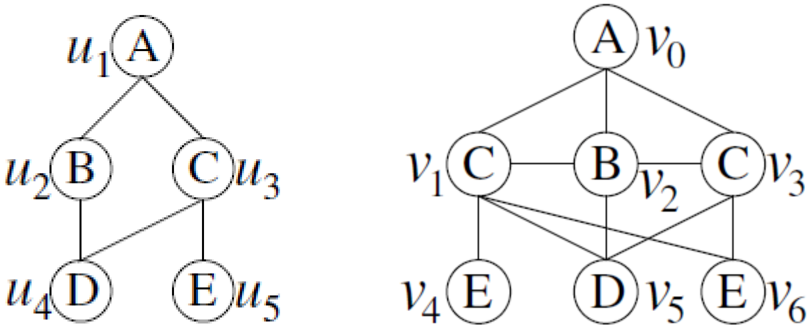
All-Matching

- **Subgraph Matching**

  Given a query ***q*** and a large data graph ***G***, the problem is to extract all subgraph isomorphic embeddings of ***q*** in ***G***.



(a) Query q (b) Data graph G

There are **three** subgraph isomorphic embeddings of q in G.

All-Matching

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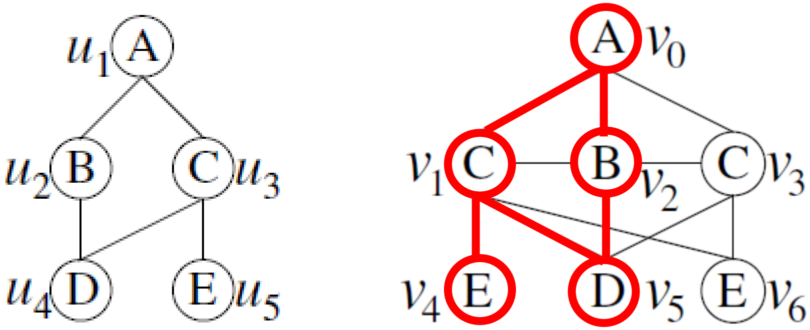

(a) Query q (b) Data graph G

Embedding One

u_1	u_2	u_3	u_4	u_5
v_0	v_2	v_1	v_5	v_4

All-Matching

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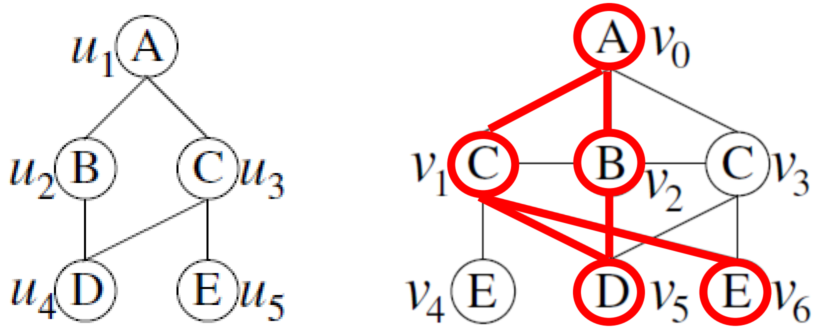

(a) Query q (b) Data graph G

Embedding Two

u_1	u_2	u_3	u_4	u_5
v_0	v_2	v_1	v_5	v_6

All-Matching

- **Subgraph Matching**

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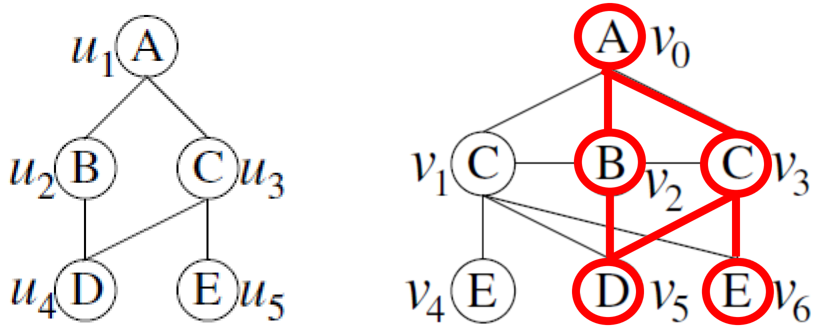


