Computing Limits.

If so, just evaluate.

1. Is the function defined and continuous.

e.g. $\lim_{x\to 2} \frac{x\sin x + e^x}{\sqrt{x^2+7}}$

$= \frac{2\cdot\sin 2 + e^2}{\sqrt{2^2+7}} = \frac{2\sin 2 + e^2}{\sqrt{11}}$

During the exam when: $\lim_{x\to x_0} f(x) = \infty$. no matter whether $\lim_{x\to x_0^+} f(x) = \lim_{x\to x_0^-} f(x)$ or not; we should conclude limit DNE.

2. Algebraic Manipulations: Transform the function into a continuous one.

e.g. $\lim_{x\to 2} \frac{x^2-4}{x^2-3x+2}$

$= \lim_{x\to 2} \frac{(x+2)(x-2)}{(x-2)(x-1)} = \lim_{x\to 2} \frac{x+2}{x-1} = 4.$

e.g. $\lim_{x\to 0} \frac{1-\sqrt{1+x}}{x}$

$= \lim_{x\to 0} \frac{(1-\sqrt{1+x})(1+\sqrt{1+x})}{x(1+\sqrt{1+x})} = \lim_{x\to 0} \frac{1-(1+x)}{x\cdot(1+\sqrt{1+x})}$

$= \lim_{x\to 0} \frac{-x}{x\cdot(1+\sqrt{1+x})}$

$= \lim_{x\to 0} \frac{-1}{1+\sqrt{1+x}}$

$= -\frac{1}{2}$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\downarrow$

$\lim_{2x\to 0} \frac{\sin 2x}{2x} = 1$

3. Reduce to a problem we have already solved.

$\lim_{x\to 0} \frac{\sin(ax)}{ax} = 1$, for any $a \neq 0$

e.g. $\lim_{x\to 0} \frac{\sin(2x)}{x}$

$= \lim_{x\to 0} \left[2\cdot \frac{\sin 2x}{2x}\right] = 2$

e.g. $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$

$= \lim_{x\to 0} \frac{2x\cdot\frac{\sin 2x}{2x}}{3x\cdot\frac{\sin 3x}{3x}} = \frac{2}{3}$

e.g. $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

$$= \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x^2\cdot(1+\cos x)} = \lim_{x\to 0} \frac{1-\cos^2 x}{x^2\cdot(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x^2\cdot(1+\cos x)} = \frac{1}{1+1} = \frac{1}{2}$$

4. Limit at infinity

e.g. $\lim_{x\to\infty}[2x^3-3x^2+7]$

$$= \lim_{x\to\infty} x^3\left[2-3\cdot\frac{1}{x}+7\cdot\frac{1}{x^3}\right]$$

$$= \infty$$

We only need to care about the biggest power and take it out.

e.g. $\lim_{x\to\infty} \frac{2x^2+x-1}{3x^2+10x+e}$

$$= \lim_{x\to\infty} \frac{x^2\left[2+\frac{1}{x}-\frac{1}{x^2}\right]}{x^2\left[3+\frac{10}{x}+\frac{e}{x^2}\right]}$$

$$= \frac{2}{3}$$

e.g. $\lim_{x\to\infty} \frac{\sqrt{x^4+1}+x^2}{\sqrt{2x^4+1}+2x}$

$$= \lim_{x\to\infty} \frac{\sqrt{x^4(1+1/x^4)}+x^2}{\sqrt{x^4(2+1/x^4)}+2x}$$

$$= \lim_{x\to\infty} \frac{x^2\left(\sqrt{1+1/x^4}+1\right)}{x^2\left(\sqrt{2+1/x^4}+\frac{2}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{\sqrt{1+1/x^4}+1}{\sqrt{2+1/x^4}+\frac{2}{x}}$$

$$= \sqrt{2}.$$