Divisibility on Integers.

1. Definition: Given a natural number $m>0$, and an integer n, we say $m|n$ if $\exists k \in \mathbb{Z}$. s.t. $n=km$.

1). $\forall m \in \mathbb{Z}$, $m>0 \Rightarrow m|0$.

Let $m \in \mathbb{Z}$. Let $m>0$.

Since $\exists k \in \mathbb{Z}$. $k=0$. $k \cdot m = 0 \cdot m = 0$, which $m|0$.

2). If $m, n \in \mathbb{Z}$. $m>0$, $m|n$ with $n=km \Rightarrow k$ is unique.

Let $m, n \in \mathbb{Z}$. $m>0$, $m|n$.

Assume for contradiction, k is not unique. which

$\exists k_1, k_2 \in \mathbb{Z}$. $k_1 \neq k_2$. $n=k_1 m$, $n=k_2 m$.

Since $n=k_1 m$, $n=k_2 m$, gives. $k_1 m = k_2 m$. since $m \in \mathbb{Z}$. $m>0$,

from cancellation law, gives. $k_1 = k_2$. contradicts.

$\longrightarrow$ We define $k=\frac{n}{m}$ as it's uniquelly guaranteed by the def^n.