Definition of Continuity.

1. Definition

1) Let $a \in \mathbb{R}$.

Let f be a function defined, at least, on an interval centered at a.

We say that f is continuous at a when

$$\lim_{x \to a} f(x) = f(a)$$

replace from L

(equivalent)

$$= \forall \varepsilon > 0, \exists \delta > 0, \text{ s.t. } |x-a| < \delta \Rightarrow |f(x) - f(a)| < \varepsilon.$$

(In the normal definition of limit when can exclude x from being exactly a, because what we need is to let x approach a. However, at here, we need x to be a, so we can simply don't restrict $|x-a|>0$).

2. Difference from limit.

1) $\lim_{x \to a} f(x) = L$ means:

$$\begin{Bmatrix} x \text{ close to } a \\ x \neq a \end{Bmatrix} \Rightarrow f(x) \text{ close to } L.$$

2) f continuous at a means:

x close to a $\Rightarrow$ $f(x)$ close to $f(a)$.

3. Continuity in conditions

1) Continuous at a point.

f continuous at c means $\lim_{x \to c} f(x) = f(c)$.

2) Continuous on an open interval.

f continuous on the interval (a, b) means:

$\forall c \in (a,b)$, f is continuous at c.

3) Continuous on a closed inteval

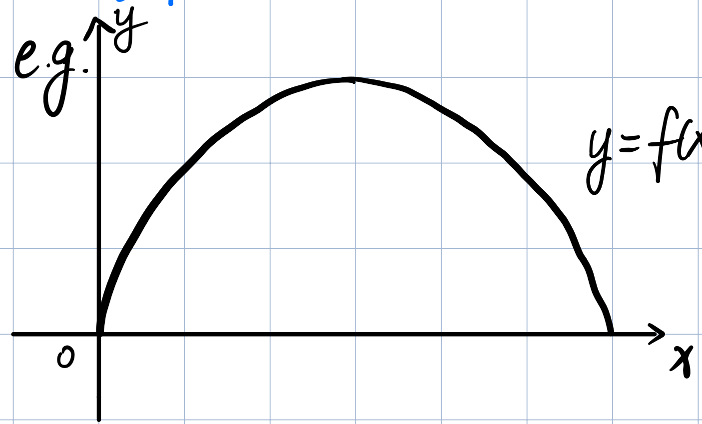
f continuous on the inteval $[a,b]$ means:

① $\lim_{x \to a^+} f(x) = f(a)$.

② $\forall c \in (a,b)$, f is continuous at c.

③ $\lim_{x \to b^-} f(x) = f(b)$

e.g.

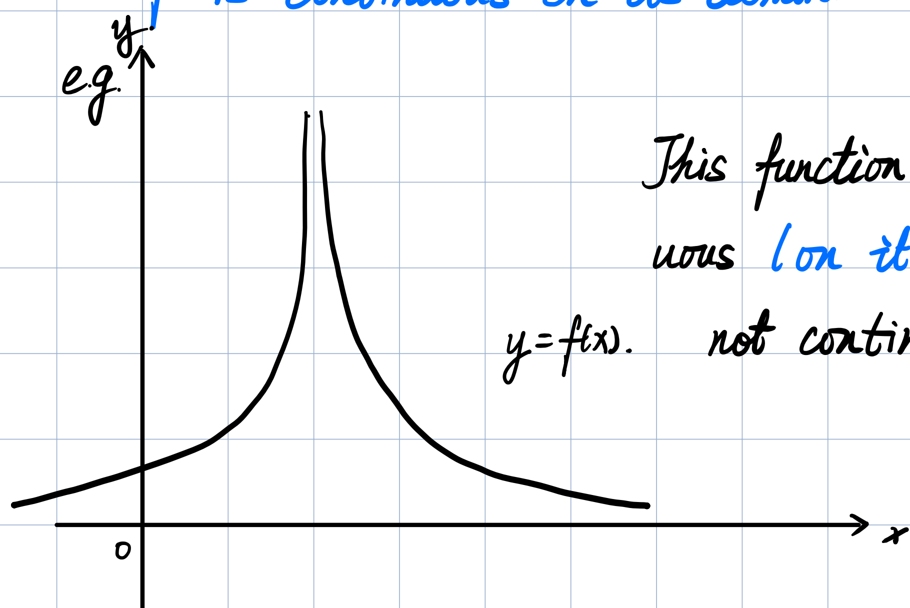


4) Just 'continuous':

f is continuous means:

f is continuous on its domin.

e.g.



This function is continuous (on it domin) but not continuous at 1.