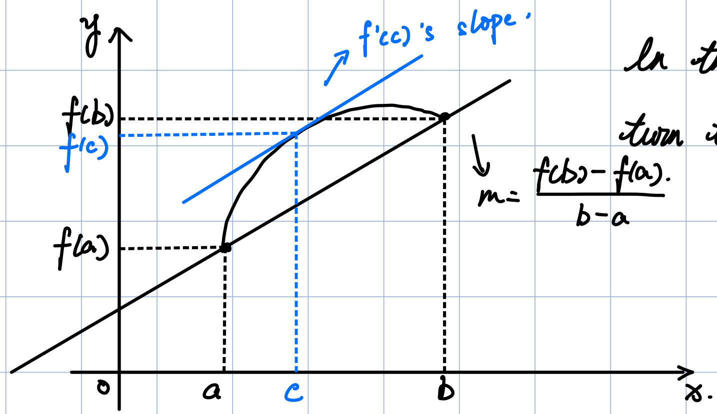
Mean Value Theorem

(Generalize Rolle's Theorem).

1. Idea.



In this case, $f(a) \neq f(b)$. however, we can turn the pic. around.

2. MVT

Let $a<b$. Let f be a function defined on $[a, b]$.

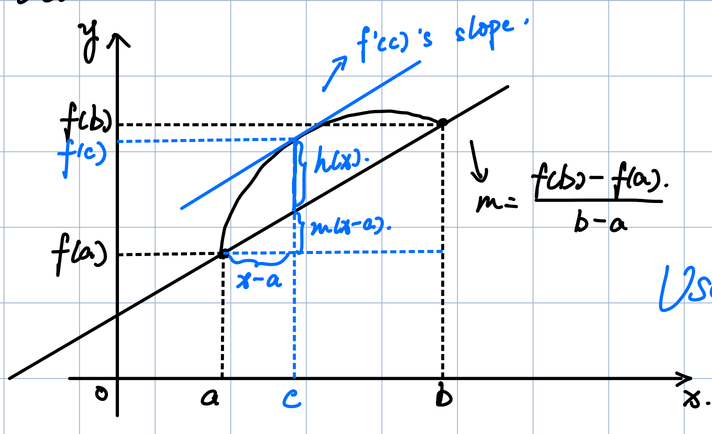
If ① f continuous on $[a, b]$.

② f differentiable on (a, b).

Then $\exists c \in (a, b)$ s.t. $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.

3. Proof.

1) Idea.



Use Rolle's Theorem on:

$h(x) = f(x) - f(a) - m(x-a)$.

2). Proof:

Let $m = \dfrac{f(b)-f(a)}{b-a}$. Define a new function h on $[a, b]$ by.

$$h(x) = f(x) - f(a) - m(x-a).$$

Apply Rolle's Theorem to h on $[a, b]$

H①: $h(x)$ is continuous on $[a, b]$.

$H②$: $h(x)$ is differentiable on (a, b).

$H③$: $h(a) = h(b) = 0$.

Therefore. $\exists c \in (a, b)$. s.t. $h'(c) = 0$.

$h'(x) = f'(x) - m$.

$\therefore f'(c) = m = \dfrac{f(b) - f(a)}{b - a}$.