Continuous Random Variables.

1. Continuous: A random variable X is continuous if $\forall x \in \mathbb{R}$, $P(X=x)=0$

2. Density Function: Let $f: \mathbb{R}^1 \to \mathbb{R}^1$ be a function. Then f is a D.F. if
"absolutely continuous". ↳ «泛函» P109. $\forall x \in \mathbb{R}$, ① $f(x) \geq 0$ and ② $\int_{-\infty}^{\infty} f(x)\,dx = 1$.
   1). If $f(x)$ is the density function for a random variable X, write it as $f_X(x)$

3. Absolute Continuous: A random variable X is A.cts. if there is a D.F. f, s.t.

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx. \quad \text{for } a \leq b.$$

   1) Let X be an A.cts random variable. Then X is a cts. random variable, i.e. $P(X=a) = P(a \leq X \leq a) = \int_a^a f(x)\,dx = 0$.

3. Uniform[L, R] Distribution: Let $L, R \in \mathbb{R}$, $L<R$. Consider a random variable X s.t.

$$P(a \leq X \leq b) = \frac{b-a}{R-L}$$ whenever $L \leq a \leq b \leq R$. with $P(X<L) = P(X<R) = 0$; The random variable X is said to have the Uniform[L, R] Distribution, write as $X \sim$ Uniform[L, R] Distribution.

   1). $X \sim$ Uniform[L, R] Density: $f_X(x) = \begin{cases} \frac{1}{R-L} & L \leq x \leq R. \\ 0 & \text{otherwise} \end{cases}$
   implies it's A.cts.

   proof: Let $a, b, L, R \in \mathbb{R}$, $L \leq a \leq b \leq R$; $X \sim$ Uniform[L, R] Dis.
   $P(a \leq X \leq b) = \frac{b-a}{R-L}$ by dfⁿ.
   $\int_a^b f_X(x)\,dx = \frac{x}{R-L}\Big|_a^b = \frac{b-a}{R-L} = P(a \leq x \leq b)$ ∎
   ↳ A.cts.

e.g. $X \sim$ Uni[5, 12] Dis.
   ① $X \sim$ Uni[5, 12] Den. $f_X(x) = \begin{cases} \frac{1}{7} & 5 \leq x \leq 12. \end{cases}$

$\quad 0 \quad$ otherwise

I. $f_X(x) \geq 0$

II. $\int_{-\infty}^{\infty} f_X(x)\,dx = \int_{-\infty}^{5} 0\,dx + \int_{5}^{12} f_X(x)\,dx + \int_{12}^{\infty} 0\,dx.$

$= \int_5^{12} \frac{1}{7}\,dx = \frac{x}{7}\Big|_5^{12} = \frac{1}{7}(12-5) = 1.$

$\therefore f_X(x)$ is a D.F.

② Let $5 \leq a \leq b \leq 12$, $P(a \leq X \leq b) = \frac{1}{7}(b-a).$

4. Exponential(λ) Distribution:

Let $\lambda > 0$ be a fixed constant. Define a function $f: \mathbb{R} \to \mathbb{R}$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0. \\ 0 & x < 0. \end{cases}$$

① $f(x) \geq 0$..

② $\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{\infty} \lambda e^{-\lambda x}\,dx = -e^{-\lambda x}\Big|_0^{\infty} = (-0)-(-1) = 1.$

Thus, $f(x)$ is a D.F.

1) Definition: Let X be a random variable have this $f(x)$.

$P(a \leq X \leq b) = \int_a^b \lambda e^{-\lambda x}\,dx = e^{-\lambda a} - e^{-\lambda b}$ whenever $0 \leq a \leq b < \infty$; The random variable X is said to have the Exponential(λ) Dis, written as, $X \sim$ Exponential(λ).

$P(X \geq a) = e^{-\lambda a}$

2) Exponential(λ) Density Function: $f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0. \\ 0 & x < 0. \end{cases}$