CSC236 Problem Set 2  Question 2.  Question 2.  Since we count the root of the time as $h=1$ , when we want the time has height $h$ , we should have at least one substree with height $h=1$ , the other would be at height $h'$ .  Case 1: both are at height $h=1$ , resulting in hours $h_1 \times h_2 + h_3 \times h_4 + h_4 \times h_5 \times h_4 \times h_5 \times h_4 \times h_5 \times h_5$			626	.236 7	rollen	Set -	2		
(a). Let $h$ $g$ $lN$ , let $g$									
Since we count the toot of the true as $h=1$ . When we want the tree has height $h$ , we should have at least one subtree with height $h-1$ , the other would be at height $h'$ .  Case 1: both are at height $h-1$ , resulting in having $bn_1 \times bn_2 + combinations$ .  Case 2: the other one is not at height $h-1$ , which means, $0 \le h' < h-1$ , since $h \in \mathbb{N}$ . The combinations can be written as: $\frac{n}{2}bi$ Since the subtree with height $h-1$ should also be counted the combination. $bn_1 \times \frac{n^2}{2}bi$ Since according to the question, mirror images are considered to be different, gives.  2 $bn_1 \times \frac{n^2}{2}bi$ combinations.  Solding Case 1 and Case 2 together gives, $bn_2 = bn_1^2 + 2bn_1 \times \frac{n^2}{2}bi$ , which. $bn_1^2 + 2bn_1 \times \frac{n^2}{2}bi$ Whe $bn_1^2 + 2bn_1 \times \frac{n^2}{2}bi$ Whe $bn_1^2 + 2bn_1^2 \times \frac{n^2}{2}bi$ When $bn_1^2 + 2bn_1^2 \times \frac{n^2}{2}bi$ When $bn_1^2 + 2bn_1^2 \times \frac{n^2}{2}bi$ Since $an_1^2 - an_1^2 - an_1^2 - an_1^2 + an_1^2$				Questio	n 2.				
should have at least one subtree with height $h-1$ , the other would be at height $h'$ .  Case 1: both are at height $h-1$ , resulting in having $bh_1 \times bh_{-1}$ combinations.  Case 2: the other one is not at height $h-1$ , which means, $0 \le h' < h-1$ , since $h \in \mathbb{N}$ . The combinations can be written as: $\sum_{k=0}^{\infty} b_k$ :  Stree the subtree with height $h-1$ should also be considered to be different, $\sum_{k=0}^{\infty} b_k$ .  Since according to the question, mirror images are considered to be different, gives.  2-bn/ $\times \stackrel{k-1}{\rightleftharpoons} b_k$ combinations.  Addling Case 1 and Case 2 together gives, $bh = bh_1^2 + 2 bh_1 \times \stackrel{k-1}{\rightleftharpoons} b_k$ , which. $h=0$ $bh$ 1 $bh_1^2 + 2 bh_1 \times \stackrel{k-2}{\rightleftharpoons} b_k$ $\forall h \in \mathbb{N}$ , $h \ge 1$ .  (b) Let $h \in \mathbb{N}$ .  WTP $\forall h \in \mathbb{N}$ , $bh_1 = ah_1^2 - ah_1^2$ .  Fase Case: $h = 0$ . or $h=1$ .  when $h=0$ , $from$ (a). $b_1=1$ .  Since $ab=1$ , $a_1=ab^2-ab^2$ .  when $h=1$ , $from$ (a). $b=1$ : $ab=1$	(a). Let h	e M, let	be be the	: number e	f elements c	of T of he	right h.		
