Summary of Convergence Tests for Series
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Test	When to Use	Conclusions
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ if $\|r\| < 1$; diverges if $\|r\| \geq 1$.
Necessary Condition	All series	If $\lim_{n\to\infty} a_n \neq 0$, then the series diverges.
Integral Test	• $a_n = f(n)$ • f is continuous, positive and decreasing. • $\int_1^{\infty} f(x)\,dx$ is easy to compute.	$\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$ both converge or both diverge.
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges for $p > 1$; diverges for $p \leq 1$.
Basic Comparison Test	$0 \leq a_n \leq b_n$	If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges
Limit Comparison Test	$a_n, b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ $(0 < L < \infty)$	$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge or both diverge.
Alternating Series Test	$\sum_{n=1}^{\infty} (-1)^n b_n$, $b_n > 0$	If • $b_n > 0$, $\forall n$ • $\{b_n\}$ is decreasing • $\lim_{n\to\infty} b_n = 0$ Then $\sum_{n=1}^{\infty} (-1)^n b_n$ is convergent.
Absolute Convergence	Series with some positive terms and some negative terms (including alternating series)	If $\sum_{n=1}^{\infty} \|a_n\|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges (absolutely).
Ratio Test	Any series (especially those involving exponentials and/or factorials)	For $\lim_{n\to\infty} \left\| \frac{a_{n+1}}{a_n} \right\| = L$ (including $L = \infty$), • If $L < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely • If $L > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges • If $L = 1$, then we can draw no conclusion.