Basic Knowledge

can't be empty

1. Sample Space: the set of all possible outcomes. (S).

   e.g. flip a coin: $S = \{Head, Tail\}$.

   1) Event A is 'any' subset $A \subseteq S$.

      ① $P(A)$: probability that A will occur.

2. Basic Properties of Probabilities

   1) If A is an event, $0 \le P(A) \le 1$.

   2) If $A = S$, $P(A) = P(S) = 1$

   3) If $A = \phi$, $P(A) = P(\phi) = 0$.

   4) Additivity: if $A_1, A_2, A_3, \dots$ are any sequence (finite/infinite) of disjoint events (i.e. $A_i \cap A_j = \phi$, whenever $i \neq j$), then

$$P(\cup_i A_i) = \Sigma_i P(A_i).$$

   e.g. $A = Heads$. $B = Tails$. $P(A \cup B) = P(A) + P(B)$

$$= P(Head) + P(Tails).$$

$$= 0.5 + 0.5 = 1.$$

3. Derived Properties of Probabilities

A^c: the set of all outcomes which are not in A.

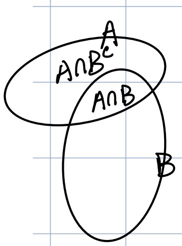
   1) Fact: If A^c is the complement of A, the $P(A^c) = 1 - P(A)$.

   proof: Since A and A^c are disjoint, gives

$$P(A \cup A^c) = P(A) + P(A^c).$$

   Since $P(A \cup A^c) = P(S) = 1$, $1 = P(A) + P(A^c)$. i.e.

$$P(A^c) = 1 - P(A)$$



   2) Fact: For any events A and B, $P(A) = P(A \cap B) + P(A \cap B^c)$

   proof: Since event $A \cap B$ and $A \cap B^c$ are disjoint, and $(A \cap B) \cup (A \cap B^c) = A$ according to the diagram.

   By additivity, $P((A \cap B) \cup (A \cap B^c)) = P(A \cap B) + P(A \cap B^c)$

$$= P(A).$$