.

Please note:

1. Ideally the PS must be written in the spaces provided, and page by page (a picture or scan of individual pages) to be uploaded to the appropriate pages of a Crowdmark file (link to be posted.) If you wish, you can write your answers on separate pieces of paper and upload them properly at each page of the Crowdmark. Please note that any mismatch in uploading the pages of the solutions may cause in parts of the solutions not getting marked.

2. Please provide your final, polished solutions in the spaces provided. If you have no access to a printer you can write your solutions on a blank sheet of paper and upload a picture of scan. Remember, it is an art to write a short yet complete solution; one learns a lot from practicing this art. Please write a complete, even though long solutions. Then inspect and edit; while editing, ask yourselves:

   - am I proving common believes and facts accepted in the literature? If yes, know which facts are common belief and which ones need an argument.
   - can I just quote some of the facts that I tried to prove? This requires a good familiarity with the literature of the subject (in our case, the textbook and the slides, and earlier questions in the same PS).
   - am I introducing too many cases in my proof, and presenting parallel arguments? Change your type of argument if possible (direct proof versus proof by cases, or proof by contradiction.)

   Editing your first complete solution is a very good way of learning. Make sure you have a good solution, then return to it and reflect on the above questions.

3. Problem set questions contain ideas complementary to the textbook and lecture materials, opportunity for reflection and deepening on the subject. The level of difficulty of the questions is elementary so that each student has chance to individually think about the problems. So, please don't let your "kind" friends take this opportunity away from you!

4. Not all questions will be marked, nor they are all of equal weight, and it is not known in advance which questions will be marked, and how heavy they will be weighed. Therefore if a question is skipped one may lose a larger portion of the PS mark than one would expect.

5. To receive full mark on the computational questions all the computation details must be provided. For questions involving proofs there is no need to prove textbook or "slides" proofs but one can simply quote them. When a theorem is applied the relevance of it to the proof must be made clear.

6. Collaboration in this work is not allowed. You may discuss the idea of a the solutions only, but each person must write their own solutions completely independently, without knowing or consulting the structure and details of the argument of another person.

   So please be careful not to share any written hint with your friends, because some people have photographic memory, and then your solutions may become subject to plagiarism.

7. By participating in this problem set you agree that you are aware of the university regulations regarding plagiarism. Individual students are expected to organize and write their own solutions independently. During the online teaching, and while students are connected through email, any assistance to your fellow classmate in the form of a written note might be directly copied and it may count as a form of plagiarism. Markers are carefully monitoring the solutions, and any incidence of plagiarism, and all the parties involved will be dealt with according to university regulations. Please carefully read the item regarding plagiarism in our course outline, and consult the university policies regarding plagiarism (not knowing the rules is not a good excuse for not obeying them):

   http://www.artsci.utoronto.ca/newstudents/transition/academic/plagiarism

0. **Failure to answer this question may result in a mark of 0 on the entire PS2. And it is not the name, signature and date, but it is the acknowledgement .. signing some statement that is intended in this part:**

Please sign below as an acknowledgment that you have read the cover page ; **(unsigned papers shall not be marked.)**

Print Name (first, last) : XUANQI, WEI., Signature: Henry Wei Date: 2023.11.18.

1. Recursively defining bijections.

a) Given a bijection $f : A \times A \longrightarrow A$, use it to define another bijection $g : A \times A \times A \longrightarrow A$. Make sure to prove g is a bijection given that f is a bijection.

Let $x_1, x_2, x_3, y_1, y_2, y_3 \in A$

Assume f is a bijection, $f(x_1, x_2) = f(y_1, y_2) \Rightarrow (x_1, x_2) = y_1, y_2$.

WTS: g is a bijection, i.e. g is injective and surjective

① g is injective.

Assume $g((x_1, x_2, x_3)) = g((y_1, y_2, y_3))$.

WTS: $x_1 = x_2$.

Since f is bijective, which f is surjective and injective. gives $f(f(x_1, x_2)), x_3) = f(f((y_1, y_2), y_3))$.

Since $f(x_1, x_2) = f(y_1, y_2)$ and $x_3 = y_3$.

Thus. $x_1 = y_1$ and $x_2 = y_2$ as well. which g is injective.

② g is surjective.

Assume $\forall b \in A$, $\exists (x_1, x_2) \in A \times A$. s.t. $f(x_1, x_2) = b$

WTS: $\forall b \in A$. $\exists (x_1, x_2, x_3) \in A \times A \times A$. s.t $g(x_1, x_2, x_3) = b$

Let $b \in A$. $\exists (x_1, x_2) \in A \times A$ s.t. $f(x_1, x_2) = b$.

WTS: $f(f(x_1, x_2), x_3) = b$.

Since $b \in A$. $\exists (y_1, y_2) \in A \times A$. s.t. $f(y_1, y_2) = b$.

Let $x_3 = y_2$. Since $y_1 \in A$. we know $\exists (x_1, x_2) \in A \times A$. s.t. $f(x_1, x_2) = y_1$. Therefore. $b = f(y_1, y_2) = f(f(x_1, x_2), x_3)$ as needed

b) Given a bijection $f : A \times A \longrightarrow A$ (as in part a), present a short recursive argument to prove for each n, there is a bijection between A^n (the Cartesian product of n copies of the set A), and the set A itself.

c) Apply a problem from PS3 to conclude that any finite Cartesian product of copies of $\mathbb{N}$ must be of the same cardinality (see Definition 10.1.7) with $\mathbb{N}$.

I'll do the complete Induction. on n. where n is the copies of $\mathbb{N}$.

Base