

Question: It is possible to define a finite basis of a vector space V as a subset $\{v_1, v_2, v_3, \dots, v_n\}$ of V that is neither too big nor too small. Define "too big" and "too small" so that the above statement is accurate.

Answer:

By 'too big', we mean the basis contains vectors that are not linearly independent to each other, which some vectors in this subset(basis) can be written as a linear combination of other vectors in the subset.

By 'too small', we mean the vectors that basis contains can't span the entire vector space.

Accurately speaking, a finite basis of a vector space V is a subset $\{v_1, v_2, v_3, \dots, v_n\}$ of V such that any vector in V can be written as a linear combination of the basis vectors, and no vector in the subset can be written as a linear combination of the other vectors in the subset.