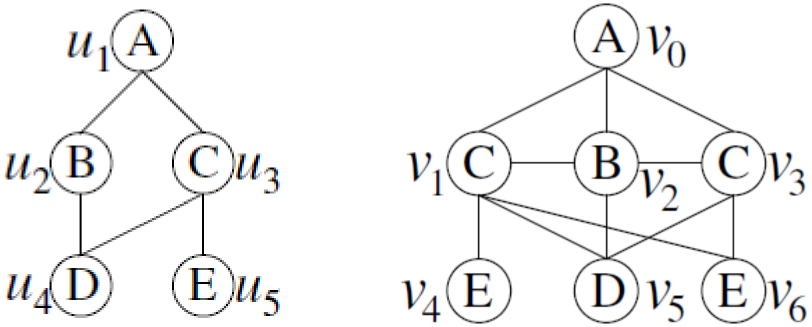
(a) Query q (b) Data graph G

Embedding Three

u_1	u_2	u_3	u_4	u_5
v_0	v_2	v_3	v_5	v_6

Existing Work

- **Ullmann's algorithm [J.ACM'76]**
  - First algorithm to enumerate all subgraph isomorphic embeddings
  - A **backtracking** algorithm that maps query vertices one by one, following a **random** order.
  - Example: A random order for query *q* could be $(u_1, u_4, u_2, u_3, u_5)$



(a) Query q (b) Data graph G

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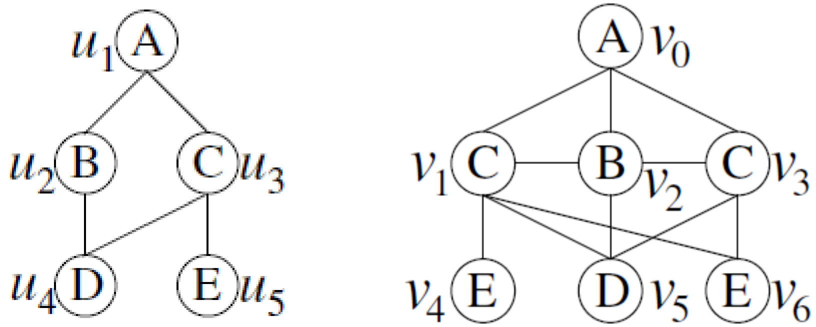
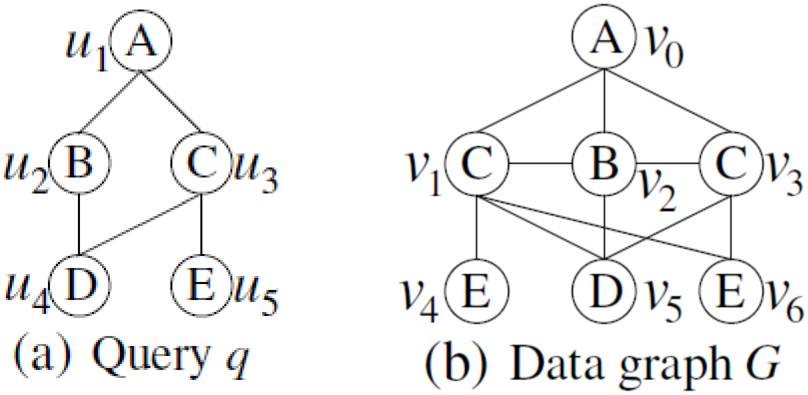


(a) Query q (b) Data graph G

Existing Work

- **VF2 [IEEE Trans'04] and QuickSI [VLDB'08]**
  - Independently propose to enforce **connectivity** of the matching order
  - Example : A connected order for query *q* could be (u_1, u_2, u_4, u_3, u_5)



  - QuickSI further removes false-positive intermediate results by first processing infrequent query vertices and edges.

