Sets, Vectors & Notation

1. Sets: an unordered collection of distinct objects

can contain mixture of objects, including other sets: $\{1, a, \{70, \infty\}, y\}$

1) Notations.

① elements:

→ $\in$: is an element of set.

$3 \in \{1,2,3\}$

→ $\notin$: is not an element of set.

$4 \notin \{1,2,3\}$

★ e.g. $A = \{1\}$
$B = \{\{1\}, \{1,2,3\}\}$
$A \in B$.
$\underline{\subseteq}$.

② empty set: the set containing no elements.

ϕ or $\{\}$

→ $\{\phi\}$ is not the empty set: the set containing the empty set.

③ sets:

→ $B \subseteq A$: set B is a subset of set A: for all $b \in B$ we also have $b \in A$; A is a superset of B.

we can write $B \subseteq B$.

→ $A = B$ (set equality): $A \subseteq B$ and $B \subseteq A$.

e.g. Let A be the set of numbers that can be expressed as $2n$ for some whole number n, and let B be the set of numbers ...... $m+1$, where m is an

* Q on piazza.

odd number. Show $A = B$.

If $A=B$, it satisfies $A \subseteq B$, therefore for any $x \in A$, gives $x \in B$.

$x = 2n = 2n-1+1 = (2n-1)+1$

when $m = 2n-1$, which is an odd number by definition. $m+1$ is an even number; $A \subseteq B$.

$B \subseteq A$. for any $x \in B$, gives $x \in A$. by definition $m = 2k+1$. $x = m+1 = 2k+1+1 = 2(k+1)$; $n = k+1$. $\therefore x \in A$. $B \subseteq A$.

④ Set-builder Notation:

⟶ $Y = \{a \in X \mid \text{some rule involving } a\}$

name of a set; take element from; such that; extra constraints.

Y is the set of element taken from X such that ......

$B = \{2x \mid x \in A\}$

B is the set of elements of the form of 2x such that x is an element of A.

⑤ Unions & Intersections.

⟶ Unions: $X \cup Y = \{a \mid a \in X \text{ or } a \in Y\}$

⟶ Intersections: $X \cap Y = \{a \mid a \in X \text{ and } a \in Y\}$

⑥ Sum of two Sets.

⟶ $A+B = \{x+y \mid x \in A \text{ and } y \in B\}$.

2) Associativity: $(A \cup B) \cup C = A \cup (B \cup C)$

3) Common Sets:

① $\phi = \{\}$

remember to write the extra bar.

② $\mathbb{N} = \{0, 1, 2, 3, \ldots\} = \{$natural numbers$\}$

③ $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, \ldots\} = \{$integers$\}$

(fractions)

④ $\mathbb{Q} = \{$rational numbers$\} = \{$quotient of integers$\}$

⑤ $\mathbb{R} = \{$real numbers$\} = \{$numbers with a deci

⑥ $\mathbb{R}^n = \{$vectors in n-dimensional Euclidean space$\}$.

4) Some and All.

漏一个就不行.

① $A = \{x \mid x = 2n$ for all $n \in \mathbb{N}\}$

$x = 2\times1 = 2\times2 = 2\times3 = 2\times4$ ......

↳ no any number can satisfy.

$A \neq \phi$

有一个就行.

② $B = \{y \mid y = 2n$ for some $n \in \mathbb{N}\}$

$2 \in B$; $2 = 2\times1$　　$4 \in B$; $4 = 2\times2$.

$6 \in B$; $6 = 2\times3$　　$\therefore B =$ set of all even number.

2. Vectors & Scalars.

1) definition.

① scalar: models a relationship between quantities.

② vector: models a relationship between points. (displacement with a direction and a magnitude)

2) Notation

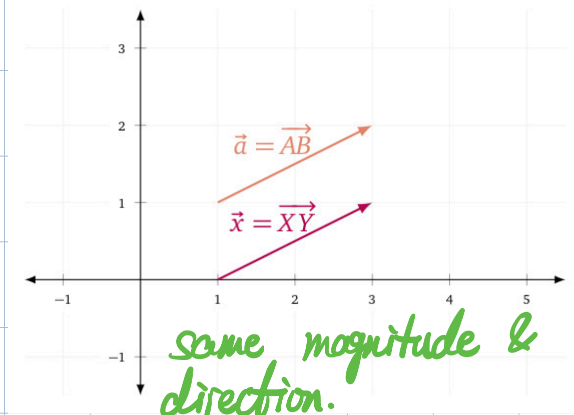
① $\overrightarrow{PQ}$: the vector from P to Q.

② $\|\vec{a}\|$: the magnitude of the vector $\vec{a}$. (norm or length is sometimes called).

3) Line segments $\not\leftrightarrow$ vector.

① represent vectors as directed line segments (with an arrow at the end)

↳ direct line segments always start somewhere, but a vector models a displacement and has no sense of 'origin'.



same magnitude & direction.

different line segments but same vector.

$\vec{a} = \vec{x}$ despit $A \neq X$.

4) treat vectors & points interchangeably.

5) zero vector: $\vec{0}$, the vector with no magnitude

① every directions / no direction / never talk about direction.

3. laws (axioms) of vector arithmetic

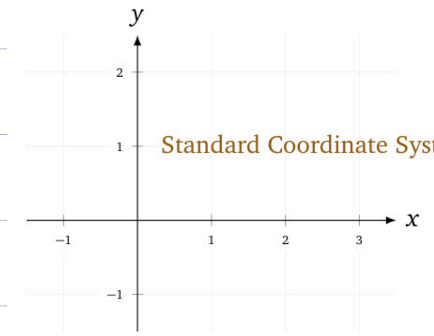
1) linear combinations of the vectors $\vec{v_1}$, $\vec{v_2}$, ..., $\vec{v_n}$ is a vector.

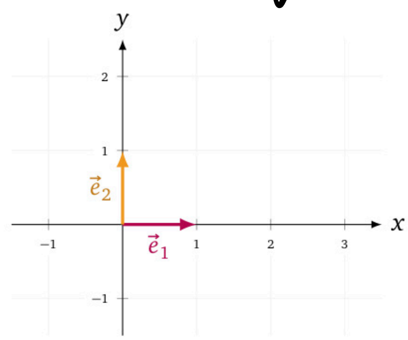
$$\vec{w} = a_1\vec{v_1} + a_2\vec{v_2} + \cdots + a_n\vec{v_n}.$$

coefficients.

4. Coordinates and the Standard Basis.

1) Euclidean plane (notated by $\mathbb{R}^2$)





2) standard basis vectors:

① $\vec{e_1}$: always points one unit in the direction of the positive x-axis

② $\vec{e_2}$: always points one unit in the direction of the positive y-axis.

3) standard coordinates:

① $\vec{w} = \alpha\vec{e_1} + \beta\vec{e_2}$, call the pair (α, β) the standard coordinates of vector $\vec{w}$.

② (α, β) ; $<\alpha, \beta>$; $[\alpha, \beta]$; $\begin{bmatrix}\alpha \\ \beta\end{bmatrix}$.

③ often write: $\vec{v} = \alpha\vec{e_1} + \beta\vec{e_2}$ in

$\vec{v} = \begin{bmatrix}\alpha \\ \beta\end{bmatrix}$. as a shorthand