CSC236 Problem Set 2

Question 1.

Let ♡ be the statement: $\forall n, a_n \le a_{n+1}$.

Let ▽ be the statement: $\forall n, \forall k \le n, a_k \le a_n$.

(a) WTP: ▽ $\Rightarrow$ ♡.

Proof: Let $n_1 \in \mathbb{N}$. Let $k \in \mathbb{N}$. $k \le n_1$. Assume $a_k \le a_{n_1}$.

Let $n_2 \in \mathbb{N}$. WTP. $a_{n_2} \le a_{n_2+1}$.

Since $n_2 \in \mathbb{N}$, $(n_2+1) \in \mathbb{N}$, which $\forall k \le n_2+1$, according to assumption, gives $a_k \le a_{n_2+1}$.

Since $n_2 \in \mathbb{N}$, and $n_2 \le n_2+1$, gives $a_{n_2} \le a_{n_2+1}$ as n_2 is an instance of such k in the statement '$\forall k \le n_2+1, a_k \le a_{n_2+1}$'.

I've proved ▽ $\Rightarrow$ ♡. ■

(b). WTP: ♡ $\Rightarrow$ ▽.

proof. Assume $\forall n_1 \in \mathbb{N}, a_{n_1} \le a_{n_1+1}$ — assumption 1.

Let $P(n_2)$: $\forall k \le n_2, a_k \le a_{n_2}$. where $n_2 \in \mathbb{N}$.

Let $n_2 \in \mathbb{N}$.

Base Case: $n_2 = 0$.

Let $k \in \mathbb{N}$. Assume $k \le 0$

Since $k \in \mathbb{N}$ and $k \le 0$, gives $k=0$, which $a_k = a_0 \le a_0$.

I've proved the base case is true.

Induction Step: Let $n_2 \in \mathbb{N}$.

Induction Hypothesis: Assume $P(n_2)$.

WTP. $P(n_2+1)$, which $\forall k' \le n_2+1, a_{k'} \le a_{n_2+1}$.

Let $k' \in \mathbb{N}$. Let $k' \le n_2+1$.

Since $n_2 \in \mathbb{N}$, according to assumption 1, gives $a_{n_2} \le a_{n_2+1}$.

① when $k' \le n_2$.

By I.H. $a_{k'} \le a_{n_2}$ as k' satisfies the I.H..

Since $a_{n_2} \le a_{n_2+1}$, gives $a_{k'} \le a_{n_2} \le a_{n_2+1}$.

② when $n_2 < k' \le n_2+1$,

Since $k' \in \mathbb{N}$, $k' = n_2+1$. gives $a_{k'} = a_{n_2+1} \le a_{n_2+1}$.

Since k must satisfy $k' \le n_2$ or $n_2 < k' \le n_2+1$.

I've proved the induction step is true.

Therefore, ♡ ⇒ ◇. ∎