Subgraph Matching

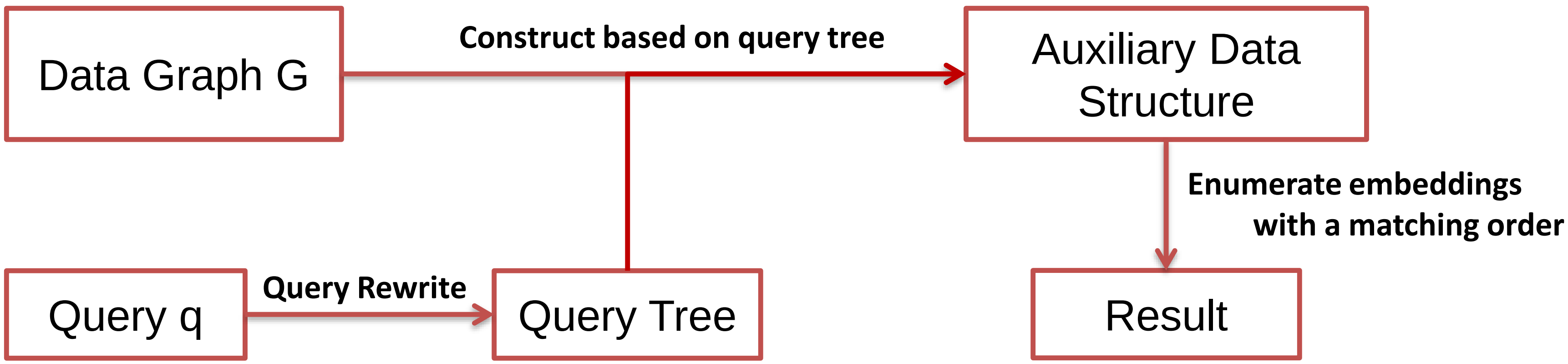
- Nevertheless, it is still very difficult to enumerate subgraph matchings.

  Due to 1) Data graph could be very large and dense.

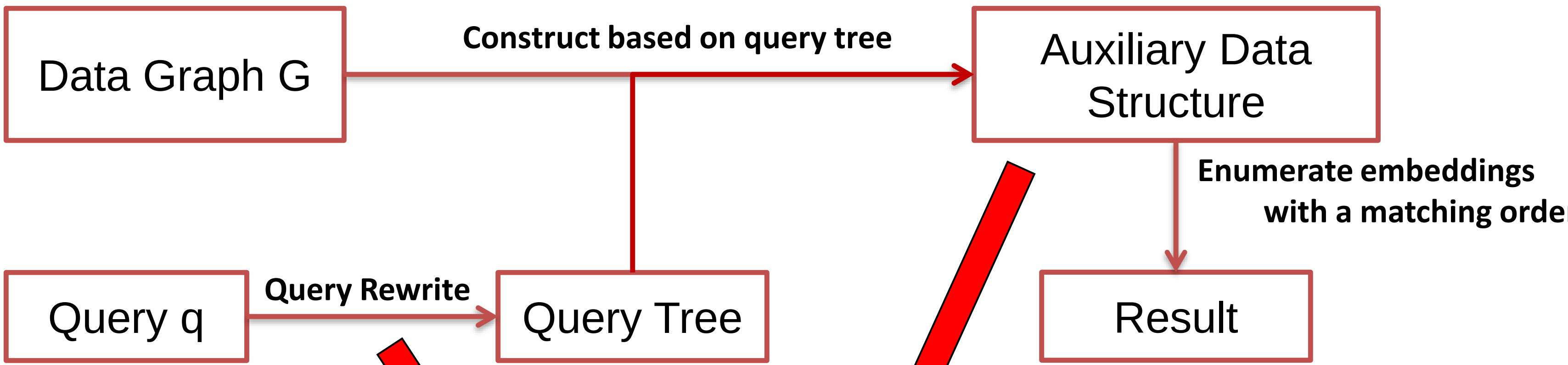
  2) There could be exponential number of embeddings of q in G.

- Two Advanced Approaches.
  - TurboISO
  - CFL-Match

TurboISO Overview



TurboISO Overview



- Key Techniques
  - ➢ Neighborhood Equivalence Class (NEC)
    - Merging vertices with same neighbors to reduce query size
    - Combine / Permute strategy to enumerate results
  - ➢ Candidate Region Exploration
    - Constructed on-the-fly based on q
    - A path-based data structure containing all embeddings of q in G

Will be explained with example in detail.