should have at least one subtree with height h-1, the other would be at height h'.  Case 1: both are at height h-1, resulting in having $\delta_{h,1} \times \delta_{h-1}$ combinations.  Case 2: the other one is not at height h-1, which means, $0 \le h' < h-1$ , since hell. the combinations can be written as: $\sum_{j=0}^{k-2} b_j$ .  Since the subtree with height h-1 should also be consted the combination: $\delta_{h-1} \times \sum_{j=0}^{k-2} b_j$ .  Since according to the question mirror images are considered to be different, gives.  2 by $\times \sum_{j=0}^{k-2} b_j$ combinations.  Addling Case 1 and Case 2 logether gives, $\delta_h = \delta_{h-1}^2 + 2 \delta_{h-1} \times \sum_{j=0}^{k-2} b_j$ , which. $\delta_h = 0$								he tree ha	s height h,
Case 1: both are at height $h-1$ , resulting in having $bh_1 \times bh_1$ combinations.  Case 2: the other one is not at height $h-1$ , which means, $0 \le h' < h-1$ , since $h \in M$ , the combinations can be unitten as: $\sum_{j=0}^{k-2} b_j$ .  Since the subtree with height $h-1$ should also be counted, the combination: $bh_1 \times \sum_{j=0}^{k-2} b_j$ .  Since according to the question, mirror images are considered to be different, gives.  2: $bh_1 \times \sum_{j=0}^{k-2} b_j$ combinations.  Solding Case 1 and Case 2 together gives, $bh_1 = bh_1^2 + 2 \cdot bh_1 \times \sum_{j=0}^{k-2} b_j$ , which. $h=0$ $bh_1^2 + 2 \cdot bh_1 \times \sum_{j=0}^{k-2} b_j$ $h \in M$			·						
Cose 2: the other one is not at height h-1, which means, $0 \le h' < h-1$ , since $h \in \mathbb{N}$ . The combinations can be written as: $\sum_{k=0}^{h-2}b_k$ .  Since the subtree with height h-1 should also be counted the combination: $bh = x = b_k$ bits being according to the question mirror images are considered to be different, gives.  2 $bh_1 \times \frac{h^2}{5}b_k$ combinations.  Adding Case I and Cose 2 together gives, $bh = bh_1^2 + 2 bh_1 \times \frac{h^2}{5}b_k$ , which. $h=0$ $bh$ $h=0$ $h=1$ $h$					T				
Combinations can be written as: $\frac{2}{100}bi$ Since the subtree with height $h-1$ should also be counted the combination: $bh-1 \times \frac{h^2}{100}bi$ Since according to the question, mirror images are considered to be different, gives.  2 $bh_1 \times \frac{h^2}{100}bi$ combinations.  Ackling Case I and Cose 2 together gives, $bh = bh_1^2 + 2 bh_1 \times \frac{h^2}{100}bi$ , which. $h=0$ $bh$ $h=1$ $bh_1^2 + 2 bh_1 \times \frac{h^2}{100}bi$ $h=0$ $h=1$ $h=0$ $h=1$ $h=0$ $h=1$ $h=0$ $h$					·				
Since the subtree with height $\Lambda$ -1 should also be counted, the combination: $bh_1 \times \sum_{i=0}^{\infty} b_i$ Since according to the question, mirror images are considered to be different, gives.  2 $bn_1 \times \sum_{i=0}^{\infty} b_i$ combinations.  Solding Case   and Case 2 together gives, $bh = bh_1^2 + 2 bh_1 \times \sum_{i=0}^{\infty} b_i$ , which. $h = 0$ $h = 1$ $h$				1.72					
Since according to the question, mirror images are considered to be different, gives.  2 bn/x $\stackrel{k=2}{\geq}$ bi combinations.  Adding Case I and Case 2 together gives, $bh = bh = 1$ + 2 $bh = 1$ x $\stackrel{k=2}{\geq}$ bi, which. $h = 0$ $bh = 1$		Since the	subtree with	height h-	: Should a	lso be cou	nted, the	combination:	bh-1 x Z b
2. $b_{1} \times \frac{2}{120}b_{1}$ combinations.  Adding Case I and Case 2 together gives, $b_{1} = b_{1}^{2} + 2 \cdot b_{1} + x \cdot \frac{b^{2}}{120}b_{1}$ , which. $b_{1} = b_{1}^{2} + 2 \cdot b_{1} + x \cdot \frac{b^{2}}{120}b_{1}$ $\forall h \in M, h \geq 1$ .  (b) Let $h \in M$ . $b_{1} = a_{1}^{2} - a_{1}^{2}$ $b_{2} = a_{2} = a_{2} = a_{2}^{2}$ $b_{3} = a_{2} = a_{3}^{2} - a_{1}^{2}$ $b_{4} = a_{2} = a_{2}^{2} - a_{2}^{2}$ $b_{5} = a_{5} = a_{5} = a_{5}^{2} - a_{5}^{2}$ $b_{5} = a_{5} = a_{5}^{2} - a_{5}^{2}$ $b_{5} = a_{5} = a_{5}^{2} - a_{5}^{2}$ $b_{5} = a_{5}^{2} + a_{5}^{2} - a_{5}^{2} = a_{5}^{2}$		Since accor	ding to the ga	uestion, nij	TOT Emages	are cons	sidered ti	be diffe	rent, gives.
