Online Quiz 9.

1. Since $z^7+z^6+z^5+z^4+z^3+z^2+z+1=0$, from long division gives.

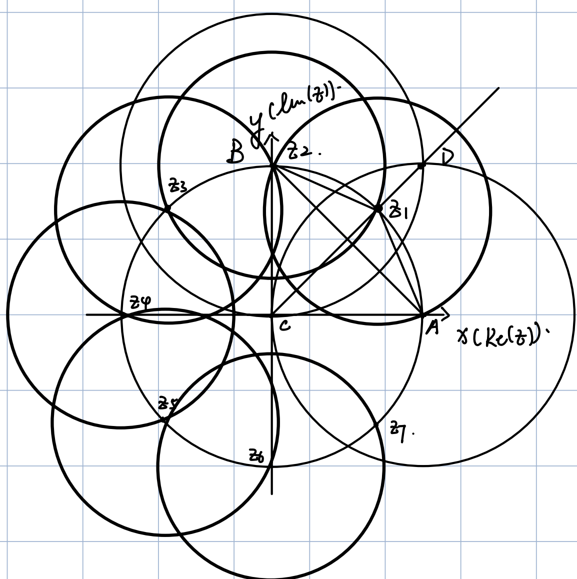
$$z^7+z^6+z^5+z^4+z^3+z^2+z+1 = \frac{z^8-1}{z-1} = 0.$$

Since $z-1 \neq 0$ as it's the denominator, $z^8-1=0$, which $z^8=1$.

Thus, we need to solve $z^8=1$, 8^{th} roots of unity.

Since $z^8=1$, from lecture 17. we obtain that solutions for $z^n=1$ are located on unit circle with next root place at $\frac{2\pi}{n}$ angle c.c.w. around the circle.

Therefore $z=\cos\frac{2k\pi}{8}+i\sin\frac{2k\pi}{8}$, where $k=1,\ldots\ldots,7$. as $z-1\neq 0$.



$z_1 = \cos\frac{\pi}{4}+i\sin\frac{\pi}{4}$

$z_2 = \cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$.

$z_3 = \cos\frac{3}{4}\pi+i\sin\frac{3}{4}\pi$

$z_4 = \cos\pi+i\sin\pi$.

$z_5 = \cos\frac{5}{4}\pi+i\sin\frac{5}{4}\pi$.

$z_6 = \cos\frac{3}{2}\pi+i\sin\frac{3}{2}\pi$.

$z_7 = \cos\frac{7}{4}\pi+i\sin\frac{7}{4}\pi$.

2. Since $(1-(1+i)z)^4=-1$, gives. $1-(1+i)z=\sqrt[4]{-1}$, which

$$\Rightarrow \quad 1-\sqrt[4]{-1}=(1+i)z$$

$$\Rightarrow \quad z=\frac{1-\sqrt[4]{-1}}{1+i}.$$, multiply $(1-i)$ gives.

$$\Rightarrow \quad z=\frac{(1-\sqrt[4]{-1})(1-i)}{(1+i)(1-i)}$$

$$\Rightarrow \quad z=\frac{1-i-\sqrt[4]{-1}+\sqrt[4]{-1}\,i}{1+1}$$

$$\Rightarrow \quad z=\frac{1-\sqrt[4]{-1}}{2}-\frac{1-\sqrt[4]{-1}}{2.}i \qquad (\star)$$

I'll then calculate $\sqrt[4]{-1}$ for plotting.

Since $-1=-1+i0=\cos(\pi)+i\sin(\pi)=e^{i\theta}$, where $\theta=\pi+2k\pi$. $k=,0,1,2,3$.

When $k=0$, $-1=e^{i\pi}\Rightarrow\sqrt[4]{-1}=e^{\frac{\pi}{4}i}=\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$ substitute into $(\star)$.

$$z_1=\frac{1-(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i)}{2.}-\frac{1-(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i)}{2.}i=\frac{1}{2}-\frac{\sqrt{2}}{4}-\frac{\sqrt{2}}{4}i-\frac{1}{2}i+\frac{\sqrt{2}}{4}i-\frac{\sqrt{2}}{4}=\frac{1}{2}-\frac{\sqrt{2}}{2}-\frac{1}{2}i=(\frac{1}{2}-\frac{\sqrt{2}}{2})+(-\frac{1}{2})i$$

When $k=1$, $-1=e^{i3\pi} \Rightarrow \sqrt[4]{-1} = e^{\frac{3}{4}\pi i} = \cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ substitute into (☆).

$$z_2 = \frac{1-(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i)}{2} - \frac{1-(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i)}{2}i = \frac{1+\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i}{2} - \frac{1+\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i}{2}i$$

$$= \frac{1}{2}+\frac{\sqrt{2}}{4}-\frac{\sqrt{2}}{4}i - (\frac{1}{2}i+\frac{\sqrt{2}}{4}i+\frac{\sqrt{2}}{4}).$$

$$= \frac{1}{2}+\frac{\sqrt{2}}{4}-\frac{\sqrt{2}}{4}i - \frac{1}{2}i - \frac{\sqrt{2}}{4}i - \frac{\sqrt{2}}{4}.$$

$$= \frac{1}{2} - \frac{1}{2}i - \frac{\sqrt{2}}{2}i = \frac{1}{2} + (-\frac{1}{2}-\frac{\sqrt{2}}{2})i$$

When $k=2$. $-1=e^{i5\pi} \Rightarrow \sqrt[4]{-1} = e^{\frac{5}{4}\pi i} = \cos\frac{5}{4}\pi + i\sin\frac{5}{4}\pi = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ substitute in (☆).

$$z_3 = \frac{1-(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i)}{2} - \frac{1-(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i)}{2}i = \frac{1+\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i}{2} - \frac{1+\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i}{2}i = \frac{1}{2}+\frac{\sqrt{2}}{2}-\frac{1}{2}i = (\frac{1}{2}+\frac{\sqrt{2}}{2})+(-\frac{1}{2})i$$

When $k=2$. $-1=e^{i7\pi} \Rightarrow \sqrt[4]{-1} = e^{\frac{7}{4}\pi i} = \cos\frac{7}{4}\pi + i\sin\frac{7}{4}\pi = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ substitute in (☆).

$$z_4 = \frac{1-(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i)}{2} - \frac{1-(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i)}{2} = \frac{1}{2}-\frac{1}{2}i+\frac{\sqrt{2}}{2}i = \frac{1}{2}+(-\frac{1}{2}+\frac{\sqrt{2}}{2})i.$$