

# Inequalities

1. Markov's Inequality: If  $X \geq 0$  and  $a > 0$ , then  $P(X \geq a) \leq \frac{E(X)}{a}$ .

(Only for non-negative)

e.g.  $X \geq 0$ .  $E(X) = 5$ .

$$P(X \geq 100) \leq \frac{E(X)}{a} = \frac{5}{100} = 0.05.$$

2. Chebychev's Inequality. For any  $a > 0$ ,  $P(|Y - \mu_Y| \geq a) \leq \frac{\text{Var}(Y)}{a^2}$ .

$$\text{proof: } P(|Y - \mu_Y| \geq a) = P((Y - \mu_Y)^2 \geq a^2) \leq \frac{E[(Y - \mu_Y)^2]}{a^2} = \frac{\text{Var}(Y)}{a^2}.$$

e.g.  $\mu_Z = 5$ .  $\text{Var}(Z) = 9$ .

$$P(Z \geq 17) = P(Z - 5 \geq 12) \leq \frac{\text{Var}(Z)}{12^2} = \frac{9}{144}.$$

