Online Quiz 5

1. Let $p.q$ be prime number; Let $n>1$.
Assume $p|n$ and $q|n$
WTS: $p\cdot q|n$.
Since $p|n$, gives $\exists k_1 \in \mathbb{Z}$. s.t. $n=p\cdot k_1$ (from def^{n} of divisibility).
Since $q|n$. gives $\exists k_2 \in \mathbb{Z}$. s.t. $n=q\cdot k_2$, gives. $p\cdot k_1 = q\cdot k_2$; Since $p\neq q$. gives $k_1 \neq k_2$.
Since $p.q$ are prime. $p\neq q$, gives. $\exists a \in \mathbb{Z}$. s.t. $k_1 = a\cdot q$, which. $p\cdot a\cdot q = q\cdot k_2$.
(Also. $\exists b \in \mathbb{Z}$, s.t. $k_2 = b\cdot p$, which $p\cdot k_1 = q\cdot b\cdot p$).
Since $q\cdot k_2 = n$, gives. $n = p\cdot a\cdot q = a\cdot(p\cdot q)$. which $p\cdot q|n$.

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2. Let $a\equiv r \pmod p$, $a \equiv r \pmod q$.
WTS: $a\equiv r \pmod{pq}$.
According to the definition, $\exists k_1 \in \mathbb{Z}$. s.t. $a = r+k_1p$; $\exists k_2 \in \mathbb{Z}$. s.t. $a=r+k_2q$, gives.
$r+k_1p = r+k_2q$, according to the cancellation rules:
$\Rightarrow \quad k_1p = k_2q$.
Similar to Q1, $\exists b \in \mathbb{Z}$. s.t. $k_1 = bq$ which. $p\cdot b\cdot q = q\cdot k_2$.
Since $a = r+k_2\cdot q = r+b\cdot(p\cdot q)$, which $a\equiv r \pmod{p\cdot q}$.

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3. Let $n\equiv 14 \pmod{17}$; $n\equiv 14 \pmod{19}$; $n \equiv 14 \pmod{23}$. gives.
From Q2, gives. $n\equiv 14 \pmod{\underset{17\times 19}{\underset{\downarrow}{323}}}$; $n \equiv 14 \pmod{\underset{17\times 23}{\underset{\downarrow}{391}}}$; $n\equiv 14 \pmod{\underset{19\times 23}{\underset{\downarrow}{437}}}$.
Thus. the possible combination from 4000 to 11000 of $n\equiv 14 \pmod{323}$ are:
$n = 14+323\times 13 = 4213$; 4536; 4859; ……; 7443; 7766; ……; 10996; ~~11319~~
$n = 14+391\times 11 = 4315$; 4706; 5097; ……; 7443; 7834; ……; 10962; ~~11353~~
$n = 14+437\times 10 = 4384$; 4821; 5258; ……; 7443; 7880; ……; 10939; ~~11376~~
Therefore $n = 7443$ is the only such number n according to Q2.