Online Quiz 6.

1. Let a, b relatively prime to m. $m > 1$.

WTS. ab relatively prime to m.

Since a relatively prime to m, gives $\gcd(a, m) = 1$, which $\exists r_1, s_1 \in \mathbb{Z}$. s.t. $r_1 a + s_1 m = 1$ (from lemma 1).

Since b relatively prime to m, gives $\gcd(b, m) = 1$. which $\exists r_2, s_2 \in \mathbb{Z}$. s.t. $r_2 b + s_2 m = 1$. (from lemma 1).

Multiplying the equations, gives, $(r_1 a + s_1 m)(r_2 b + s_2 m) = 1$

$$\Rightarrow r_1 r_2 ab + r_1 a \cdot s_2 m + s_1 m \cdot r_2 b + s_1 \cdot s_2 m = 1$$

$$\Rightarrow (r_1 \cdot r_2) ab + (r_1 s_2 a + r_2 s_1 b + s_1 \cdot s_2) m = 1.$$

Since $r_1, r_2 \in \mathbb{Z}$. gives $r_3 = r_1 \cdot r_2$, $r_3 \in \mathbb{Z}$.

Since $r_1, s_1, r_2, s_2, a, b \in \mathbb{Z}$. gives. $(r_1 s_2 a + r_2 s_1 b + s_1 \cdot s_2) = s_3$, $s_3 \in \mathbb{Z}$.

Therefore, ab is relatively prime to m. as $r_3 \cdot (ab) + s_3 m = 1$. from lemma 0. ∎

2. Let $n, k, r \in \mathbb{N}$, $n > 1$, $1 \le r < n$.

WTS: $\gcd(nk+r, n) = \gcd(n, r)$ (if n, r are relative prime, $nk+r, n$ are relatively prime)

Assume n and r are relatively prime. which $\gcd(n, r) = 1$, by lemma 0.

Say $\gcd(nk+r, n) = q$. gives. $\exists t_1, s_1 \in \mathbb{Z}$. s.t. $q = t_1(nk+r) + s_1 n$. by lemma 0, which $q = t_1 nk + t_1 r + s_1 n = (t_1 k + s_1) n + t_1 r$. since. $t_1, k, s_1 \in \mathbb{Z}$, gives.

$q = \gcd(n, r)$. by lemma 1; $\gcd(n, r) = q = \gcd(nk+r, n)$.

Since from the assumption. $\gcd(n, r) = 1$, gives $\gcd(nk+r, n) = 1$.

Therefore. $nk+r$ and n are relatively prime. ∎