The Point of Truth Tables

1. Truth Value Assignments (TVA)

   1) Truth Tables show which Conditions make a statement True or False.

   2) Such conditions are called: TVA.

   3) A particular TVA is also called a Valuation.

   eg. what TVA makes the following True?

   $P \land Q$.

P	Q	$P \land Q$.
T	T	T
T	F	F
F	T	F
F	F	F

   $\therefore P = T \quad Q = T$

2. Truth Table allow us to analyze the Semantic Properties of:

   1) Statements

   2) Sets of Statements, and.

   3) Argument.

3. A Compact Style of Truth Table.

P	Q	((P	∧	~	Q)	→	~	(Q	∨	P))
T	T	T	F	F	T	T	F	T	T	T
T	F	T	T	T	F	F	F	F	T	T
F	T	F	F	F	T	T	F	T	T	F
F	F	F	F	T	F	T	T	F	F	F

— : 1st step — : final step.
— : 2nd step
— : 3rd step

4. Tautology: a sentence that is true in every row of the truth table.

e.g. ~(P∧Q) ↔ (~P ∨ ~Q)

P	Q	~	(P	∧	Q)	↔	(~	P	∨	~	Q)
T	T	F	T	T	T	**T**	F	T	F	F	T
T	F	T	T	F	F	**T**	F	T	T	T	F
F	T	T	F	F	T	**T**	T	F	T	F	T
F	F	T	F	F	F	**T**	T	F	T	T	F

5. Satisfiable: A sentence is true in some row

e.g. ((P∧~Q) → ~(Q∨P))

P	Q	((P	∧	~	Q)	→	~	(Q	∨	P))
T	T	T	F	F	T	**T**	F	T	T	T
T	F	T	T	T	F	**F**	F	F	T	T
F	T	F	F	F	T	**T**	F	T	T	F
F	F	F	F	T	F	**T**	T	F	F	F

1) Unsatisfiable: false in every row of the truth table.

2) Jointly Satisfiable (two or more sentences): if there is at least one row of the truth table such that, all of the sentences are true in that row

6. Invalidating Row: a row of the truth table in which all of the premises are true and the conclusion is false.

e.g. $P \rightarrow Q \vee R$. $((\sim Q \rightarrow P) \rightarrow P) \wedge (R \rightarrow P)$ ∴ $P \vee R$

			Premise 1:					Premise 2:										Conclusion:		
P	Q	R	P	→	Q	∨	R	((∼	Q	→	P)	→	P)	∧	(R	→	P)	P	∨	R
T	T	T	T	**T**	T	T	T	F	T	T	T	T	T	**T**	T	T	T	T	**T**	T
T	T	F	T	**T**	T	T	F	F	T	T	T	T	T	**T**	F	T	T	T	**T**	F
T	F	T	T	**T**	F	T	T	T	F	T	T	T	T	**T**	T	T	T	T	**T**	T
T	F	F	T	**F**	F	F	F	T	F	T	T	T	T	**T**	F	T	T	T	**T**	F
F	T	T	F	**T**	T	T	T	F	T	T	F	F	F	**F**	T	F	F	F	**T**	T
F	T	F	F	**T**	T	T	F	F	T	T	F	F	F	**F**	F	T	F	F	**T**	F
F	F	T	F	**T**	F	T	T	T	F	F	F	T	F	**F**	T	F	F	F	**T**	T
F	F	F	F	**T**	F	F	F	T	F	F	F	T	F	**T**	F	T	F	F	**F**	F

1) **Tautologically Valid Argument**: an argument with no invalidating row.

2) Tautologically Invalid Argument: an argument with at least one invalidating row.

7. Shortened Truth Tables
   1). Steps.
   ① Identify the main connectives and the truth value that needed (under the relevant main connectives)
   ② Work the way backwards from what need to what can establish.

③ Prioritize connectives with the fewest poss-
ible cases first — don't guess unless have to.
④ If establish the truth value for an atomic sen-
tence anywhere in shortened truth table: apply that
truth value everywhere in STT
——> an atomic sentence: a type of declarative
sentence which is either true or false and can-
not be broken down into other simple sentences.
     e.g. The dog ran : atomic.
       The dog ran and the cat hid : molecular.

2) STT example 1:

$(P \leftrightarrow \sim(Q \vee R)) \wedge \sim(P \rightarrow Q)$ is Satisfiable.

$$P \leftrightarrow \sim (Q \vee R)) \wedge \sim (P \rightarrow Q)$$

	T	T		F			T	T		F	
P	⟷	~	(Q	∨	R))	∧	~	(P	→	Q)
T	T	T	F	F	F	T	T	T	F	F

— : 1st.   — : 4th.

— : 2nd.   — : 5th.

— : 3rd

TVA: P = T, Q = F, R = F

3) Example 2:
the argument $P \vee \sim Q$. $P \rightarrow Q$ ∴ $\sim Q$ is invalid.

P   V   ~   Q || P   →   Q || ~   Q

(left margin, blue): Use STT when question already says the final truth – value.

(annotations above formula): 2nd, 3rd, 4th, 1st, 2nd, 3rd, write what needed first.

(Example 2 annotations): 1st T, 1st T, 1st F

$$T \quad T \quad F \quad T \parallel T \quad T \quad T \parallel F \quad T$$

TVA: P = T, Q = T