Separating Variables.

1. Separable DE: Can be written as:

$$M(x) + N(x)\frac{dy}{dx} = 0.$$

↳ $\frac{dy}{dx} = f(x)\cdot g(y)$; 无真正意义上的加&减 (x&y之间)

Also for solve first order ODE.
↓
常用在 non-Linear
因为有 $y\cdot y'$ 出现.

2. Solve separable ODE.

e.g. $(1+x)\,dy - y\,dx = 0.$

1). Write in form: $N(y)\,dy = -M(x)\,dx$; 考虑 eq. sol$^{\pm}$

$(1+x)\,dy = y\,dx.$

$\Rightarrow \frac{1}{y}\,dy = \frac{1}{1+x}\,dx.$ $y\neq 0.$

$\frac{dy}{dx} = \frac{y}{1+x}.$

$\Rightarrow y=0$, $\frac{dy}{dx}=0$ 为 eq. sol$^{\pm}$.

2). Integral both sides: $\int N(y)\,dy = -\int M(x)\,dx.$

$\int \frac{1}{y}\,dy = \int \frac{1}{1+x}\,dx.$

$\Rightarrow \ln|y| = \ln|1+x| + C.$ ← 非零两侧都加. 相当于只加一侧.

3). Organize.

有 l.v. 无须考虑 eq. sol$^{\pm}$

$y = \pm e^{\ln|1+x|}\cdot \underbrace{e^{C}}_{>0}.$

$\Rightarrow y = \pm C\cdot(1+x).$ $C>0.$

$\Rightarrow y = C\cdot(1+x).$ $C>0$ or $C<0$ or $y=0.$

$\therefore y = C\cdot(1+x).$ $C\in\mathbb{R}.$ ← 合并.

e.g. $y' = x\sqrt{y}.$

$\frac{dy}{dx} = x\cdot y^{\frac{1}{2}}.$

$\Rightarrow \frac{1}{\sqrt{y}}\,dy = x\,dx.$

$\Rightarrow \int \frac{1}{\sqrt{y}}\,dy = \int x\,dx.$

$\Rightarrow 2y^{\frac{1}{2}} = \frac{1}{2}x^2 + C.$

$\Rightarrow \sqrt{y} = \frac{1}{4}x^2 + \frac{1}{2}C.$ $y\geq 0$

$\Rightarrow y = (\frac{1}{4}x^2 + \frac{1}{2}C)^2$ $y>0,\ x\neq 0$ or $y=0.$

when $\frac{dy}{dx} = x\cdot y^{\frac{1}{2}} = 0.$

$\Rightarrow y = 0.$