1 Question 1

(a) According to the definition of P:

$$(\forall g \in G_1,\ \exists t \in T_1,\ t\ tiles\ g) \implies (\forall g \in G_2,\ \exists t \in T_2,\ t\ tiles\ g)$$

(b) Firstly, assume

$$\forall g_1 \in G_1,\ \exists t_1 \in T_1,\ t_1\ tiles\ g_1, \text{ which is the antecedent.}$$

Secondly, I will do the consequent part, which:

$$Let\ g_2 \in G_2.\ Let\ t_2 = [\text{a tiling that satisfies the condition}]$$

Then, I want to prove that

$$t_2 \in T_2\ and\ t_2\ tiles\ g_2$$

by selecting a satisfying element t_2 from T_2 and prove the element t_2 satisfies $t_2\ tiles\ g_2$.

(c) The diagram above illustrates one instance of G_2 grids, which being tiled by triominoes.

Firstly, we already know that for P(1), the statement $\forall g_1 \in G_1,\ \exists t_1 \in T_1,\ t_1\ tiles\ g$ is true which is the antecedent of this direct proof.

Secondly, the above diagram is an element of the set of all $2^2 \times 2^2$ grid with one square removed, which is an element of G_2. By visulising those colorful triominoes, we see a combination triominoes, t_2, which is an element of the set of all tilings of elements of G_2 using triominoes, belonging to T_2, exists and tiles g_2.

Therefore, the diagram above illustrates an instance of that direct proof.

(d) Given the statement to prove: $\forall n \in \mathbb{N},\ P(n)$, which any $2^n \times 2^n$ grid with one cell missing can be tiled using only triominoes.

Proof: We prove this by Simple Induction on n.

Base Case: Let $0 \leq n \leq 1$.

When $n = 0$, since G_0 is the set of $2^0 \times 2^0$ grid with one cell removed, which G_0 does not contain any grid. This is actually an edge case, which $g \in G_0$ is empty, so g can definitely be tiled by a $t \in T_0$, which is empty as well. $P(0)$ is True.

When $n = 1$, since G_1 is the set of all $2^1 \times 2^1$ grids with one cell removed, which by definition is a single triominoe. $P(1)$ is True.

I've shown the base case is true.

Induction Step: Let $n \in \mathbb{N}$.

Induction Hypothesis: Assume that $P(n)$ is true.

Since $g_{n+1} \in G_{n+1}$ is a $2^{n+1} \times 2^{n+1}$ grid with one cell missing, I will siplit it into four $2^n \times 2^n$ quadrants.

By Induction Hypothesis, we know that $P(n)$ is true, which $\forall g_n \in G_n,\ \exists t_n \in T_n,\ t_n\ tiles\ g_n$ is true. I will take 3 different g_ns, the first with right button corner square missing, the second with right top corner square missing, and the third with left top corner square missing. I will make the missing corners in these 3 g_ns face inwards and add a triomino which will result in getting a 'L' shape. The remaining $\frac{1}{4}$ place is missing a cell to form a g_{n+1}, which can actually be an arbitraty element from G_n. By Induction Hypothesis, since $\forall g_n \in G_n,\ \exists t_n \in T_n,\ t_n\ tiles\ g_n$ is true, the remaining G_n place can be covered by trimonoes, proving the $P(n+1)$ is true.

Therefore, we've proved $\forall n \in \mathbb{N},\ P(n)$ is true.

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