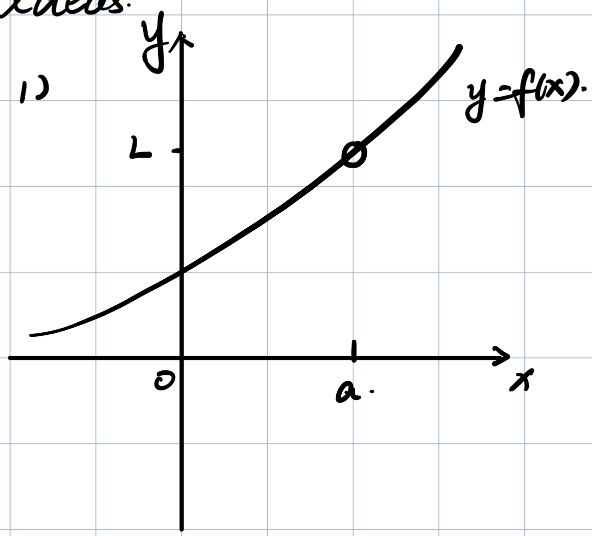
Formal Definition of Limit.

1. Ideas.

1)



$\lim_{x \to a} f(x) = L.$

If x is close to a ($x \neq a$), then $f(x)$ is close to L.

2) 'x is close to a' ($x \neq a$)

① $|x-a|$ is 'small'

$|x-a|$ distance between x & a.

② $|x-a| < \delta$

③ $0 < |x-a| < \delta.$

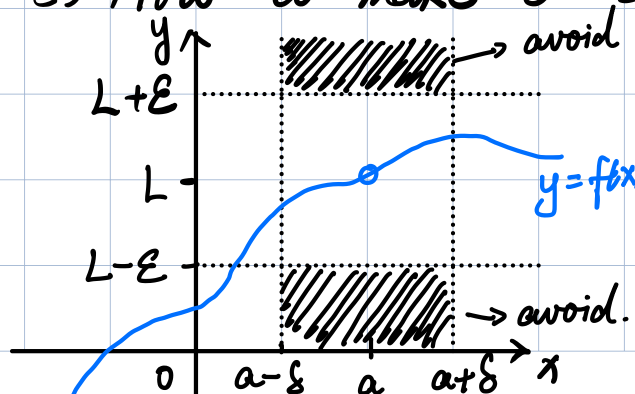
3) $f(x)$ is close to L.

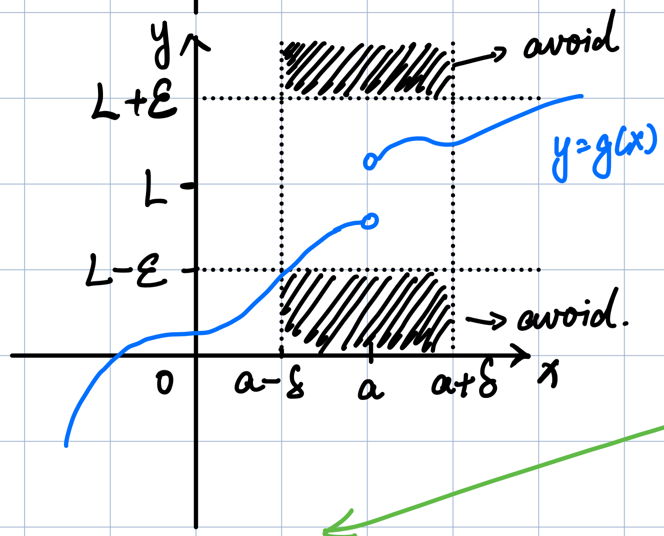
① $|f(x) - L| < \varepsilon.$

4) If x is close to a, ($x \neq a$) then $f(x)$ is close to L.

$$0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon.$$

5) How to make δ & ε small?





$\lim_{x \to a} g(x)$ DNE.

making δ small is not going to help. but making ε smaller does.

$\therefore \forall \varepsilon > 0, \exists \delta > 0$

↳ making ε smaller can find a value of δ smaller that works.

6) a, L are real numbers

7.) Don't care if $f(a)$ is defined or not, but f needs to be defined for values of x close to a. (at least defined on an interval centered at a, except maybe at a).

2. The formal definition of limit.

We say that

$$\lim_{x \to a} f(x) = L$$

when

$$\forall \varepsilon > 0, \exists \delta > 0 \text{ s.t. } (\forall x \in \mathbb{R},)$$

$$0 < |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon.$$

we should be careful!!! if $f(x)$ is not continuous function with domin $(-\infty, b) \cup (b, +\infty)$. where $b \in \mathbb{R}$ and $b \neq a$ Then we should write. $x \in D$.

where the domin of $f(x)$ is.

$\lim_{x\to\infty} f(x) = L$

(x large $\to f(x)$ approaching L)

3. Limit at infinity.

We say that:

$$\lim_{x\to\infty} f(x) = L$$

recognize as positive ∞.

when

$$\forall \varepsilon>0,\ \underset{\star}{\exists M\in\mathbb{R}} \text{ s.t. } x>M \Rightarrow |f(x)-L|<\varepsilon.$$

when $x\to\infty$, M definitely should ≥ 0; however, we should briefly mention when $M\leq 0$. we say: Since $x\to\infty$, x clearly ≥ 0, therefore, $M\leq 0$ doesn't have any impact or restrain on x. we only need to think about when $M>0$.