Definition of Derivatives

1. Definition

Let $a \in \mathbb{R}$.

Let f be a function defined, at least, on an inteval centred at a.

1) The derivative of f at a is the number

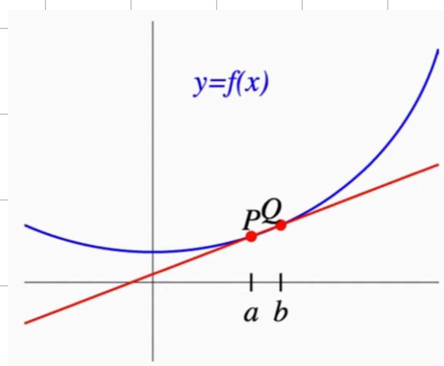
$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

2) $h = x - a$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

'f is differentiable at a' when this limit exists.

'f is differentiable' means at all points on its domin.



2. Tangent Line Revisited.

The line tangent to the graph of $y = f(x)$ at the point with x-coordinate a is the line

- through the point $(a, f(a))$
- with slope $f'(a)$

$$y = f(a) + f'(a)(x-a)$$

3. Example of calculating derivative from definition.

e.g. $f(x) = x^2 - 4x$.

$$f'(1) = \lim_{h\to 0} \frac{f(h+a) - f(a)}{h}$$
$$= \lim_{h\to 0} \frac{(h+a)^2 - 4(h+a) - a^2 + 4a}{h}$$
$$= \lim_{h\to 0} \frac{h^2 + a^2 + 2ah - 4h - 4a - a^2 + 4a}{h}$$
$$= \lim_{h\to 0} \frac{h^2 + 2ah - 4h}{h}$$
$$= \lim_{h\to 0} [h + 2a - 4] = -2.$$

4. Derivatives as rate of change.

e.g. physical quantities Q, x; Q depends on x.

average rate of change: $\frac{\Delta Q}{\Delta x}$.

instantaneous rate of change: $\lim_{\Delta x \to 0} \frac{\Delta Q}{\Delta x} = \frac{dQ}{dx}$.

derivative of Q with respect to x.

e.g. t = time, x = position $x = f(t)$.

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{t_2 \to t_1} \frac{f(t_2) - f(t_1)}{t_2 - t_1} = f'(t_1).$$