Quantifiers

universal quantifiers
existential quantifiers

1. Quantifiers

1) $\forall$ = 'for all / every'

2) $\exists$ = 'there exist/is' (at least one)

e.g. $\forall x \in \mathbb{Z}, x > \pi$ FALSE

$\exists x \in \mathbb{R}$ such that $x^2 = 5$

benefit reading

e.g. The graph of f intercepts the x-axis

$\exists x \in \mathbb{R}$. s.t. $f(x) < 0$.

f is not the zero function.

$\exists x \in \mathbb{R}$. s.t. $f(x) \neq 0$.

The equation $f(x) = 0$ has no solutions.

$\forall x \in \mathbb{R}$, $f(x) \neq 0$; $\phi = \{x \in \mathbb{R} \mid f(x) = 0\}$.

2. Double Quantifiers

eg. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}. y > x$.

For every real number x, there exists a real number y which is bigger than x.

$\Leftrightarrow$ There is no largest real number.

1) Order of quantifiers:

① $\forall x \forall y = \forall y \forall x$; $\exists x \exists y = \exists y \exists x$

② may not $\forall x \exists y = \exists x \forall y$.

2) Negating Quantifiers: negates all parts.

① Statement | Negation

Statement	Negation
$\forall x, P(x)$	$\exists x, \neg P(x)$
$\exists x, P(x)$	$\forall x, \neg P(x)$

3. Simple Proof

e.g. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}$ s.t. $x<y$.

pf. Let $x \in \mathbb{Z}$ means fix an arbitrary value of x ($x \in \mathbb{Z}$)

Take $y = x+1$

Verify $y \in \mathbb{Z}$

$y > x$. ■

when taking value of y, x must be sth. specific.

any x. not need to choose.

not able to try all the x out.

4. Quantifiers & the empty set.

1) $\forall x \in \phi$, $x > 0$. True.

able to proof its truth; not able to proof its false.

① To proof sth. true, need to verify all elements satisfy

② To proof sth. false, need to find one element doesn't satisfy.

2) $\exists x \in \phi$ s.t. $x>0$. False.