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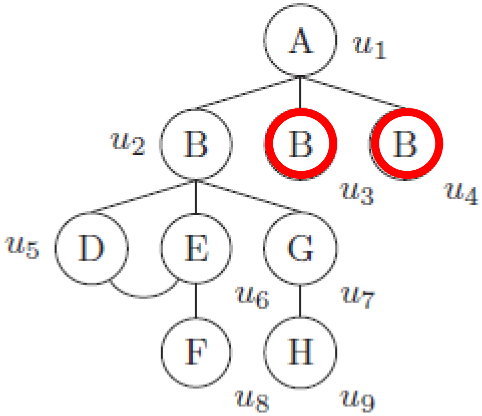
Neighborhood Equivalence Class(NEC)

- Definition

  Let $\simeq$ be an *equivalence relation over all query vertices in q such that,* $u_i(\in V(q)) \simeq u_j(\in V(q))$ *if for every embedding m that contains* (u_i, v_x) *and* (u_j, v_y) $(v_x, v_y \in V(g))$ *, there exists an embedding m' such that*

$$m' = m - \{(u_i, v_x), (u_j, v_y)\} \cup \{(u_i, v_y), (u_j, v_x)\}$$

- Example



u_3 and u_4 are equivalent.

The Neighborhood Equivalence Class(NEC) of a query vertex u is a set of query vertices, which are equivalent to u.

Query Rewrite

- Given a query q, we rewrite it into a NEC Tree in following steps:
  1) Root Node Selection

     Ranking function $Rank(u) = \frac{freq(g, L(u))}{deg(u)}$

$freq(g, l)$: number of **data** vertices in *g* that have label *l*

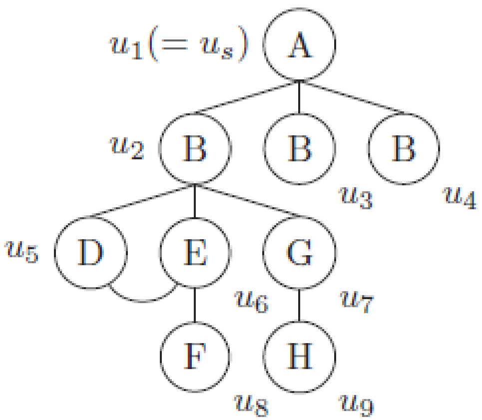
$deg(u)$: the degree of *u*

The vertex with **smallest** ranking value will be selected as the root node.

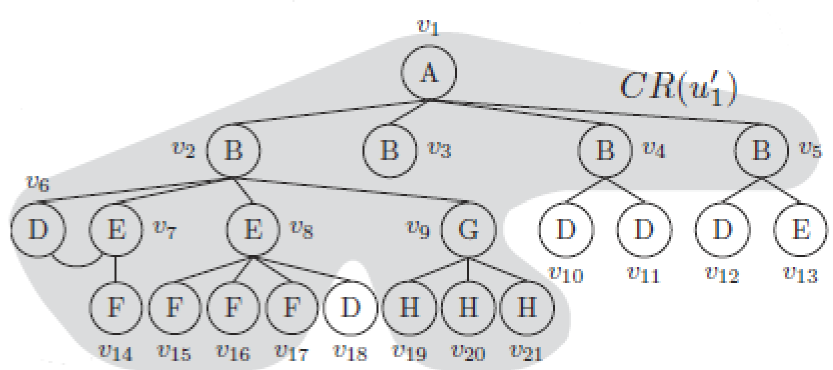
Query Rewrite

- Given a query q, we rewrite it into a NEC Tree in following steps:
  1) Root Node Selection

