Complex Numbers.

1. $z = a+bi$, $a,b \in \mathbb{R}$.

1). $Re(z) = a$, $Im(z) = b$.

2) $i^2 = -1$

3). $\mathbb{C} = \{a+bi : a,b \in \mathbb{R},\ i^2=-1\}$.

4). Proposition:

①. $a+bi = c+di$ iff $a=c,\ b=d$.

②. $(a+bi)+(c+di) = (a+c)+(b+d)i$

③ $(a+bi)(c+di) = (ac-bd)+(ad+bc)i$.

④. If $a+bi \neq 0$. then $\frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}i$ is the multiplicative inverse of $a+bi$.

Suppose. $(a+bi)(c+di) = 1$. multiplying $\overline{(a+bi)}$ gives.

$$(a^2+b^2)(c+di) = a-bi$$

$$\Rightarrow \quad c+di = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}i.$$

2. Complex Conjugate: $a-bi$ is the C.C. of $a+bi$. which denote as $\overline{a+bi}$

$z\cdot\bar{z} = a^2+b^2$.
$= |z|^2$.
$\Rightarrow \bar{z} = \frac{|z|^2}{z}$.
useful in polar forms.

1). z is real iff. $z = \bar{z}$.

2). Let $s,t \in \mathbb{R}$. Then $\overline{sz_1 \pm tz_2} = s\bar{z}_1 \pm t\bar{z}_2$.

3). $\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$.

4). $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$.

5). $\overline{z^n} = (\bar{z})^n$.

6). Given polynomial $p(z)$ with real coefficient: $\overline{p(z)} = p(\bar{z})$.

3. Modulus & Argument.

1). Modulus of $a+bi$ is $r = \sqrt{a^2+b^2}$ denote as. $|a+bi|$.

①. $z=0$ iff $|z|=0$.

②. $|z| = |\bar{z}|$.

③. $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$.

④. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$.

⑤. $|z^n| = |z|^n$.

2). Argument: An angel of deviation from polar-axis. measured. c.c.w.

①. $\arg(\bar{z}) = -\arg(z) = 2\pi - \arg(z)$.

②. $\arg(z_1 \cdot z_2) = \arg(z_1) + \arg(z_2)$. (Thm. 9.2.4).

4. Polar Form: $z = a+bi = r(\cos\theta + i\sin\theta)$.

5. De Moivre's Theorem. $\forall n \in \mathbb{N}$.

$$(r(\cos\theta + i\sin\theta))^n = r^n \cdot (\cos n\theta + i\sin n\theta).$$

6. n^{th} root of unity: (Solve $z^n = 1$ using. D.M.T.).

$\vdots$

$\cos n\theta + i\sin n\theta = 1 = \cos 0 + i\sin(0)$.

$$\begin{cases} \cos n\theta = 1. \\ \sin n\theta = 0. \end{cases} \Rightarrow \begin{cases} n\theta = 2k\pi. \\ n\theta = k\pi \end{cases} \Rightarrow n\theta = 2k\pi \Rightarrow \theta = \frac{2k\pi}{n}.$$

$k = 0, 1, \dots, n-1$.

All sol^{n}: $1, \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}, \cos\frac{4\pi}{n} + i\sin\frac{4\pi}{n}, \dots, \cos\left(\frac{2\pi(n-1)}{n}\right) + i\cdot\sin\left(\frac{2\pi(n-1)}{n}\right)$.