MAT 246, PS1   Due: Fri. Sep 22   Time 10:00 pm.   Crowdmark

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Please note:

1. Ideally the PS must be written in the spaces provided, and page by page, (a picture or scan of individual pages) to be uploaded to the appropriate pages of a Crowdmark file (link to be posted.) If you wish, you can write your answers on separate pieces of paper and upload them properly at the allocated page of Crowdmark. Please note that any mismatch in uploading the pages of the solutions may result in parts of the solutions not getting marked.

2. Please provide your final, polished solutions in the spaces provided. Sometimes the spaces are very small, which indicates a short solution is possible or expected. Sometimes the break point of a question forces us to leave a large space, which doesn't necessarily mean a long solutions is expected. If you have no access to a printer you can write your solutions on a blank sheet of paper and upload a picture of scan. Remember, it is an art to write a short yet complete solution; one learns a lot from practicing this art. Please write a complete, even though long solutions. Then inspect and edit; while editing, ask yourselves:

   - am I proving common believes and facts accepted in the literature? If yes, know which facts are common belief and which ones need an argument.

   - can I just quote some of the facts that I tried to prove? This requires a good familiarity with the literature of the subject (in our case, the textbook and the slides, and earlier questions in the same PS).

   - am I introducing too many cases in my proof, and presenting parallel arguments? Change your type of argument if possible (direct proof versus proof by cases, or proof by contradiction.)

   Editing your first complete solution is a very good way of learning. Make sure you have a good solution, then return to it and reflect on the above questions.

3. Problem set questions contain ideas complementary to the textbook and lecture materials, opportunity for reflection and deepening on the subject. The level of difficulty of the questions is elementary so that each student has chance to individually think about the problems. So, please don't let your "kind" friends take this opportunity away from you!

4. Not all questions will be marked, **nor** are they all of equal weight. It is not known in advance which questions will be marked, and how heavy they will be weighed. Therefore if a question is skipped one may lose a larger portion of the PS mark than one would expect.

5. To receive full mark on the computational questions all the computation details must be provided. For questions involving proofs there is no need to prove textbook or "slides" proofs but one can simply quote them. When a theorem is applied the relevance of it to the proof must be made clear.

6. Collaboration in this work is not allowed. You may discuss the idea of a the solutions only, but each person must write their own solutions completely **independently**, without knowing or consulting the structure and details of the argument of another person.

   So please be careful not to share any written hint with your friends, because some people have photographic memory, and then your solutions may become subject to plagiarism. You can give them your general ideas if you wish, but absolutely no details and format of writing.

7. By participating in this problem set you agree that you are aware of the university regulations regarding plagiarism. Individual students are expected to organize and write their own solutions independently. During the online teaching, and while students are connected through email, any assistance to your fellow classmate in the form of a written note might be directly copied and it may count as a form of plagiarism. Markers are carefully monitoring the solutions, and any incidence of plagiarism, and all the parties involved will be dealt with according to university regulations. Please carefully read the item regarding plagiarism in our course outline, and consult the university policies regarding plagiarism (not knowing the rules is not a good excuse for not obeying them):

   http://www.artsci.utoronto.ca/newstudents/transition/academic/plagiarism

0. **Failure to answer this question may result in a mark of 0 on the entire PS1. And it is not the name, signature and date, but it is the acknowledgement .. signing some statement that is intended in this part:**

Please sign below as an acknowledgment that you have read the cover page ; **(unsigned papers shall not be marked.)** If you don't have access to a printer, please produce the same acknowledgement by hand on a piece of paper. Your name alone is not an acknowledgement that you are aware of the policies discussed in the cover page.

Print Name (first, last) : <u>Xuanqi, Wei</u> , Signature: <u>Jerry Wei</u> Date: <u>09/20/2023</u> .

