CSC236 Problem Set 2

Question 1.

Let $\heartsuit$ be the statement: $\forall n,\ a_n \le a_{n+1}$.

Let $\Diamond$ be the statement: $\forall n,\ \forall k \le n,\ a_k \le a_n$.

(a) WTP: $\Diamond \Rightarrow \heartsuit$.

Proof: Let $n \in \mathbb{N}$. Let $k \in \mathbb{N}$. $k \le n$.

Assume $a_k \le a_n$.

Proof by contradiction, which $\exists n.\ a_n > a_{n+1}$.

Since $k \le n$, gives. $k \in S$, where $S = \{n, n-1, n-2, \dots, n-n\}$.

Since $a_n > a_{n+1}$, $a_n \ge a_k$, which $\exists k$, s.t. $k' = n+1$. which $k' \in S$, gives $(n+1) \in S$.

However, $\forall s \in S$, $S_{max} = n$. and $S_{max} \ge S$, since $n \in \mathbb{N}$. $(n+1) > n$. and $(n+1) \in S$ contradicts

Therefore, we've proved. $(\forall n,\ \forall k \le n,\ a_k \le a_n) \Rightarrow (\forall n,\ a_n \le a_{n+1})$, i.e. $\Diamond \Rightarrow \heartsuit$.

$\blacksquare$

(b). WTP: $\heartsuit \Rightarrow \Diamond$ (by simple induction).

Proof: Let $n \in \mathbb{N}$. assume $a_n \le a_{n+1}$. — assumption 1.

Base Case: when $k=0$, $a_k = a_0$ which $a_0 \le a_n$. since $a_0 \le a_1 \le a_2 \le \dots \le a_n \le a_{n+1}$ from $\heartsuit$.

I've proved the base case is true.

Induction Step. Let $n \in \mathbb{N}$. Let $k \in \mathbb{N}$.

Induction Hypothesis: Assume $\forall k \le n$, $a_k \le a_n$

WTP: $\forall k \le n+1$, $a_k \le a_{n+1}$.

From induction hypothesis. $\forall k \le n$, $a_k \le a_n$.

From assumption 1, $a_n \le a_{n+1}$. which gives. $a_k \le a_n \le a_{n+1}$. when $k \le n$.

When $k = n+1$. $a_k = a_{n+1} \le a_{n+1}$. gives. $\forall k \le n+1$, $a_k \le a_{n+1}$.

I've proved the induction step is true.

Therefore. $\heartsuit \Rightarrow \Diamond$

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