     Ranking function $Rank(u) = \frac{freq(g, L(u))}{deg(u)}$



query graph q



data graph g

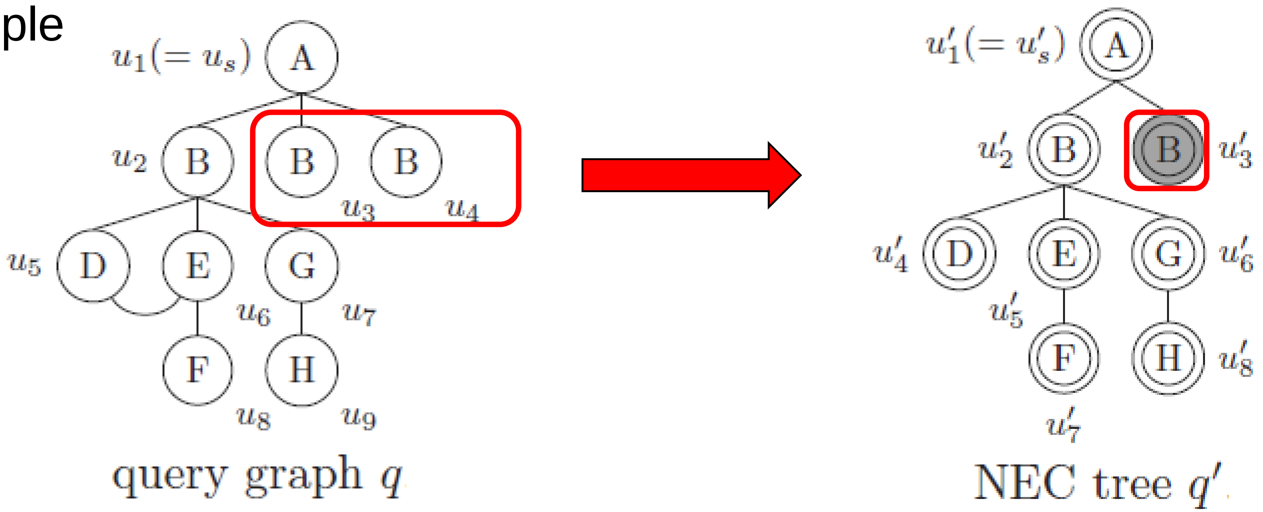
Rank(u_1) = 1 / 3
Rank(u_2) = 4 / 4
Rank(u_3) = 4 / 1
Rank(u_4) = 4 / 1
Rank(u_5) = 5 / 2
Rank(u_6) = 3 / 3
Rank(u_7) = 1 / 2
Rank(u_8) = 4 / 1
Rank(u_9) = 3 / 1

Hence, u_1 is selected as the root node.

Query Rewrite

- Given a query q, we rewrite it into a NEC Tree in following steps:
  2) Rewrite to NEC Tree by
     I. Performing BFS from the root node
     II. Merging vertices from same NEC into a single vertex

Example

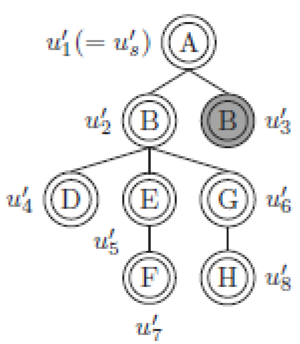


u_1 has been selected as the root node in the previous step.

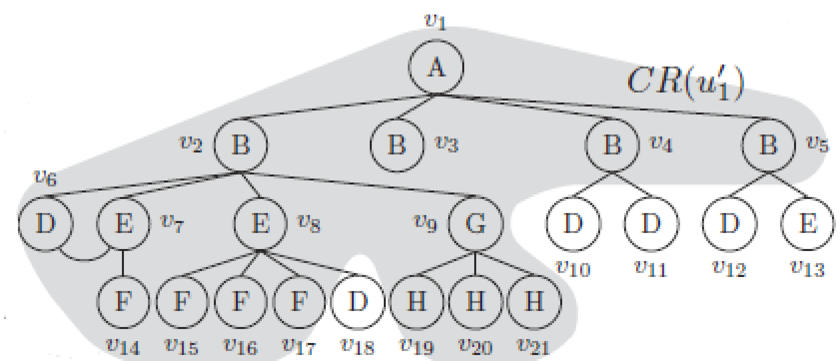
Candidate Region Exploration

- Candidate Subregion

  Candidate subregion denoted as *CR*(*u, v*), contains data vertices such that they match *u* and are child vertices of *v* in the DFS tree, where *u* is an NEC vertex, and *v* is a data vertex.



NEC tree q'



data graph g

CR	candidate vertices
$\mathrm{CR}(u'_1, v^*_s)$	$\{v_1\}$
$\mathrm{CR}(u'_2, v_1)$	$\{v_2\}$
$\mathrm{CR}(u'_3, v_1)$	$\{v_2, v_3, v_4, v_5\}$
$\mathrm{CR}(u'_4, v_2)$	$\{v_6\}$
$\mathrm{CR}(u'_5, v_2)$	$\{v_7, v_8\}$
$\mathrm{CR}(u'_6, v_2)$	$\{v_9\}$
$\mathrm{CR}(u'_7, v_7)$	$\{v_{14}\}$
$\mathrm{CR}(u'_7, v_8)$	$\{v_{15}, v_{16}, v_{17}\}$
$\mathrm{CR}(u'_8, v_9)$	$\{v_{19}, v_{20}, v_{21}\}$

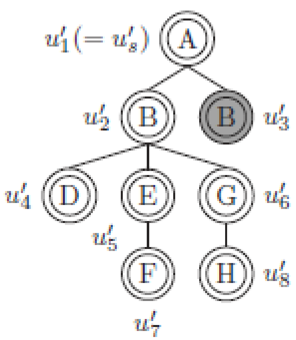
candidate subregions.

