Sum, Supremum & Infimum

1. Sum Notation

1) The summation index is a 'dummy index'.

no intrinstic meaning.

e.g. $\sum_{j=1}^{3} \frac{j}{k} = \frac{1}{k} + \frac{2}{k} + \frac{3}{k} = \frac{6}{k}$.

$\sum_{k=1}^{3} \frac{j}{k} = \frac{j}{1} + \frac{j}{2} + \frac{j}{3} = \frac{11}{6}j$;

$\sum_{\mu=1}^{3} \frac{i}{k} = \frac{i}{k} + \frac{i}{k} + \frac{i}{k} = \frac{3i}{k}$.

2. Supremum. (To solve open interval issues).

1) Goal: maximum must to be in the interval.

① maximum: A number $c \in \mathbb{R}$ is the maximum of a set A when: $c \in A$; $\forall x \in A$, $x \le c$.

e.g. 2 is the maximum of the set $[0, 2]$.

2 <u>isn't</u> the maximum of $(0, 2)$ $(2 \notin (0,2))$.

2) Definition: (Set)

Let $A \subseteq \mathbb{R}$, Let $c \in \mathbb{R}$.

① c is an upper bound of A $(\forall x \in A,\ x \le c)$

② c is the least upper bound (lub) or supremum (sup) of A (c is an upper bound of A, and if b is an upper bound of A, the $c \le b$).

③ If the supremum of A is in A, it's called maximum

④ A is bounded above means it has (at least) one upper bound.

e.g.

set	upper bounds	supremum	maximum	bounded above?
$[0, 2]$	2, 2.1, ...	2	2	✓

$(0,2)$	$2, 2.1, \ldots\ldots$	2	X	✓
$\mathbb{Z}$	X	X	X	X

3. Infimum

1) Definition: (Set).

Let $A \subseteq \mathbb{R}$, Let $c \in \mathbb{R}$.

① c is an lower bound of A ($\forall x \in A$, $x \geq c$)

② c is the least lower bound (llb) or infimum (inf) of A (c is an lower bound of A, and if b is an lower bound of A, the $c \geq b$).

③ If the infimum of A is in A, it's called minimum.

④ A is bounded below means it has (at least) one lower bound.

4. Bounded.

1) 'A set is bounded' means it is both bounded above and bounded below.

1) [Theorem] The L.U.B Principle.

Let $A \subseteq \mathbb{R}$. If

① A is bounded above, and.

② A is not empty.

Then A has a least upper bound.

consequence

5. Suprenum of a function (suprenum of its range).

1). Suprenum of a function f (on domain I)

$= \sup \{ f(x) \mid x \in I \}$.

$= \sup\limits_{x \in I} f(x)$.

2) [Theorem] Consequence of the LUB Principle

Let f be a function defined on a domain $I \neq \emptyset$

① If f is bounded above on I.

② Then f has a supremum on I.