Derivative of the Inverse of a Function.

1. Theorem of differentiable f^{-1}.

Let f be a function defined on an interval I.

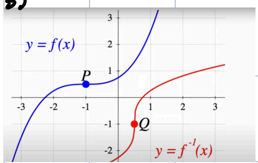
If 1) f has an inverse (injective)

2) f is differentiable ($\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ exists).

3) For all $x \in I$, $f'(x) \neq 0$.

Then f^{-1} is differentiable.

3)



not differentiable at Q.

2. Derivative

1) Derivation.

$$\frac{d}{dy}[f(f^{-1}(y))] = \frac{d}{dy}(y)$$

$$\Rightarrow f'(f^{-1}(y)) \cdot (f^{-1}(y))' = 1 \qquad \text{since } f^{-1}(y) = x$$

$$\Rightarrow f'(x) \cdot (f^{-1}(y))' = 1$$

$$\Rightarrow (f^{-1}(y))' = \frac{1}{f'(x)}$$

f and f^{-1} are evaluated at different points.