Integrals as Limits.

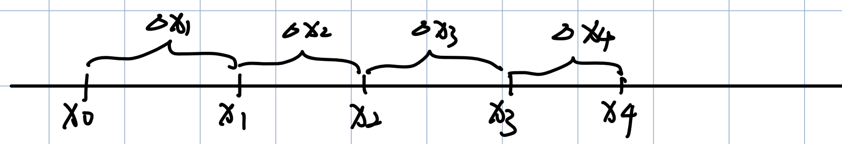
1. Norm of Partition

Let $P = \{x_0, x_1, x_2, \dots, x_n\}$ be a partition of $[a, b]$.

For each i, let $\Delta x_i = x_i - x_{i-1}$.

The norm of P is:

$$\|P\| = \max\{\Delta x_1, \Delta x_2, \dots, \Delta x_n\}$$



We want to compute limit as $\|P\| \to 0$.

2. Reframing Def.

1) $\underline{I}_a^b(f) = \sup\{L_P(f) \mid P \text{ is a partition of } [a, b]\}$

$\underline{I}_a^b(f) = \lim_{\|P\| \to 0} L_P(f)$.

↳ $\forall \varepsilon > 0, \exists \delta > 0, \forall$ partition P of $[a, b]$,

$\|P\| < \delta \Rightarrow |\underline{I}_a^b(f) - L_P(f)| < \varepsilon$.

$\overline{I}_a^b(f) = \inf\{U_P(f) \mid P \text{ is a partition of } [a, b]\}$.

$\overline{I}_a^b(f) = \lim_{\|P\| \to 0} U_P(f)$.

2) Pick a sequence of partitions $P_1, P_2, P_3, \dots\dots$ satisfying

$$\lim_{n \to \infty} \|P_n\| = 0.$$

(e.g. P_n: break the interval $[a, b]$ into n subintervals of equal length).

Then $\underline{I}_a^b(f) = \lim_{n \to \infty} L_{P_n}(f)$.

$\overline{I}_a^b(f) = \lim_{n \to \infty} U_{P_n}(f)$.