Expectation.

1. Discrete: X is a discrete random variable.

$$E(X) = \sum_{x \in \mathbb{R}} x P(X=x) = \sum_{x \in \mathbb{R}} x\, p_X(x)$$

1). Constant: $X = c$, $E(X) = c$.

2). $X \sim \text{Bernoulli}(\theta)$: $E(X) = \theta$.

3). $X \sim \text{Binomial}(n, \theta)$: $E(X) = n \cdot \theta$.

4) $X \sim \text{Geometric}(\theta) = \frac{1-\theta}{\theta}$.

5). $X \sim \text{Poisson}(\lambda) = \lambda$.

6). Expectation of Functions.:

Z is also discrete.

① $Z = g(X)$., $E(Z) = \sum_{z \in \mathbb{R}} z \cdot P(Z=z) = \sum_{x \in \mathbb{R}} g(x) \cdot P(X=x)$.

e.g. $X \sim \text{Binomial}(3, \frac{1}{4})$.

$E(5X^2) = \sum_{x \in \mathbb{R}} 5x^2 P(X=x) = \sum_{k=0}^{3} 5k^2 \cdot \binom{3}{k} \cdot (\frac{1}{4})^k \cdot (\frac{3}{4})^{3-k}$.

② $Z = h(X, Y)$, $E(Z) = \sum_{z \in \mathbb{R}} z \cdot P(Z=z) = \sum_{x,y \in \mathbb{R}} h(x,y) P(X=x, Y=y)$.

7) Linear Property: $Z = aX + bY$, $E(Z) = aE(X) + bE(Y)$.

8). Monotonicity: If $X \le Y$, then $E(X) \le E(Y)$.

9). Suppose X, Y are independent: $E(XY) = E(X) \cdot E(Y)$.

Inverse is false.

2. A.cts.: X is an A.cts. variable

$$E(X) = \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx.$$

1) $X \sim \text{Uniform}(L, R)$: $E(X) = \frac{1}{2}(R+L)$.

2) $X \sim \text{Exponential}(\lambda)$: $E(X) = \frac{1}{\lambda}$.

3) $X \sim \text{Normal}(\mu, \sigma^2)$: $E(X) = \mu$.

4). Expectation of Functions.

① $Z = g(X)$. $E(Z) = E(g(X)) = \int_{-\infty}^{\infty} g(x) f_X(x) dx$.

② $Z = h(X, Y)$. $E(Z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y) f_{X,Y}(x,y)\, dx dy$.

5) Linear Property: $Z = aX + bY$, $E(Z) = aE(X) + bE(Y)$.

6) Monotonicity: If $X \le Y$, then $E(X) \le E(Y)$.

7) Suppose X, Y are independent: $E(XY) = E(X) \cdot E(Y)$.

Inverse is false.