ODE Intro

w.r.t.

1. Differential Equations describe how a quantity change with respect to other quantity

2. Ordinary Differential Equations involving only derivative (up to a finite order) of a single variable.

$$F(t, y, y', y'', \dots, y^{(n)}) = 0.$$

自变量 (independent variable) ↖ 因变量 (dependent variable)

e.g. $F(t, y, y^{(n)}) = 0$.

3. Classification of ODE.

n^{th} derivative

$y^{(n)} = \frac{d^n y}{dt^n}$.

most of the time will be t.

1) systems of ODE: multiple functions that depends on t.

e.g. $\begin{cases} \frac{dx}{dt} = x(a - by). \\ \frac{dy}{dt} = y(x - d). \end{cases}$

2). order of ODE: highest order of derivative that appears.

e.g. $\frac{dy}{dt} + ty^2 = 0$. — 1st order.

3). Homogenous v.s. non-Homo

有无与 y 及其 derivative 无关的项.

① Homo: Involve only derivatives of y and terms involving y.

RHS always. 0.

e.g. $\frac{d^4y}{dy^4} + x\frac{d^2y}{dy^2} + y^2 = 0$.

② Non-Homo: Can have terms involving only x and const. on R.H.S.

e.g. $\frac{d^4y}{dy^4} + x\frac{d^2y}{dy^2} + y^2 = 6x + 3$.

4). linear v.s. non-linear

① linear: linear in y and its derivatives $y', y'', \dots, y^{(n)}$.

$g(t) = 0$ Homo;

$g(t) \neq 0$ Non-Homo

$$a_n(t)y^{(n)} + a_{n-1}(t)y^{(n-1)} + \cdots + a_0 y = g(t).$$

e.g. $y'' + \frac{1}{4}y' + y = 2\cos t$.

② non-linear

e.g. $y' = ry(1 - \frac{y}{k})$.

e.g. $y \cdot y'$