Derived Derivative

1. Type 1: Negation of basic rules: eliminate ~ rather than use contradiction generator

1) Negation of Conditional (NC)

$$\frac{\sim(\phi \to \psi)}{\phi \land \sim\psi} \qquad \frac{\phi \land \sim\psi}{\sim(\phi \to \psi)}$$

2) Negation of Biconditional (NB)

$$\frac{\sim(\phi \leftrightarrow \psi)}{(\phi \leftrightarrow \sim\psi)} \qquad \frac{(\phi \leftrightarrow \sim\psi)}{\sim(\phi \leftrightarrow \psi)}$$

3) De Morgan's Law (DM): 对于"∧"和"∨"，把外面的 negation 乘进去，并将中间的连接符取相反（∧变∨；∨变∧）

$$\frac{\sim(\phi \lor \psi)}{\sim\phi \land \sim\psi} \qquad \frac{\sim(\phi \land \psi)}{\sim\phi \lor \sim\psi}$$

$$\frac{\sim(\sim\phi \lor \sim\psi)}{\phi \land \psi} \quad \text{直接跳过DN} \quad \frac{\sim(\sim\phi \land \sim\psi)}{\phi \lor \psi}$$

$$\frac{\sim\phi \land \sim\psi}{\sim(\phi \lor \psi)} \qquad \frac{\sim\phi \lor \sim\psi}{\sim(\phi \land \psi)}$$

$$\frac{\phi \land \psi}{\sim(\sim\phi \lor \sim\psi)} \qquad \frac{\phi \lor \psi}{\sim(\sim\phi \land \sim\psi)}$$

e.g. $W \to \sim(S \wedge T)$. $\sim S \to \sim Z$. $\sim T \to \sim Z$

$\therefore (\sim W \vee \sim Z)$

1	~~show~~ $\sim W \vee \sim Z$	
2	$\sim(\sim W \vee \sim Z)$	1 ASS ID.
3	$W \wedge Z$	2 DM
4	W	3 S
5	Z	3 S
6	$\sim(S \wedge T)$	4 Pr1 MP
7	$\sim\sim Z$	5 DN
8	S	Pr2 7 MT DN
9	T	Pr3 7 MT DN
10	$\sim S \vee \sim T$	6 DM
11	$\sim T$	8 DN 10 MTP
12		9 11 ID

2. Type 2: Situational Rules

1) Separation of Cases (SC)

$$\frac{\phi \vee \psi \quad \phi \to \chi \quad \psi \to \chi}{\chi} \qquad \frac{\phi \to \chi \quad \sim\phi \to \chi}{\chi}$$

2) Conditional as Disjunction (CDJ) (however, we see '$\to$' we use CD naturally than use CDJ.)

$$\frac{\phi \to \psi}{\sim\phi \vee \psi} \qquad \frac{\sim\phi \to \psi}{\phi \vee \psi}$$

$$\frac{\phi \vee \psi}{\sim\phi \to \psi} \qquad \frac{\sim\phi \vee \psi}{\phi \to \psi}$$

e.g. $(X \to \sim R) \to \sim(Z \to W)$. $\sim(P \vee W) \vee Q$.
$\sim W \to \sim(Z \wedge Q)$. $\therefore \sim P \vee (Q \leftrightarrow R)$.

1	~~show~~ $\sim P \vee (Q \leftrightarrow R)$	
2	$\sim(\sim P \vee (Q \leftrightarrow R))$	ASS ID
3	$\sim\sim P \wedge \sim(Q \leftrightarrow R)$	2 DM
4	P	3 S DN
5	$\sim(Q \leftrightarrow R)$	3 S
6	$Q \leftrightarrow \sim R$	5 NB
7	$P \vee W$	4 ADD
8	Q	7 DN Pr2 MTP
9	$Q \to \sim R$	6 BC
10	$\sim R$	9 MP
11.	~~show~~ $X \to \sim R$	
12.	X	ASS CD
13	$\sim R$	10 R.
14		12 13 CD.
15	$\sim(Z \to W)$	11 Pr1 MP
16.	$Z \wedge \sim W$	15 NC
17.	Z	16 S
18.	$\sim W$	16 S
19.	$\sim(Z \wedge Q)$	18 Pr3 MP

见到∧先simplify

没思路时一点点来. 全把Pr分析完, 总有contradiction出现.

20.	$\sim Z \vee \sim Q$	19 DM
21.	$\sim\sim Z$	17 DN
22.	$\sim Q$	20 21 MTP
23.		8 22 ID