3} \rfloor$.

Given on line 6 that

$$\begin{aligned} T(n) = T(e-b) &= T(c-b) + T(e-c) + 1 \\ &= T(\lfloor \tfrac{2b+e+2}{3} \rfloor - b) + T(e - \lfloor \tfrac{2b+e+2}{3} \rfloor) + 1. \\ &= T(\lfloor \tfrac{2b+e+2-3b}{3} \rfloor) + T(e - \lfloor \tfrac{2b+c+2}{3} \rfloor) + 1. \quad (\star). \end{aligned}$$

Since $c = \lfloor \frac{2b+e+2}{3} \rfloor$, gives $\frac{2b+e+2}{3} - 1 < c \leq \frac{2b+e+2}{3}$.

$$\Rightarrow -\frac{2b+e+2}{3} \leq -c < 1 - \frac{2b+e+2}{3}.$$

$$\Rightarrow e - \frac{2b+e+2}{3} \leq e - c < e + 1 - \frac{2b+e+2}{3}.$$

$$\Rightarrow e - c = \lceil e - \frac{2b+e+2}{3} \rceil = \lceil \frac{3e-2b-e-2}{3} \rceil = \lceil \frac{2e-2b-2}{3} \rceil$$

$$\begin{aligned} \text{Thus } (\star) &= T(\lfloor \tfrac{e-b+2}{3} \rfloor) + T(\lceil \tfrac{2e-2b-2}{3} \rceil) + 1 \\ &= T(\lfloor \tfrac{n+2}{3} \rfloor) + T(\lceil \tfrac{2n-2}{3} \rceil) + 1 \text{ as } n = e - b. \end{aligned}$$