MAT137Y1 - Calculus!
Test 1. — October 21st, 2016
Time: 100 minutes

Please complete this cover page with ALL CAPITAL LETTERS.

Last name .....

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Instructions: (READ CAREFULLY!)

- Do not write or draw anything on the QR code on the top corner of any page.
- This exam booklet contains 10 pages including this one. It consists of 8 questions. The maximum score is 40 points.
- For questions where we provide you with a box labelled "Your answer", please write your final answer in the box, along with any justifications, explanations, or calculations you need underneath. In order to get any points, you need your final answer to be correct and you need a justification.
- If you need scratch paper, use the back of the pages. **We will only read and grade what you write on the front of each page.**
- **If you need extra space for a question, you may use Page 10 for this purpose.** If you do so, clearly indicate it on the corresponding problem page.
- No calculators, cellphones, or any aids are allowed.
  If you have any, place them underneath your seat, or in the front of the room.
- Do not turn over this page until the invigilators instruct you to do so. Good luck!

1. *[6 points]* Calculate each of the following limits. If a limit does not exist, indicate whether it is ∞, $-\infty$, or neither.

(a) $\lim_{x \to -1} \dfrac{x^2 + 3x + 2}{x^2 - 1}$ **Your answer:**

(b) $\lim_{x \to 4^+} \dfrac{3 + x}{4 - x}$ **Your answer:**

(c) $\lim_{x \to 1} \dfrac{x^2 - 2x + 1}{x^2 + 1}$ **Your answer:**

2. *[4 points]* Calculate each of the following limits. If a limit does not exist, indicate whether it is ∞, $-\infty$, or neither.

(a) $\lim_{x \to 4} \dfrac{2 - \sqrt{x}}{x - 4}$

Your answer:

(b) $\lim_{x \to 0} \dfrac{2 \sin^2(2x^5)}{x^{10}}$

Your answer:

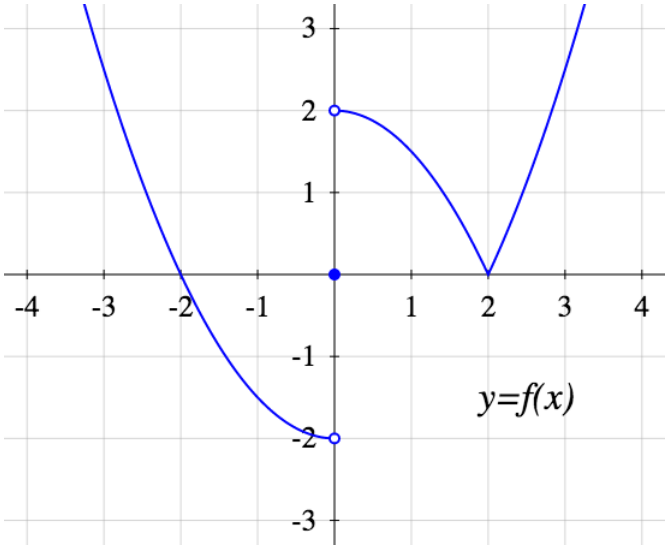
3. *[6 points]* In each of the following cases we ask you to give an example of a function with a certain property. Provide an equation for the function and sketch its graph. It is okay to use piece-wise-defined functions. You do not need to prove anything.

   (a) A function f that is continuous on $\mathbb{R}$.

   (b) A function g that is continuous on $\mathbb{R}$, except at 0, where it has a removable discontinuity.

   (c) A function h that is continuous on $\mathbb{R}$, except at 0, where it has a non-removable discontinuity.

4. *[6 points]* Below is the graph of the function f:



Find the following limits for f:

(a) $\lim_{x \to 0} f(x)$ **Your answer:**

(b) $\lim_{x \to 0} f(f(x))$ **Your answer:**

(c) $\lim_{x \to 2} f(f(x))$ **Your answer:**

Hint: The three answers are different.

5. *[6 points]* Let f be a function with domain $(-\infty, \infty)$. Let $a, L \in \mathbb{R}$. Write the ε–δ definition of the following statements:

   (a) $\lim_{x \to a} f(x) = L$

   (b) $\lim_{x \to a^+} f(x) = L$

   (c) $\lim_{x \to a^-} f(x) = -\infty$

6. *[4 points]* Let A, B be subsets of $\mathbb{R}$. Consider the following two statements:

   (I) $\forall x \in A, \exists y \in B$ such that $x < y$.

   (II) $\forall y \in B, \exists x \in A$ such that $x < y$.

   Are statements (I) and (II) equivalent?

Answer: (circle one) Yes No

   If you answered "Yes", prove it.
   If you answered "No", give an example of a pair of sets that satisfy one of the conditions but not the other, and prove that this is the case.

7. *[4 points]*

(a) Write the precise statement of the Intermediate Value Theorem.

(b) Use the Intermediate Value Theorem to prove that the equation

$$x^5 - 2x = 100$$

has a positive solution.

8. *[4 points]* Let f be a function with domain $(-\infty, \infty)$. Assume that

$$\lim_{x \to a} f(x) = 2$$

Prove that $\lim_{x \to a} (2f(x) - 1) = 3$.

Do a formal proof directly from the ε–δ definition of limit. Do not use any of the limit laws.

Use this page if you run out of space for one or more problems. If you do so, clearly indicate in the corresponding problem page that you will continue here.