Adding Case I and Case 2 together gives, $bh = bh_1^2 + 2bh_1 \times \sum_{i=0}^{2} b_i$ , which.  S I $bh \mid I$ $bh_1^2 + 2bh_1 \times \sum_{i=0}^{2} b_i$ $bh \mid Mh \mid $			2.144	/ > bi co	whin a time				
bh   1	Adding	Case 1 and	d Cose 2 .	together a	ives, bh	= bh-1 <sup>2</sup> +	2. bh-1 ×	$\frac{h^{-2}}{\sum_{i=1}^{n}b_{i}}$	lich.
(b) Let he M.  LOTP: Whe W, bht = $a_{rj}^2 - a_n^2$ Base Case: $h = 0$ or $h = 1$ .  Lohen $h = 0$ , From (a). $b_1 = 1$ .  Since $a_0 = 1$ , $a_1 = a_0^2 + 1 = 1$ , which $a_1^2 - a_0^2 = 1^2 - 0^2 = 1$ .  Thus $b_1 = 1 = a_1^2 - a_0^2$ .  Lohen $h = 1$ , From (a). $b_2 = b_1^2 + 2 \cdot b_1 \cdot \frac{2}{100} b_1 = 1^2 + 2 \cdot 1 \cdot 1 = 3$ . $a_3^2 - a_1^2 = (a_1^2 + 1)^2 - a_1^2 = ((a_0^2 + 1)^2 + 1)^2 - (a_0^2 + 1)^2$ $= (1^2 + 1)^2 - 1 = 3$ .		\		/		/		120	
(b) Let he M.  LOTP: Whe W, bht = $a_{r}^{2} - a_{r}^{2}$ Base Case: $h = 0$ or $h = 1$ .  Lohen $h = 0$ , From (a). $b_{1} = 1$ .  Since $a_{0} = 1$ , $a_{1} = a_{0}^{2} - a_{0}^{2}$ .  When $h = 1$ , From (a). $h = 1$ is $h = 1$ .  Lohen $h = 1$ , From (a). $h = 1$ is $h = 1$ is $h = 1$ in $h = 1$ . $a_{2}^{2} - a_{1}^{2} = (a_{1}^{2} + 1)^{2} - a_{2}^{2} = ((a_{2}^{2} + 1)^{2} + 1)^{2} - (a_{0}^{2} + 1)^{2} = (1^{2} + 1)^{2} - 1 = 3$ .	Son			h=1					
(b) Let he M.  LOTP: Whe W, bht = $a_{rj}^2 - a_n^2$ Base Case: $h = 0$ or $h = 1$ .  Lohen $h = 0$ , From (a). $b_1 = 1$ .  Since $a_0 = 1$ , $a_1 = a_0^2 + 1 = 1$ , which $a_1^2 - a_0^2 = 1^2 - 0^2 = 1$ .  Thus $b_1 = 1 = a_1^2 - a_0^2$ .  Lohen $h = 1$ , From (a). $b_2 = b_1^2 + 2 \cdot b_1 \cdot \frac{2}{100} b_1 = 1^2 + 2 \cdot 1 \cdot 1 = 3$ . $a_3^2 - a_1^2 = (a_1^2 + 1)^2 - a_1^2 = ((a_0^2 + 1)^2 + 1)^2 - (a_0^2 + 1)^2$ $= (1^2 + 1)^2 - 1 = 3$ .		12h-1 + )	1/2 x \( \frac{h-2}{2} \)	Yhe	W h>1.				
