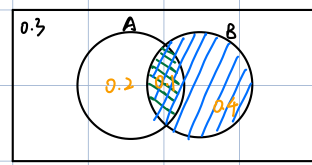
Conditional Probability.

1. Conditional Probability: If A and B are two events, then the conditional probability of A given B is written as $P(A|B)$, represents the fraction of the times when B occurs, in which A also occurs, which.



$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

e.g. Roll 3 fair dice. $P(\text{first die is } 3 \mid \text{at least one } 3)$

S: all possibility of rolling 3 dice.

$|S| = |S_1| \cdot |S_2| \cdot |S_3| = 6^3 = 216$.

A: first die is 3; B: at least one 3.; B^C: no 3.

$B^C = \{1, 2, 4, 5, 6\}^3 = 5^3 = 125$.; $P(B^C) = \frac{125}{216}$; $P(B) = 1 - P(B^C) = \frac{91}{216}$.

$A \cap B$: first die is 3 and at least one 3.

Since $A \subseteq B$. $P(A \cap B) = P(A)$; $|A| = |S_4| \cdot |S_5| = 6^2 = 36$. (first 3 is fixed, $3xx$).

$P(A) = \frac{|A|}{|S|} = \frac{36}{216} = \frac{1}{6}$.

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{91}{216}} = \frac{36}{91} \doteq 0.396$

2. Conditional Multiplication Formula: Since $P(A|B) = \frac{P(A \cap B)}{P(B)}$, gives.

$$P(A \cap B) = P(B) \cdot P(A|B). = P(A) \cdot P(B|A).$$

☆ $\hookrightarrow$ $(P(A \cap B) = P(B \cap A))$.

$\longrightarrow$ Also gives $P(A|B) = \frac{P(A)}{P(B)} P(B|A)$.

3. Law of Total Probability — Conditioned Version: Suppose $A_1, A_2, \ldots$ are a sequence (finite or infinite events), which form a partition of S. i.e. they are disjoint ($A_i \cap A_j = \emptyset$ for all $i \neq j$). and their union equals the entire sample space ($\cup_i A_i = S$), and let B be any event. Then,

$P(B) = \sum_i P(A_i) \cdot P(B|A_i)$ $\quad$ $(P(A_i \cap B) = P(B) \cdot P(A_i|B)$

$= P(A) \cdot P(B|A_i))$.

e.g. 3 cards; $C_1: B-B$; $C_2: Y-Y$; $C_3: B-Y$. Pick a card at random. Then pick a side of the card, at random. $P(C_2|\text{side } Y)$.

$B = \{\text{the side is } Y\}$. $\quad$ $A = \{\text{the card is } C_2\}$.

$P(B) = P(C_1)P(B|C_1) + P(C_2)P(B|C_2) + P(C_3)P(B|C_3)$.

$= \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{2}$.

$P(A \cap B) = P(A) \cdot P(B|A) = P(C_2) \cdot P(B|C_2) = \frac{1}{3} \cdot 1 = \frac{1}{3}$.

$\hookrightarrow$ C.M.

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2}{3}$.