Properties & Confinement.

1. Binary Predicate Properties.

1) Symmetry: anything stands in a relationship of the predicate stands in the opposite as well.

i.e. Π^2 is symmetry iff $\forall\alpha\forall\beta(\Pi(\alpha\beta)\leftrightarrow\Pi(\beta\alpha))$

e.g. a loves b; b loves a. (not symmetry)

a touches b $\leftrightarrow$ b touches a (symmetry).

2) Transitivity:

Π^2 is transitive iff $\forall\alpha\forall\beta\forall\gamma(\Pi(\alpha\beta)\wedge\Pi(\beta\gamma)\rightarrow\Pi(\alpha\gamma))$

e.g. a is taller than b, b is taller than c $\longrightarrow$ a is taller than c.

3) Reflexivity: anything stands in a relationship of itself.

i.e. Π^2 is reflexive iff $\forall\alpha\,\Pi(\alpha\alpha)$.

e.g. red is the same color of red.

e.g. $\exists x\exists y(R(xy)\wedge x\neq y)$. $\forall x\forall y(R(xy)\rightarrow R(yx))$. $\forall x\forall y\forall z(R(xy)\wedge R(yz)\rightarrow R(xz))$

$\therefore \forall x R(xx)$ Invalid.

Pr1: some pair in R and the member in pair are not the same.

Pr2: if (x, y) in R then (y, x) in R. (Symmetry).

Pr3: if (x, y) and (y, z) in R, then (x, z) in R. (Transitive).

$\sim$C: $\sim\forall x R(xx)$ (not reflexive) $=\exists x\sim R(xx)$.

There is sth. in UD that not (x, x).

UD: {0, 1, 2}

R: {(0, 1), (1, 0), (0, 0), (1, 1)}

① noting since it's 'for all', x might be same as z.

② (0, 1), (1, 0) follows Pr3; don't forget (1, 0).

$(0,1)$.

2. Prenex Form.

1) All quantifiers in the front (Confinement).

① Conjunctions & Disjunctions: Directly take to the front.
(restriction: α can't be free in ϕ).

$(\forall\alpha \Pi\alpha \wedge \phi) \longleftrightarrow \forall\alpha(\Pi\alpha \wedge \phi)$. $(\exists\alpha \Pi\alpha \wedge \phi) \longleftrightarrow \exists\alpha(\Pi\alpha \wedge \phi)$.

$(\forall\alpha \Pi\alpha \vee \phi) \longleftrightarrow \forall\alpha(\Pi\alpha \vee \phi)$ $(\exists\alpha \Pi\alpha \vee \phi) \longleftrightarrow \exists\alpha(\Pi\alpha \vee \phi)$.

e.g. $\exists x(Fx \wedge \forall y(Fy \to Gy)) \equiv \exists x \forall y(Fx \wedge (Fy \to Gy))$.

② Condiontionals. (restriction: α can't be free in ϕ).

$\longrightarrow$ In consequence: Directly take to the front.

$(\phi \to \exists\alpha \Pi\alpha) \longleftrightarrow \exists\alpha(\phi \to \Pi\alpha)$; $(\phi \to \forall\alpha \Pi\alpha) \longleftrightarrow \forall\alpha(\phi \to \Pi\alpha)$.

there exist y Ay
$\exists y Ay \to \forall x(Fx \wedge Gx)$
$\not\equiv$
$\exists y(Ay \to \forall x(Fx \wedge Gx))$
if there is Ay.

$\longrightarrow$ In antecedent: When taking out, the quantifier changes.

$(\exists\alpha \Pi\alpha \to \phi) \longleftrightarrow \forall\alpha(\Pi\alpha \to \phi)$; $(\forall\alpha \Pi\alpha \to \phi) \longleftrightarrow \exists\alpha(\Pi\alpha \to \phi)$.

③ Biconditional (Use the rules above).

$(\phi \leftrightarrow \psi) \equiv (\phi \to \psi) \wedge (\psi \to \phi)$.

2) Each quantifier has a different letter.

3) No biconditionals.

e.g. $\exists x Fx \to \forall x Hx \wedge \sim \forall y Gy$ into Prenex Form.

$\equiv \exists x Fx \to \forall x Hx \wedge \exists y \sim Gy$.

$\equiv \exists x Fx \to \forall x \exists y(Hx \wedge \sim Gy)$

$= \exists x Fx \to \forall z \exists y(Hz \wedge \sim Gy)$.

$= \forall x \forall z \exists y(Fx \to (Hz \wedge \sim Gy))$.

In single predicate it doesn't matter because one doesn't relate to other.
(change the consequence of quantifiers)
However, in multiplace, $\forall x \exists y F(xy)$ generic.
$\not\equiv \exists y \forall x F(xy)$ specific