Mistakes

1. Negation of: I have a friend all of whose former boyfriends had at least two siblings with exactly three different vowels in their name.

   Each of my friends has a former boyfriend who had at most one siblings with exactly three different vowels in their name.

2. x's domin: $\forall a > 0, x \leq a$.

   $x \leq 0$.

3. Formally prove:

   $$\forall a > 0, \exists b \in \mathbb{R}, \text{ s.t. } (b + \sin b) \cdot a > 7.$$

   when doing rough work, we need to figure out the value of b. but not reformulate the question.

   pf. Let $a > 0$. (fix a).
   Take $b = \frac{8}{a} + 1$.
   Since $\sin b \in [-1, 1]$, indeed,
   $(b + \sin b) a \geq (b - 1) \cdot a = \frac{8}{a} \cdot a = 8 > 7$.
   Therefore. ...... ∎

4. Prove:

   Suppose that $a \in \mathbb{Z}$. If a^2 is even, then a is even.

   反证法别忘记.

   pf. Let $a \in \mathbb{Z}$.
   The negation is: $\exists a \in \mathbb{Z}$, a^2 is even and a is odd.

Since $a=2k+1$, $k\in\mathbb{Z}$. when a is odd.

$a^2=(2k+1)^2=4k^2+4k+1=2(2k^2+2k)+1$ is odd.

The negation is false.

Hence. the statement is true $\blacksquare$

5. Prove:

The sum of a rational number and an irrational number is irrational.

pf. Let $a\in\mathbb{Q}$, $b\notin\mathbb{Q}$

Take $c=a+b$. indeed $b=c-a$

The negation: ······ is rational.

Therefore, $c-a$ is a rational number, negates b is irrational.

To conclude, ······ $\blacksquare$

6. Let $S=\{2,3,5,7,11\}$. $\forall x\in S, \exists y\in S$ s.t. $x+y$ is even.

True! Since x, y are variables. they can be the same number in set S.

7.