1. Use truth tables to determine whether the following statements are equivalent. In case they are not equivalent, determine an instance in which they differ (this would be a counter-example demonstrating the pairs are not equivalent.) One can try to include all cases in one or two truth tables to save sapace. The truth tables must be accompanied by some explanation, which could be minimal, describing what information to be extracted from the truth table, and how this information translates to results that you are suggesting.

a) $\neg(p \implies q)$ and $p \implies \neg q)$

b) $\neg(p \implies q)$ and $p \land \neg q$

c) $\neg(p \implies q)$ and $\neg p \implies \neg q$

d) $\neg(p \implies q)$ and $\neg p \implies q$

e) $p \implies q$ and $\neg(q \implies p)$

f) $p \implies q$ and $q \lor \neg p$

g) $p \implies q$ and $\neg q \lor p$

h) $p \implies q$ and $\neg q \implies \neg p$

a). Not equivalent : col. 6 & col. 7 $(p = F, q = T)$ $(p = F, q = F)$

b) Equivalent: col. 6 & col. 8.

c). Not equivalent: col. 6 & col. 9. $(p = T, q = T)$ $(p = F, q = F)$

d) Not equivalent: col. 6 & col. 10. $(p = T, q = T)(p = F, q = T)$

e). Not equivalent: col. 5 & col. 12. $(p = T, q = T)(p = F, q = F)$.

f). Equivalent: col. 5 & col. 13.

g). Not equivalent: col. 5 & col. 14. $(p = T, q = F)(p = F, q = T)$.

h) Equivalent: col. 5 & col. 15.

I've shown the truth-table below. I will explain column by column.

col. 1	col. 2	col. 3	col. 4	col. 5	col. 6	col. 7	col. 8	col. 9	col. 10	col. 11	col. 12	col. 13	col. 14.	col. 15.
P	q	$\neg p$	$\neg q$	$p \Rightarrow q$	$\neg(p \Rightarrow q)$	$p \Rightarrow \neg q$	$p \land \neg q$	$\neg p \Rightarrow \neg q$	$\neg p \Rightarrow q$	$q \Rightarrow p$	$\neg(q \Rightarrow p)$	$q \lor \neg p$	$\neg q \lor p$	$\neg q \Rightarrow \neg p$
T	T	F	F	T	F	F	F	T	T	T	F	T	T	T
T	F	F	T	F	T	T	T	T	T	T	F	F	T	F
F	T	T	F	T	F	T	F	F	T	F	T	T	F	T
F	F	T	T	T	F	T	F	T	F	T	F	T	T	T

column 1-2: To show whether they are the same. I'll list all possibility of statement p and q.

column 3-4: I'll show the negation of P. q.

column 5: I'll get it by using column 1 and 2, which when the 'p' is T, the 'q' should be T to make the whole T, when 'p' is F, the whole is vacuously T.

column 6: Get it from the negation of col. 5.

column 7: I'll get it by using column 1 and 4, which when the 'p' is T, the '$\neg q$' should be T to make the whole T, when 'p' is F, the whole is vacuously T.

column 8: The whole is T only when both 'p' and '$\neg q$' are T.

column 9: I'll get it by using column 3 and 4, which when the '$\neg p$' is T, the '$\neg q$' should be T to make the whole T, when '$\neg p$' is F, the whole is vacuously T.

column 10: I'll get it by using column 3 and 2, which when '$\neg p$' is T, the 'q' should be T to make the whole T, when '$\neg p$' is F, the whole is vacuously T.

column 11: I'll get it by using column 1 and 2, which when 'q' is T, the 'P' should be T to make the whole T, when 'q' is F, the whole is vacuously T.

column 12: Get it from the negation of column 11.

column 13: The whole is T when either 'q' or '$\neg p$' is T.

column 14: The whole is T when either '$\neg q$' or 'p' is T.

column 15: I'll get it by using column 4 and 3., which when '$\neg q$' is T, the '$\neg p$' should be T to make the whole T, when '$\neg q$' is F, the whole is vacuously T.