# Solve Recur (A.O.)

1、Homo:

1) 解 Advance operator (degree ≤ 3) 得到 General sol$^{n}$.

① $A_1, A_2 \in \mathbb{R}$, $A_1 \neq A_2$.

$$a_c = c_1(A_1)^n + c_2(A_1)^n$$

② $A_1, A_2 \in \mathbb{R}$, $A_1 = A_2$

$$a_c = c_1(A_1)^n + c_2 \cdot n \cdot (A_2)^n$$ 重复几次，前面乘几个.

2) 代入 Initial Condition. 解出 $c_1, c_2$.

2. Non-homo.

1) 解 homo. e.g. $A_1 = 3$, $A_2 = 7$.

2) 看 non-homo 部分 ($g(n)$) 有无 $(A_i)^n$.

① 没有:

→ $g(n)$ 为 poly: 设 $a_p$ 为 poly (最高次相同).

$g(n) = 5 \to a_p = c_1$ ; $g(n) = 3n^2 - 2 \to a_p = c_1 n^2 + c_2 n + c_3$.

→ $g(n)$ 中有 $q \cdot B^n$ : 设 $a_p$ 为 $b \cdot B^n$.

$g(n) = 7 \cdot 11^n \to a_p = c_1 \cdot 11^n$.

② 有 $A_i^n$.

→ 有一个: 设 $a_p$ 为 $b \cdot n \cdot B^n$. 不要擅自更改.

$g(n) = 2(3^n) - 8(9^n) \to a_p = c_1 \cdot n \cdot 3^n + \underline{c_2 \cdot 9^n}$.

$g(n) = 4(3^n) - 3(7^n) \to a_p = c_1 \cdot n \cdot 3^n + c_2 \cdot n \cdot 7^n$.

→ 有 $t$ 个: 设 $a_p$ 为 $b \cdot n^t \cdot B^n$.

e.g. $A_1 = A_2 = -2$.

$g(n) = 5(-2)^n \to a_p = c_1 \cdot n^2 \cdot (-2)^n$.

$g(n) = 7n(-2)^n \to a_p = (c_1 n + c_2) \cdot n^2 \cdot (-2)^n$.

$g(n) = -11n^2(-2)^n \to a_p = (c_1 n^2 + c_2 n + c_3) \cdot n^2 \cdot (-2)^n$.

3) 把 $a_p$ 代入原式，解出 $c_i$

3. Exercise:

e.g. $(A+2)(A-6)f = 3^n$.

① Solve for homo.

$(A+2)(A-6)f = 0 \Rightarrow (A+2)(A-6) = 0 \Rightarrow A_1 = -2,\ A_2 = 6$.

$\therefore a_c = c_1 \cdot (-2)^n + c_2 \cdot 6^n$.

②. Set particular sol$^{\underline{n}}$

Assume $a_p = b_1 \cdot 3^n$.

Since $(A+2)(A-6)f = (A^2-4A-12)f = b_1 \cdot 3^{n+2} - 4b_1 \cdot 3^{n+1} - 12b_1 \cdot 3^n$

$= 3^n$, gives.

$\Rightarrow 3^2 \cdot b_1 \cdot 3^n - 4 \cdot 3 \cdot b_1 \cdot 3^n - 12 \cdot b_1 \cdot 3^n = 3^n$, gives,

$\Rightarrow 9b_1 - 12b_1 - 12b_1 = 1 \Rightarrow -15b_1 = 1 \Rightarrow b_1 = -\frac{1}{15}$.

$\therefore a_p = -\frac{1}{15} 3^n$.

$\therefore a = c_1 \cdot (-2)^n + c_2 \cdot 6^n - \frac{1}{15} 3^n$.

e.g. $a_n - 3a_{n-1} = n$ for $n \geq 1$ and $a_0 = 1$.

① Solve for homo.

$a_n - 3a_{n-1} = 0 \Rightarrow a_{n+1} - 3a_n = 0 \Rightarrow (A-3)a_n = 0 \Rightarrow A = 3$.

$a_c = c_1 \cdot 3^n$.

Since $a_0 = 1$, gives $a_0 = c_1 \cdot 3^0 = 1 \Rightarrow c_1 = 1$.

$\therefore a_c = 3^n$.

② Solve for non-homo.

Let $a_p = b_1 n + b_2$.

$a_{n+1} - 3a_n = n \Rightarrow b_1(n+1) + b_2 - 3(b_1 n + b_2) = n$.

$\Rightarrow b_1 n + b_1 + b_2 - 3b_1 n - 3b_2 = n$.

$\Rightarrow n(b_1 - 3b_1) + (b_1 - 2b_2) = n$.

$$\begin{cases} b_1 - 3b_2 = 1 \\ b_1 - 2b_2 = 0 \end{cases} \Rightarrow \begin{cases} b_1 = -2 \\ b_2 = -1 \end{cases}$$

$\therefore a_p = -2n-1.$

$\therefore a_n = 3^n - 2n - 1.$