Sets & Intervals

1. Sets: an unordered collection of distinct objects

can contain mixture of objects, including other sets: $\{1, a, \{70, \infty\}, y\}$

1) Notations.

① elements:

$\longrightarrow$ $\in$: is an element of set.

$3 \in \{1, 2, 3\}$

$\longrightarrow$ $\notin$: is not an element of set.

$4 \notin \{1, 2, 3\}$

② empty set: the set containing no elements.

ϕ or $\{\}$

$\longrightarrow$ $\{\phi\}$ is not the empty set: the set containing the empty set.

③ sets:

we can write $B \subseteq B$.

$\longrightarrow$ $B \subseteq A$: set B is a subset of set A: for all $b \in B$ we also have $b \in A$; A is a superset of B.

$\longrightarrow$ $A = B$ (set equlity): $A \subseteq B$ and $B \subseteq A$.

e.g. Let A be the set of numbers that can be expressed as $2n$ for some whole number n, and let B be the set of numbers ..... $m+1$, where m is an

odd number. Show $A = B$.

If $A = B$. it satisfies $A \subseteq B$, therefore. for any $x \in A$, gives $x \in B$.

$x = 2n = 2n-1+1$

★ Q on piazza.

$B = \{2x \mid x \in A\}$

B is the set of elements of the form of $2x$ such that x is an element of A.

④ Set-builder Notation:

⟶ $Y = \{a \in X \mid \text{some rule involving } a\}$

name of a set — take element from — such that — extra constraits.

Y is the set of element taken from X such that .....

⑤ Unions & Intersections.

⟶ Unions: $X \cup Y = \{a \mid a \in X \text{ or } a \in Y\}$

⟶ Intersections: $X \cap Y = \{a \mid a \in X \text{ and } a \in Y\}$

2) Associativity: $(A \cup B) \cup C = A \cup (B \cup C)$

3) Common Sets:

① $\emptyset = \{\}$

remember to write the

extra bar.

② $\mathbb{N} = \{0, 1, 2, 3, \dots\} = \{\text{natural numbers}\}$

③ $\mathbb{Z} = \{\dots, -3, -2, -1, 0, \dots\} = \{\text{integers}\}$ (fractions)

④ $\mathbb{Q} = \{\text{rational numbers}\} = \{\text{quotient of integers}\}$

⑤ $\mathbb{R} = \{\text{real numbers}\} = \{\text{numbers with a deci}$

⑥ $\mathbb{R}^n = \{\text{vectors in } n\text{-dimensional Euclidean space}\}$.

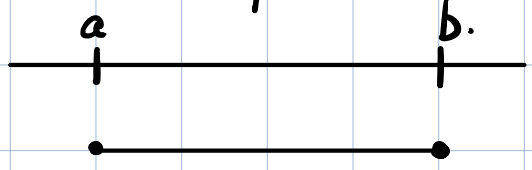
2. Intervals.

$[0,0] = \{0\}$

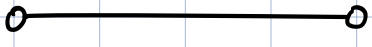
$(0,0) = \emptyset$

Let $a, b \in \mathbb{R}$.

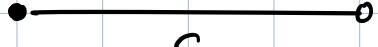
1) $[a, b] = \{x \in \mathbb{R} \mid a \le x \le b\}$



2) $(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$



3) $[a, b) = \{x \in \mathbb{R} \mid a \le x < b\}$



4) $(a, \infty) = \{x \in \mathbb{R} \mid a < x\}$