Derivative of Trigonometry

1. Overall

1) $\frac{d}{dx}\sin x = \cos x$

2) $\frac{d}{dx}\tan x = \sec^2 x$.

3) $\frac{d}{dx}\sec x = \sec x \tan x$.

4) $\frac{d}{dx}\cos x = -\sin x$.

5) $\frac{d}{dx}\cot x = -\csc^2 x$.

6) $\frac{d}{dx}\csc x = -\csc x \cot x$.

2. Plan.

1) Obtain $\frac{d}{dx}(\sin x) = \ldots$ from definition.

2) Use $\cos x = \sin(\frac{\pi}{2} - x)$ to obtain $\frac{d}{dx}\cos x = \ldots$

3) Obtain the other four derivatives from

$\tan x = \frac{\sin x}{\cos x}$, $\sec x = \frac{1}{\cos x}$, ……

3. Proof: $f(x) = \sin x$.

$$f'(x) = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x\cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \left[\sin x \cdot \frac{\cos h - 1}{h} + \cos x \cdot \frac{\sin h}{h}\right]$$

$$= \sin x \lim_{h\to 0}\left[\underbrace{\frac{\cos h - 1}{h}}_{\to 0}\right] + \cos x \cdot \lim_{h\to 0}\left[\underbrace{\frac{\sin h}{h}}_{\to 1}\right]$$

$$= \cos x$$