Normal Distribution

fact:
$\int_{-\infty}^{\infty} \phi(x)\,dx = 1$

1. Standard Normal Distribution

1) S.N.D. Density func.: $\forall x \in \mathbb{R}$, $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

2). S.N.D.: Let X be a random variable having the density func. ϕ,

$$P(a \le X \le b) = \int_a^b \phi(x)\,dx = \int_a^b \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx, \text{ whenever } -\infty < a < b < \infty$$

The random variable X is said to have $N(0,1)$ dis., written as. $X \sim N(0,1)$.

2. Normal Distribution

Let $\mu \in \mathbb{R}$, $\sigma > 0$. Let f definded by,

$$f(x) = \frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

① $f(x) \geq 2$.

② Let $y = \frac{x-\mu}{\sigma}$. $\Rightarrow dy = \sigma^{-1} dx \Rightarrow \sigma \cdot dy = dx$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} \sigma^{-1} \cdot \phi\left(\frac{x-\mu}{\sigma}\right) dx = \int_{-\infty}^{\infty} \sigma^{-1} \cdot \phi(y) \cdot \sigma\,dy$$
$$= \int_{-\infty}^{\infty} \phi(y)\,dy = 1.$$

Thus, $f(x)$ is a density function.

1) N.D.: Let X be a random variable having this density func. f. The random variable X is said to have the $N(\mu, \sigma^2)$ Dis. written as $X \sim N(\mu, \sigma^2)$.

2) N.D. Density Func: $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, $\forall x \in \mathbb{R}$.

3). chang μ shift it; $\nearrow \sigma$ makes it fatter.

3. Independence:

If $X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$, X and Y are independent. then $X + Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$.