

# Conditional Statements.

1. Proposition:  $P(x)$  has a true value.

1) negation: not  $P(x) = \neg P(x)$ ; whenever  $P(x)$  is true,  $\neg P(x)$  is not true.

2. 'If, then' statement:

1) 'If  $P(x)$  then  $Q(x)$ ' is true <sup>if</sup> whenever  $P(x)$  is true then  $Q(x)$  is true; when  $P(x)$  is false, we don't care about  $Q(x)$  part.

e.g. Assume  $x \in A \Rightarrow x > 0$ . conclude?

•  $x \notin A$

No conclusion. (no  $P(x)$ ; don't care)

•  $x > 0$

No conclusion.

•  $x \leq 0 \Rightarrow x \notin A$ . (contrapositive)

e.g.  $0 > 1 \Rightarrow 103574289$  is prime; T/F?

True! when  $P(x)$  is false, don't care about  $Q(x)$

$P(x)$

$Q(x)$

$P(x) \Rightarrow Q(x)$

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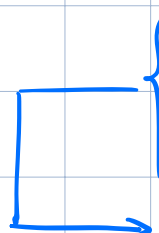
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$P(x) \Rightarrow Q(x)$

$\neq Q(x) \Rightarrow P(x)$

$= \neg Q(x) \Rightarrow \neg P(x)$



e.g. If it's sunny, I wear my sun-glasses. If  $P(x)$  is false,  $Q(x)$  is true, this is saying that it's not sunny.

but I wore my sunglasses anyway.  
(This doesn't invalid the original statement)

Not being sunny and me not wearing my sunglasses. (doesn't invalid)

2)  $P(x) \Rightarrow Q(x)$

① = 'If  $P(x)$  then  $Q(x)$ '

② converse:  $Q(x) \Rightarrow P(x)$

→ not logically equivalent.

③ contrapositive:  $\neg Q(x) \Rightarrow \neg P(x)$

→ equivalent

→ considering using the contrapositive.  
when it's hard to proof the original statement.

e.g. Let  $x$  be a positive integer. If  $x \neq 1$ , then  $x \neq x^2$ .

Solution:

When showing the question in contrapositive gives:

'Let ..... If  $x = x^2$ , then  $x = 1$ .'

Of course, this follows:

$$x \geq x.$$

$$\Leftrightarrow x^2 - x \geq 0.$$

$$\Leftrightarrow x(x-1) \geq 0.$$

$$\Leftrightarrow x=0 \text{ or } x=1.$$

there's always an implicit quantifier; but it comes to negation, should be explicitly.

and since we said  $x$  is a positive integer,  $x$  must be 1.

3. If and only if:

$$1) P(x) \Leftrightarrow Q(x)$$

$$\cdot P(x) \Rightarrow Q(x) \text{ and } Q(x) \Rightarrow P(x)$$

$\cdot P$  and  $Q$  are equivalent.

$\cdot P$  and  $Q$  both true / false.

4. Negation of a conditional statement.

e.g. let  $A \subseteq \mathbb{R}$ .

$\rightarrow (\forall x \in \mathbb{R})$  If  $x \in A$ , then  $x > 0$ .

means.

$$\rightarrow \forall x \in \mathbb{R} \left\{ \begin{array}{l} x \in A \text{ and } x > 0. \text{ or.} \\ x \notin A \text{ and } x > 0 \text{ or.} \\ x \notin A \text{ and } x \leq 0 \end{array} \right.$$

Negation:  $\exists x \in \mathbb{R}$ . s.t.  $x \in A$  and  $x \leq 0$ .