WTP: $\forall h \in \mathbb{N}$ , $b_{h+1} = a_{h+1}^2 - a_h^2$ .  \$\frac{1}{2} ase \text{Case}: \h = 0. \text{ or } \h = 1.  \text{Ushen } \hat{h} = 0, \text{ from } (a). \hat{b}_1 = 1.  \text{Since } a_0 = 1,  a_1 = a_0^2 + 1 = 1, \text{ which } a_1^2 - a_0^2 = 1^2 - 0^2 = 1.  \text{Thus. } \hat{b}_1 = 1 = a_1^2 - a_0^2.  \text{Ushen } \hat{h} = 1, \text{ from } (a). \hat{b}_2 = \hat{b}_1^2 + 2 \dots \hat{b}_1 \dots \frac{2}{10} \hat{b}_1 = 1^2 + 2 \dots 1 \dots 1 = 3.  \alpha^2 - a_1^2 = (a_1^2 + 1)^2 - a_1^2 = ((a_0^2 + 1)^2 + 1)^2 - (a_0^2 + 1)^2 = ((1^2 + 1)^2 - 1) = 3.			i=0	V	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,				
WTP: $\forall h \in \mathbb{N}$ , $b_{h+1} = a_{h+1}^2 - a_h^2$ .  \$\frac{1}{2} ase \text{Case}: \h = 0. \text{ or } \h = 1.  \text{Ushen } \hat{h} = 0, \text{ from } (a). \hat{b}_1 = 1.  \text{Since } a_0 = 1,  a_1 = a_0^2 + 1 = 1, \text{ which } a_1^2 - a_0^2 = 1^2 - 0^2 = 1.  \text{Thus. } \hat{b}_1 = 1 = a_1^2 - a_0^2.  \text{Ushen } \hat{h} = 1, \text{ from } (a). \hat{b}_2 = \hat{b}_1^2 + 2 \dots \hat{b}_1 \dots \frac{2}{10} \hat{b}_1 = 1^2 + 2 \dots 1 \dots 1 = 3.  \alpha^2 - a_1^2 = (a_1^2 + 1)^2 - a_1^2 = ((a_0^2 + 1)^2 + 1)^2 - (a_0^2 + 1)^2 = ((1^2 + 1)^2 - 1) = 3.	16 Yet	heW							
Base Case: $h = 0$ . or $h = 1$ .  Ushen $h = 0$ , From (a). $b_1 = 1$ .  Since $a_0 = 1$ , $a_1 = a_0^2 + 1 = 1$ , which $a_1^2 - a_0^2 = 1^2 - 0^2 = 1$ .  Thus $b_1 = 1 = a_1^2 - a_0^2$ .  When $h = 1$ , From (a). $b_2 = b_1^2 + 2 \cdot b_1 \cdot \sum_{i=0}^{2} b_i = 1^3 + 2 \cdot 1 \cdot 1 = 3$ . $a_2^2 - a_1^2 = (a_1^2 + 1)^2 - a_1^2 = ((a_0^2 + 1)^2 + 1)^2 - (a_0^2 + 1)^2$ $= (1^2 + 1)^2 - 1 = 3$ .			\$h+1 = ah+1 =	- Q4					
when $h=0$ , From $(a)$ . $b_1 = 1$ .  Since $a_0 = 1$ , $a_1 = a_0^2 + 1 = 1$ , which $a_1^2 - a_0^2 = 1^2 - 0^2 = 1$ .  Thus, $b_1 = 1 = a_1^2 - a_0^2$ .  when $h=1$ , From $(a)$ . $b_2 = b_1^2 + 2 \cdot b_1 \cdot \sum_{i=0}^{n} b_i = 1^2 + 2 \cdot 1 \cdot 1 = 3$ . $a_2^2 - a_1^2 = (a_1^2 + 1)^2 - a_1^2 = ((a_0^2 + 1)^2 + 1)^2 - (a_0^2 + 1)^2$ $= (1^2 + 1)^2 - 1 = 3$ .	\$ 100	nea. h -	1 - h -	- 1					
Since $a_0 = 1$ , $a_1 = a_0^2 + 1 = 1$ , which $a_1^2 - a_0^2 = 1^2 - 0^2 = 1$ .  Thus $b_1 = 1 = a_1^2 - a_0^2$ .  when $b_1 = 1$ , from $b_2 = b_1^2 + 2 \cdot b_1 \cdot \sum_{i=0}^{n} b_i = 1^2 + 2 \cdot 1 \cdot 1 = 3$ . $a_2^2 - a_1^2 = (a_1^2 + 1)^2 - a_1^2 = (a_0^2 + 1)^2 + 1)^2 - (a_0^2 + 1)^2$ $= (1^2 + 1)^2 - 1 = 3$ .									
