UNIVERSITY OF TORONTO : Faculty of Arts & Science
April 2019 Examinations

MAT301H1-S

Duration: 3 hours
Aids Allowed: None

First name (please write as legibly as possible within the boxes)

Last name

Student ID number

- Do not begin writing the actual exam until the announcements have ended and the Exam Facilitator has started the exam.
- As a student, you help create a fair and inclusive writing environment. If you possess an unauthorized aid during an exam, you may be charged with an academic offence.
- Turn off and place all cell phones, smart watches, electronic devices, and unauthorized study materials in your bag under your desk. If it is left in your pocket, it may be an academic offence.
- When you are done with your exam, raise your hand for someone to come and collect your exam. Do not collect your bag and jacket before your exam is handed in.
- If you are feeling ill and unable to finish your exam, please bring it to the attention of an Exam. Facilitator so it can be recorded before leaving the exam hall.
- In the event of a fire alarm, do not check your cell phone when escorted outside.
- **Hand in all materials at the end of the exam. Good luck!**

Questions	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Total
Points	11	11	11	11	11	11	11	11	12	100

1. **(11 points)**

   Let $H \subset S_3$ be the subgroup of permutations $\{e, (12)\}$. List all of the distinct cosets $aH, a \in S_3$. Is H normal in S_3?

2. a) (**6 points**) Let $H \subset \mathbb{Z}_6 \oplus \mathbb{Z}_6$ be the subset of elements of order ≤ 3. Is H a subgroup?

   b) (**5 points**) Let $K \subset \mathbb{Z}_6 \oplus \mathbb{Z}_6$ be the subset of elements of order ≤ 2. Is K a subgroup?

3. **(11 points)**

Compute the order of $\mathrm{Aut}(\mathbb{Z}_3 \oplus \mathbb{Z}_5)$.

4. **(11 points)**

List all abelian groups of order 48, up to isomorphism.

5. Suppose G is a finite group and $N \subset G$ is a normal subgroup. Let $x \in G$, and suppose $xN \in G/N$ has order n.

   a) (**5 points**) Show that $\langle x \rangle \cap N = \langle x^n \rangle$.

   b) (**6 points**) Show that G has an element of order n.

6. **(11 points)**

If M and N are normal subgroups of G, show that $M \cap N$ is also a normal subgroup.

7. **(11 points)**

Find two distinct subgroups of order 44 in $\mathbb{Z}_{22} \oplus \mathbb{Z}_{22}$.

8. **(11 points)**

Consider the normal subgroup A_3 of even permutations of S_3, i.e. permutations that are a product of an even number of 2-cycles.

Show that S_3/A_3 is isomorphic to $\mathbb{Z}_2$.

9. Consider the dihedral group D_4 of symmetries of the square.

   a) (**6 points**) Find two elements x, y of D_4 such that $xy \neq yx$.

   b) (**6 points**) Show that there is no surjective homomorphism $\mathbb{Z}_{16} \to D_4$.

Scrap Paper (page 1). Please indicate if you wish this to be graded.

Scrap Paper (page 2). Please indicate if you wish this to be graded.

Scrap Paper (page 3). Please indicate if you wish this to be graded.

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