d) Prove the following fact about subtraction of natural numbers (whenever applies): Suppose $m < n$. For any natural number w, prove that $n - m < w$ implies $n < w + m$. (Can you now explain to an elementary school student why we need to change the sign we drag the number across the equal or inequality sign?)

proof: Let $m \in \mathbb{N}$. $n \in \mathbb{N}$. $m < n$.

Let $w \in \mathbb{N}$.

Assume $n - m < w$.

Let $k = S_{min}$. which $S_{min} \in S$, and S_{min} is the smallest element in S. ($S = \{a \in \mathbb{N} ; n \leq m + a\}$)

From (c). we have $n = m + k$. when k is the least element in S, which $n - m = k$.

From assumption. we have $n - m = k < w$, which $k < w$.

Therefore $n \leq m + k < m + w$. since $k < w$. gives. $n < m + w$.

We've proved that $n - m < w$ implies $n < w + m$.

■

e) prove the following distributivity for subtraction (using the distribuitivity for addition): For natural numbers $m < n$ and w, prove that $w(n - m) = wn - wm$.

proof: Let $m \in \mathbb{N}$. $n \in \mathbb{N}$. $m < n$.

Let $w \in \mathbb{N}$.

Since $m \in \mathbb{N}$. $w \in \mathbb{N}$, gives. $wm \in \mathbb{N}$. which.

$w(n-m)$

$\Rightarrow w(n-m) + wm$. (adding wm. since $wm \in \mathbb{N}$).

$\Rightarrow w(n - m + m)$. (from the distribuitivity for addition).

$\Rightarrow wn$ (substracting wm since we've added at step 2).

$\Rightarrow wn - wm$

Therefore. we've proved that $w(n-m) = wn - wm$.

■