CSC236 Problem Set 2

Question 4.

(a). After the last occurance of a, if a is at the last or the number of bs after last occurance of a are even, then I end up in circle L, otherwise, I'm end up in R.

(b). Let S be the set of all finite strings containing only as and/or bs.

Let $s \in S$. Let $P(s)$ be the claim in a.

Base Case: $s = \varepsilon$. or $s = a$ or $s = b$.

when $s=\varepsilon$, Since ε refer to an empty string, we do not move from the starting point, which is L.

when $s=a$, since a is at the last, we're in L, matches the situation.

when $s=b$. since there's no occurance of a and the number of bs is 1, we're in R, matches.

Induction Step: Let $s \in S$.

Induction Hypothesis: Assume $P(s)$. WTP. $P(sa)$ and $P(sb)$.

Case 1. $P(sa)$.

Since 'a' represents stay in or move to L, no matter what 's' is, we now at L, satisfying $P(sa)$.

Case 2. $P(sb)$

① s end with an even number of bs.

Since, from I.H., 'a' is at last or the number of bs after the last occurance of a are even, adding a 'b' causes the number of bs after the last occurance of 'a' be odd. And since 'b' represent move to the circle not standing in and I'm standing in L according to I.H. I'll then move to R, which in this condition still holds

② s end with an odd number of bs.

Since, from I.H., 'a' is at last or the number of bs after the last occurance of a are even, adding a 'b' causes the number of bs after the last occurance of 'a' be odd. However. since the number of bs after 'a' is odd, adding a 'b' will cause the num of bs after 'a' to be even. Since when the number of bs are odd, I'm at R, according to I.H. And since 'b' represent move to the circle not standing in and I'm standing in R according to I.H. I'll then move to L, which in this condition still holds.

Therefore. I've proved the claim from (a). ∎