Thus $b_1 = 1 = a_1^2 - a_0^2$ .  When $h=1$ , from $(a)$ . $b_2 = b_1^2 + 2 \cdot b_1 \cdot \sum_{j=0}^{9} b_j = 1^2 + 2 \cdot 1 \cdot 1 = 3$ . $a_2^2 - a_1^2 = (a_1^2 + 1)^2 - a_1^2 = ((a_0^2 + 1)^2 + 1)^2 - (a_0^2 + 1)^2$ $= (1^2 + 1)^2 - 1 = 3$ .	Cortain 7C-	J		- 12 <sup>2</sup> -+1 -	1 . 1.20/	2 - 2	- 1 <sup>2</sup> 0	ر _ د	
when $h=1$ , from $(a)$ . $b_2 = b_1^2 + 2 \cdot b_1 \cdot \sum_{j=0}^{0} b_j = 1^2 + 2 \cdot 1 \cdot 1 = 3$ . $a_2^2 - a_1^2 = (a_1^2 + 1)^2 - a_1^2 = ((a_2^2 + 1)^2 + 1)^2 - (a_0^2 + 1)^2$ $= (1^2 + 1)^2 - 1 = 3$ .					I, wach	<i>a</i> <sub>1</sub> <i>c</i> <sub>0</sub>	- 1 - 0	71.	
$a_{3}^{2} - a_{4}^{2} = (a_{4}^{2} + 1)^{2} - a_{4}^{2} = ((a_{6}^{2} + 1)^{2} + 1)^{2} - (a_{6}^{2} + 1)^{2}$ $= (1^{2} + 1)^{2} - 1 = 3.$			<b>'</b>	n	R 1 <sup>2</sup> + ).	1.1 - 2			
$= (1^{2} + 1)^{2} - 1 = 3.$	when h	i l		•			•		
		$u_{s} - u_{q} =$	(ay + 1) - ay						
$/ms$ $v_2 = z = u_2 - u_3$		71 0	_ 2 _ 2		7 1 = 3.				
L've proved the base case is true.	0.			<b>'</b>					

Induction Step: Let he M. h > 0.
Induction Hypothesis. Let LEW, 0 < l <h, -="" ai<="" but="airy" td=""></h,>
$LOTP$ : $Unt_1 = ant_1 - ai$
From (a), gives, $b_1 = a_1^2 - a_0^2$
bhy = bh2+2.bh x = bi (fry l.H., take ==h-1, gives, bh = ah2-ah)
$\Rightarrow b_{h+j} = (a_h^2 - a_{h-i}^2)^2 + 2 \cdot (a_h^2 - a_{h-j}^2) \cdot (b_0 + b_1 + \dots + b_{h-1}) (f_{yy} l + h \cdot l \in [0, h), \text{ take } l_2 = h + 2.$
$\Rightarrow bh+1 = (ah^2 - ah^2)^2 + 2 - (ah^2 - ah^2) \cdot (1 + a_1^2 - a_0^2 + a_2^2 - a_1^2 + a_3^2 - a_2^2 + \dots + a_{h-1}^2 - a_{h-2}^2)  (l_3 = h-3 - \dots - l_{h-1} = 0, gives$
$\Rightarrow b_{h+1} = (a_h^2 - a_{h-1}^2)^2 + 2 (a_h^2 - a_{h-1}^2) \cdot (a_h^2 + 1) \cdot (According to def = of a_{h+1} = a_h^2 + 1).$
$\Rightarrow b_{h+1} = (a_{h+1} - 1 - (a_h - 1))^{2} + 2 \cdot (a_{h+1} - 1 - (a_h - 1)) \cdot (a_h - 1 + 1).$
$\Rightarrow b_{h+1} = (a_{h+1} - a_h)^2 + 2 \cdot (a_{h+1} - a_h) \cdot a_h$
$\Rightarrow bh+j = ah+j + ah-2ah+j ah+2ah+j ah-2ah^{2}$
$\Rightarrow bhet = aht - ah$
l've proved the inclusion step is true.
Therefore, buy = any - and for he M.