Binomial Theorem.

$x, y.\ x+y \neq 0.$

1. $x, y \in \mathbb{R}.\ n \in \mathbb{Z}^+.$

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$$ ← 和为 n.

2. $x_i \in \mathbb{R}.\ x_i \neq 0.\ n \in \mathbb{Z}^+.$

$$(x_1 + \cdots + x_r)^n = \sum_{k_1 + \cdots + k_r = n} \binom{n}{k_1, k_2, \ldots, k_r} \cdot x_1^{k_1} \cdot x_2^{k_2} \cdot \cdots \cdot x_r^{k_r}.$$ where.

$$\binom{n}{k_1, k_2, \ldots, k_r} = \frac{n!}{k_1! k_2! \cdots k_r!}.$$

3. $\forall n \in \mathbb{N}.$

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} \cdot x^k \ ; \quad (1-x)^n = \sum_{k=0}^{n} \binom{n}{k} (-1)^{n-k} \cdot x^k.$$

4. Newton's Binomial Theorem.: $p \in \mathbb{R}.\ p \neq 0.\ k \in \mathbb{Z}.$

$$(1+x)^p = \sum_{k=0}^{\infty} \binom{p}{k} x^k \quad \text{where} \quad \binom{p}{k} = \frac{p(p-1)\cdots(p-k+1)}{k!}.$$

e.g. $$\binom{-1}{k} = \frac{(-1)(-1-1)\cdots(-1-k+1)}{k!} = (-1)^k \cdot \frac{k!}{k!} = (-1)^k.$$