Graph. $G(V,E)$.

解 graph 问题核心：画图进行分析。(先跟题目条件列出对应事件。)

1. Regarding degree

1). $G=(V,E)$, $\sum_{v\in V} \deg_G(v) = 2|E|$.

2) For any graph, # of vertices with odd degree is even.

从 deg 出发

2. Eulerian & Hamilton Graph.

1). Eulerian: traverse all edges, vertices can repeate.

① Eulerian $\Leftrightarrow$ connected, every vertex has even deg.

2) Hamilton: traverse all vertices once (edge may not all covered).

①. n vertices, each v has at least $\lceil \frac{n}{2} \rceil$ neighbours. $\longrightarrow$ hamilton.

Complete K_n: $\chi(K_n) = n$.

3. Graph Coloring. (相邻两点颜色不一).

1). chromatic number of G ($\chi(G)$): least t required for coloring.

2). Bipartite Graph: $V = V_1 \cup V_2$. $V_1 \cap V_2 = \phi$. $G(V,E)$, all edges are between V_1, V_2.

① 2 colorable $\leftrightarrow$ bipartite.

② 2 colorable / bipartite $\leftrightarrow$ doesn't contain an odd cycle.

③ Complete bipartite: all edges between V_1, V_2 are connected.

3). Clique number: (包含最大的 complete graph): $\omega(G)$.

① $\forall t \geq 3$. $\exists G_t$. s.t. $\omega(G_t) = 2$, $\chi(G_t) = t$.

no triangle.

4) Greedy Coloring.

5). Interval Graph:

① Interval Graph $\longrightarrow \chi(G) = \omega(G)$.

4. Planar Graph: Edges only intersect at vertices.

F: faces: bounded by edges & vertices.

1) Euler's Formula.

① $V - E + F = 2$. (connected).

② $V - E + F = 1 +$ # of components.

2). $|V| \geq 3$. then $|E| \leq 3|V| - 6$.

① G has no cycle of size 3 (triangle), then $|E| \leq 2|V| - 4$

3) K_5 & $K_{3,3}$ has no planar.

4). planar $\Leftrightarrow$ doesn't contain K_5 / $K_{3,3}$.

5) Every planar graph is 4-colourable; $\chi(G) \leq 4$.