

Functional Dependency

8.Functional Dependency

A “good” database schema should not lead to *update anomalies*.

- update anomalies,
- functional dependencies,
- Armstrong Axioms,
- closures.

8.1 Update Anomalies

- Redundancy in a database means storing a piece of data more than once.
- Redundancy is often useful for efficiency and semantic reasons, but creates the potential for consistency problems.
- A poor *redundancy control* may cause update anomalies.
- Consider the example relation below (adapted from “An Introduction to Database Systems” by Desai):

STUDENTS					
Name	Course	Phone_no	Major	Prof	Grade
Jones	353	237-4539	Comp Sci	Smith	A
Ng	329	427-7390	Chemistry	Turner	B
Jones	328	237-4539	Comp Sci	Clark	B
Martin	456	388-5183	Physics	James	A
Dulles	293	371-6259	Decision Sci	Cook	C
Duke	491	823-7293	Mathematics	Lamb	B
Duke	356	823-7293	Mathematics	Bond	UN
Jones	492	237- 4539	Comp Sci	Cross	UN
Baxter	379	839-0827	English	Broes	C

- *Modification anomalies*: e.g. Jones's phone number appears 3 times. When a phone number is changed, it must be changed in all 3 places, or the data will be inconsistent.

Insertion anomalies:

- If Jones enrolls in another course, and a different phone number is entered, again the data will be inconsistent.
- Also, if the only way that the association between course and professor is stored in this relation, we can only enter the association when someone enrolls in the course.

Deletion anomalies: If the last student in a course is deleted, the association between professor and course is lost.

8.2 Functional dependencies

A function f from S_1 to S_2 has the property

$$\text{if } x, y \in S_1 \text{ and } x = y, \text{ then } f(x) = f(y) .$$

A generalization of keys to avoid design flaws violating the above rule.

Let X and Y be sets of attributes in R .

X (*functionally*) determines Y , $X \rightarrow Y$, iff $t_1[X] = t_2[X]$ implies $t_1[Y] = t_2[Y]$.

i.e., $f(t(X)) = t[Y]$

We also say $X \rightarrow Y$ is a *functional* dependency, and that Y is *functionally* dependent on X .

X is called the *left side*, Y the *right side* of the dependency.

Examples:

- For every Name, there is a unique Phone_no and Major, assume Name is unique
- For every Course, there is a unique Prof
- For every Name and Course, there is a unique Grade

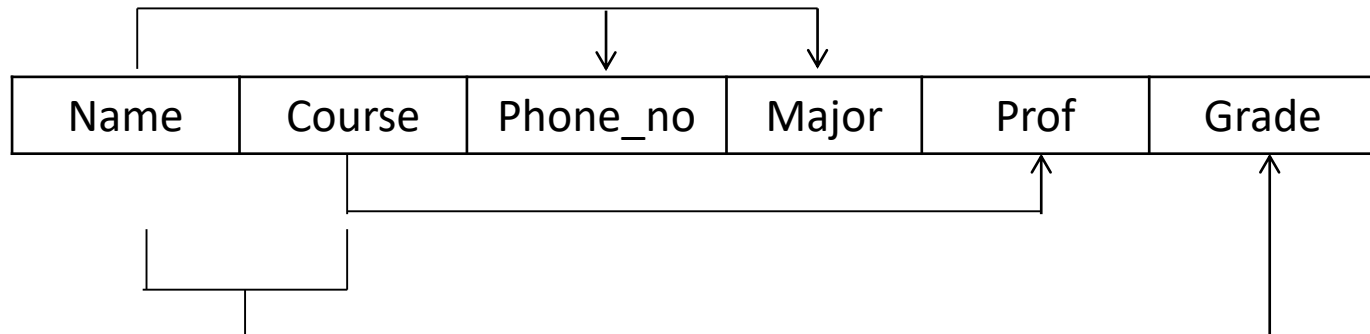
- In this example:

$$\{Name\} \rightarrow \{Phone_no, Major\}$$

$$\{Course\} \rightarrow \{Prof\}$$

$$\{Name, Course\} \rightarrow \{Grade\}$$

- We can also show these in a diagram like this one:



- Notice that other FD's follow from these:

$$\{Name\} \rightarrow \{Major\}$$

$$\{Course, Grade\} \rightarrow \{Prof, Grade\}$$

- Let F be a set of FD's.

Definition 1: $X \rightarrow Y$ is inferred from F (or that F infers $X \rightarrow Y$), written in

$$F \models X \rightarrow Y$$

- if any relation instance satisfying F must also satisfy $X \rightarrow Y$.
- Impossible to list every relation to verify if $X \rightarrow Y$ is inferred from F .
- A set ρ of derivation rules are required, such that:
 - a $X \rightarrow Y$ is inferred from F according to Definition 1 iff it can be derived using ρ .

8.3 Armstrong's axioms (1974)

Notation: If X and Y are sets of attributes, we write XY for their union.

e.g. $X = \{A, B\}$, $Y = \{B, C\}$, $XY = \{A, B, C\}$

- F1 (Reflexivity) If $X \supseteq Y$ then $X \rightarrow Y$.
- F2 (Augmentation) $\{X \rightarrow Y\} \models XZ \rightarrow YZ$.
- F3 (Transitivity) $\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$.

