3. WOP.

a) Apply the WOP to prove that there cannot be an infinite strictly decreasing sequence of natural numbers.

Let $S = \{n \in \mathbb{N}$: infinite strictly decreasing sequence starting with $n\}$, which $S \subseteq \mathbb{N}$.
WTS: $S = \phi$.
I'll use prove by contradiction. Assume $S \neq \phi$.
Since $S \neq \phi$, $S \subseteq \mathbb{N}$, according to WOP, there exist $n_0 \in S$ s.t. $\forall s' \in S$, $n_0 \leq s'$
However, Since S is a strictly decreasing sequence, gives, starting from n_0, we have, $n_0 > n_0' > n_0'' > \cdots > n_0^{'\cdots'}$ in S, contradicts to $\forall s' \in S$, $n_0 \leq s'$ (e.g when $s' = n_0'$ it contradicts).
Therefore, $S = \phi$., which there can not be an infinite strictly decreasing sequence of natural numbers.

b) Apply WOP to prove that, for any integers n and $d > 0$, there are unique integers $r \in \{0, 1, 2, \ldots, d-1\}$ and q such that

$$n = dq + r.$$

Note: How is this a proof of the fact given in the paragraph preceeding Theorem 3.1.4?

Let $n \in \mathbb{Z}$. Let $d \in \mathbb{Z}$., $d > 0$
WTP: ① $r \in \{0, 1, 2, \ldots, d-1\}$, q exists ② r, d is unique.
Proof: ① Since $d \in \mathbb{Z}$, $d > 0$, gives $d \in \mathbb{N}$.
Let $S = \{n - dq$, where $q \in \mathbb{Z}$, $n - dq \geq 0\}$, I'll show S is not an empty set, and $S \subseteq \mathbb{N}_0$.
Case ① $n \geq 0$..
When $n \geq 0$, take $q \in \mathbb{N}_0$. Since $n - dq \geq 0$, gives $n - dq \in \mathbb{N}_0$., which $S \subseteq \mathbb{N}_0$. Take $q = 0$. gives $n - dq = n \geq 0$, which is in S, gives $S \neq \phi$.
Case ② $n < 0$.
When $n < 0$. take $q = n$, $q \in \mathbb{Z}$. $q < 0$. Since $d > 0$. $d \in \mathbb{N}$, gives $n - dq = n - d \cdot n = (1-d)n \geq 0$. which is in S, gives $S \neq \phi$. Since $n - dq \geq 0$, gives $n - dq \in \mathbb{N}_0$. which $S \subseteq \mathbb{N}_0$.
Thus, I'll use WOP on S as S is non-empty and $S \subseteq \mathbb{N}_0$.
By WOP, S contains smallest element called r', gives $r' \geq 0$.
Since $r' \in S$, gives $\exists q' \in \mathbb{N}$ s.t. $r' = n - d \cdot q'$, which $n = r' + d \cdot q'$.
WTP: $r' < d$.
I'll use prove by contradiction.
Assume that $r' \geq d$, gives $r' - d \geq 0$
Since $r' - d = n - d \cdot q' - d = n - d \cdot (q' + 1)$. Since $q' + 1 \in \mathbb{N}$, $d \in \mathbb{N}$, gives $r' - d \in S$, called $r'' = r' - d$
Since $d > 0$, $d \in \mathbb{N}$. gives $r'' < r'$. contradicts to r' is the smallest element in S.
Therefore. I've proved the existence of $r \in [0, d)$. and q.

②. Since $d \in \mathbb{Z}$, $d > 0$, gives $d \in \mathbb{N}$
I'll prove by using contradiction
Assume. $\exists q_1, q_2 \in \mathbb{Z}$. $\exists r_1, r_2 \in [0, d-1)$, $r_1, r_2 \in \mathbb{Z}$. which $q_1 \neq q_2$, $r_1 \neq r_2$, that $n = d \cdot q_1 + r_1$ and $n = d \cdot q_2 + r_2$.
From assumption. gives. $d \cdot q_1 + r_1 = d \cdot q_2 + r_2$, which.
$d \cdot q_1 - d \cdot q_2 + r_1 - r_2 = 0 \Rightarrow d \cdot (q_1 - q_2) + (r_1 - r_2) = 0$.
Since from the definition of divisibility, $d \mid 0$, as $d > 0$. $d \in \mathbb{N}$.
Gives. $d \cdot (q_1 - q_2) + (r_1 - r_2)$ should also divides d, which $d \mid (r_1 - r_2)$.
Since $0 \leq r_1, r_2 < d$. gives $-d < r_1 - r_2 < d$, which when $r_1 - r_2 = 0$. $d \mid (r_1 - r_2)$, that $r_1 = r_2$.
Since $r_1 - r_2 = 0$, $d > 0$, $d \in \mathbb{N}$. gives $q_1 - q_2 = 0$, which $q_1 = q_2$.
Thus. contradicts to $r_1 \neq r_2$, $q_1 \neq q_2$.
Therefore. I've prove the uniqueness.