PMI & PCMI.

$S = \{x \in \mathbb{N} \mid P(x) \text{ is True}\}$

1. PMI. $(\forall n \in \mathbb{N}.\ P(n))$.

1). If S is any set of natural numbers with the properties that:

① 1 is in S, and

② $k+1$ is in S whenever k is any number in S.

then S is the set of all natural numbers.

2). Generalized: $m \in \mathbb{N}$. S is any set of natural numbers with the properties that:

① m is in S, and

② $k+1$ is in S whenever k is in S and is greater than or equal to m.

then S contains every natural number greater than or equal to m.

3). Structure.

① Given the statement to prove: $\forall n \in \mathbb{N}, \overset{(n \geq m)}{\vee} P(n)$ is True, i.e. $P(n)$:

$S = \{n \in \mathbb{N} \mid (n \geq m). P(n)\}$.

② I'll apply induction on n.

③ B.C. $n = 1$ (or m)., $1 \in S$ ($m \in S$).

④ Induction Step. Let $n \in \mathbb{N}$. $n \geq 1$ ($n \geq m$).

⑤ Induction H.: Assume $P(n)$.

⑥ WTP: $P(n+1)$., ... $n+1 \in S$. ∎

2. PCMI ($\forall n.\ P(n)$).

1). If S is any set of natural numbers with the property that.

① 1 is in S, and.

② $k+1$ is in S whenever k is a natural number and all of natural number from 1 to k is in S.

S is the set of all natural numbers.

2). Generalized: If S is any set of natural numbers with properties that

① m is in S, and

② $k+1$ is in S whenever k is a natural number greater or equall to m. and all of natural number from m to k is in S.

S is the set of all natural numbers geq. m.

3). Structure.

① Given the statement to prove: $\forall n \in \mathbb{N}$, $(n \geq m)$. $P(n)$ is True, i.e. $P(n)$:

$S = \{ x \in \mathbb{N} \mid (n \geq m), P(n) \}$.

② Will apply induction on n.

③ B.C. $n = 1$ (or m).

④ Induction Step. Let $n \in \mathbb{N}$. $n \geq 1$ $(n \geq m)$.

⑤ Induction H.: Assume $\forall k \in \mathbb{N}$. $1 \leq k \leq n$, $(m \leq k \leq n)$, $P(k)$

⑥ WTP: $P(n+1)$.

------ $n+1 \in S$. ■