Combinatorial Proof

1. Steps:
    1). 设定情景
    2). 解释每个 term
    3). Conclude.

2. 常见构造
    1). 看到 $\Sigma(\quad)$: 想 additional rule (一件事, 不同方式).
    2). 看到 $\Sigma(\quad)(\quad)$: 中有 times, 想 multiplication rule (一件事, 不同阶段).
    3). $\boxed{\triangle} = \Sigma \boxed{(\quad)(\quad)}$:
         ↘   ↙
         同一件事. (不是和 $\Sigma\square$).
    4). k^n : 创造 # of X-string, length $= n$, $X = \{1, \dots, k\}$.
        e.g. $7^n = \sum_{i=1}^{n} \binom{n}{i} 6^i$

        The number of X-string constructed from $X = \{1, \dots, 7\}$ with length n.
        LHS: construct number of X_1-string, length n where $X_1 = \{1, \dots, 7\}$.
        RHS: 6^i : construct number of X_2-string, length i, where $X_2 = \{1, \dots, 6\}$.
        $\binom{n}{i} 6^i$: # of X-string, length $= n$; $n-i$ positions are 7, and the rest
        i position 1-6.
        $\sum_{i=1}^{n} \binom{n}{i} \cdot 6^i$: # of X-string length n. $X = \{1, \dots, 7\}$, the positions of 1-6.
        from 1 to n covering all cases.

        Conclude:

    5). $k \cdot \binom{n}{k}$: ① choose k from n; ② choose 1 from k.
        $n \cdot \binom{n-1}{k-1}$: ① choose 1 from n; ② choose k-1 from n-1.

    set a & b to diff
              ↘ category.
    6). $\binom{a+b}{k}$: a blue, b red, choose k from a+b.

    7). $(-1)^k$: Inclusion - Exclusion.