# First Order Linear ODE.

1. First Order Linear: linear $y$ function of $t$.

1). Std. Form: $\frac{dy}{dx} + p(x)y = g(x)$

2. $Sol^{n}$ to FOLODE.

e.g. $x\ln(x)\,y' + y = xe^x$

若 $y' - p(x)\cdot y = g(x)$.

$p^*(x)$. 须包括负号.

1). Transform to std. form: $\frac{dy}{dx} + p(x)y = g(x)$

$y' + \frac{1}{x\ln x}y = \frac{e^x}{\ln x}$.

2). Calculate Integrating Factor: $\mu(x) = e^{\int p(x)dx}$. 这里可不用加 C.

$\mu(x) = e^{\int \frac{1}{x\ln x}dx} = e^{\ln(\ln(x))} = \ln x$.

3). Multiply $\mu(x)$ on both sides: $\mu(x)\cdot\frac{dy}{dx} + p(x)\cdot\mu(x)\cdot y = g(x)\cdot\mu(x)$.

$(\ln(x)\cdot y)' = \frac{e^x}{\ln x}\cdot\ln x$.

$\Updownarrow$

$(\mu(x)\cdot y)' = g(x)\cdot\mu(x)$.

4). Integral on both sides: $\mu(x)\cdot y = \int g(x)\cdot\mu(x)\,dx$.

$\ln(x)\cdot y = \int e^x dx$.

5). Solve it. (Don't forget C): $\mu(x)\cdot y = F(x) + C$.

$\ln(x)\cdot y = e^x + \underline{\underline{C}}$. ← 在这里 +C.

6). Organize: $y = \frac{F(x)+C}{\mu(x)}$.

$y = \frac{e^x + C}{\ln(x)}$.

7). substitute I.V. if there is.

e.g. $y' - y = 2te^{2t}$ with $y(0) = 1$.

$p(t) = -1$.

$\mu(t) = e^{\int p(t)dt} = e^{\int -1\,dt} = e^{-t}$.

$(e^{-t}\cdot y)' = 2te^{2t}\cdot e^{-t}$

$\Rightarrow e^{-t}\cdot y = 2\int t\cdot e^t\,dt$.

$\int u\frac{dv}{dt}dt = uv - \int v\cdot\frac{du}{dt}dt$. let $u = t$. $\frac{dv}{dt} = e^t \Rightarrow \frac{du}{dt} = 1$. $v = e^t$

$= t\cdot e^t - \int e^t dt = 2e^t(t-1) + C$.

$$\Rightarrow \quad e^{-t} \cdot y = 2e^{t}(t-1) + c.$$

$$\Rightarrow \quad y = 2e^{2t}(t-1) + e^{t} \cdot C.$$

$$y(0) = 2e^{0}(-1) + e^{0} \cdot C = 1.$$

$$\Rightarrow. \quad -2 + C = 1$$

$$\Rightarrow \quad C = 3$$

$$\therefore \ y = e^{2t}(t-1) + 3 \cdot e^{t}$$