- F4 (Additivity) $\{X \rightarrow Y, X \rightarrow Z\} \models X \rightarrow YZ$.
- F5 (Projectivity) $\{X \rightarrow YZ\} \models X \rightarrow Y$.
- F6 (Pseudotransitivity)

$$\{X \rightarrow Y, YZ \rightarrow W\} \models XZ \rightarrow W.$$

Example: Given $F = \{A \rightarrow B, A \rightarrow C, BC \rightarrow D\}$, derive $A \rightarrow D$:

1. $A \rightarrow B$ (given)
2. $A \rightarrow C$ (given)
3. $A \rightarrow BC$ (by F4, from 1 and 2)
4. $BC \rightarrow D$ (given)
5. $A \rightarrow D$ (by F3, from 3 and 4)

- F4 (Additivity) $\{X \rightarrow Y, X \rightarrow Z\} \models X \rightarrow YZ$.
- F5 (Projectivity) $\{X \rightarrow YZ\} \models X \rightarrow Y$.
- F6 (Pseudotransitivity)

$$\{X \rightarrow Y, YZ \rightarrow W\} \models XZ \rightarrow W.$$

In fact, F4, F5, and F6 can be derived from F1-F3.

Example: Prove $\{X \rightarrow Y, X \rightarrow Z\} \models X \rightarrow YZ$.

- 1) $X \rightarrow Y$ is given.
- 2) $XX \rightarrow XY$ (by F2); that is, $X \rightarrow XY$
- 3) $X \rightarrow Z$ is given.
- 4) $XY \rightarrow YZ$ (by F2)
- 5) $X \rightarrow YZ$ (by F3, 2) and 4))

We can prove that Armstrong's axioms are sound and complete:

- Sound: if F derives $A \rightarrow B$ by using Armstrong's axioms, then $F \models A \rightarrow B$ by Definition 1.
- Complete: if $F \models M \rightarrow N$ by Definition 1, then F derives $M \rightarrow N$ by using Armstrong's axioms.

8.4 Algorithm to Check a FD

Given F , how do we check if $X \rightarrow Y$ is in F^+ ?

F^+ denotes the smallest set of FD's that

- contains F , and
- is *closed* under Armstrong's axioms.

F^+ is the *closure* of F .

$$F = \{ A \rightarrow B, B \rightarrow C, A \rightarrow C \}$$

$$F^+ = \{ AB \rightarrow A, AB \rightarrow B, AB \rightarrow C, AC \rightarrow A, AC \rightarrow B, AC \rightarrow C, ABC \rightarrow A, ABC \rightarrow B, ABC \rightarrow C, AB \rightarrow AB, AB \rightarrow BC, AB \rightarrow AC, \dots \}$$

F^+ always has an exponential size regarding $|F|$.

- Too expensive to compute F^+ to verify a membership.
- Instead we can compute the *closure* X^+ of X under F ,
 X^+ is the largest set of attributes functionally
determined by X .

It can be proven (using additivity) that

$$S1: X^+ = \cup_{X \rightarrow A \in F^+} A.$$

$$S2: X \rightarrow Y \in F^+ \text{ iff (if and only if) } Y \subseteq X^+.$$

$F = \{ A \rightarrow B, BC \rightarrow D, A \rightarrow C \}$, compute $\{A\}^+$

1st scan of F:

$X^+ := \{A\}$

$X^+ := \{A, B\}$

$X^+ := \{A, B, C\}$

2nd scan of F:

$X^+ := \{A, B, C, D\}$

3rd scan of F: no change, therefore the algorithm terminates.

$\{A\}^+ := \{A, B, C, D\}$

- Algorithm to compute X^+ :
 $X^+ := X$;
 change := true;
 while change do
 begin
 change := false;
 for each FD $W \rightarrow Z$ in F do
 begin
 if $(W \subseteq X^+) \text{ and } (Z \not\subseteq X^+)$ then do
 begin
 $X^+ := X^+ \cup Z$;
 change := true;
 end
 end
 end
 end
 end

8.5 Algorithm to Compute a Candidate Key

- Given a relational schema R and a set F of functional dependencies on R .
- A key X of R must have the property that $X^+ = R$.

- **Algorithm to compute a candidate key**

Step 1: Assign X a superkey in F .

Step 2: Iteratively remove attributes from X while retaining the property $X^+ = R$ till no reduction on X .

The remaining X is a key.

$R = \{A, B, C, D\}$ and $F = \{A \rightarrow B, BC \rightarrow D, A \rightarrow C\}$

$X = \{A, B, C\}$ if the left hand side of F is a super key.

A cannot be removed because $\{BC\}^+ = \{B, C, D\} \neq R$

B can be removed because $\{AC\}^+ = \{A, B, C, D\} = R$

→ $X = \{A, C\}$

C can be further removed because $\{A\}^+ = \{A, B, C, D\}$

→ $X = \{A\}$