$\mathrm{CR}(u_2', v_1)$ represents the candidates for u_2' of NEC tree q' in g, with u_1' (the parent of u_2' in q') mapped to v_1.

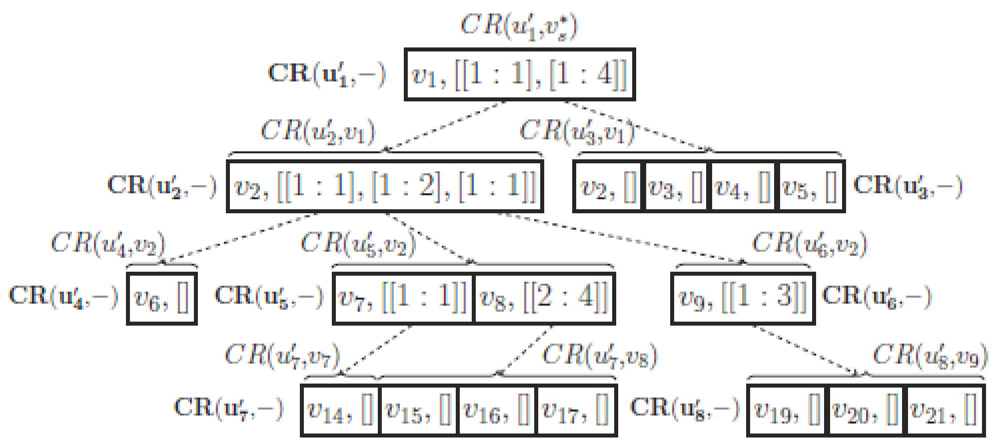
Candidate Region Exploration

- Candidate Region
  - Organize all candidate subregions together
    - It has the same structure as the NEC tree
    - To facilitate the embedding enumeration.

- The candidate region of the previous example



NEC tree q'

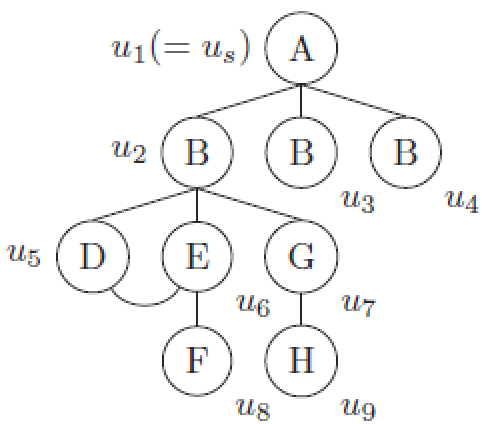


Candidate Region Exploration

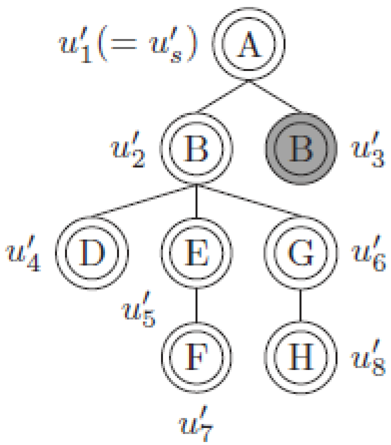
To enumerate embeddings of query q in data graph G,

- Only traverse the candidate region
- Probe G only for non-tree edges validation

Non-tree edges are the edges in query graph q but not in NEC Tree q'.



query graph q



NEC tree q'

(u_5, u_6) is a non-tree edge.

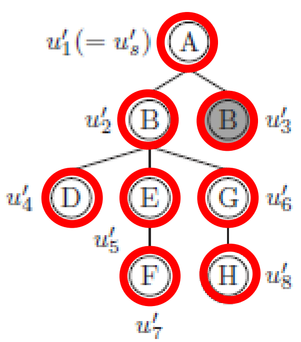
Candidate Region Exploration

- To enumerate embeddings of q in G,
  - ➢ Only traverse the candidate region
  - ➢ Probe G only for non-tree edges validation

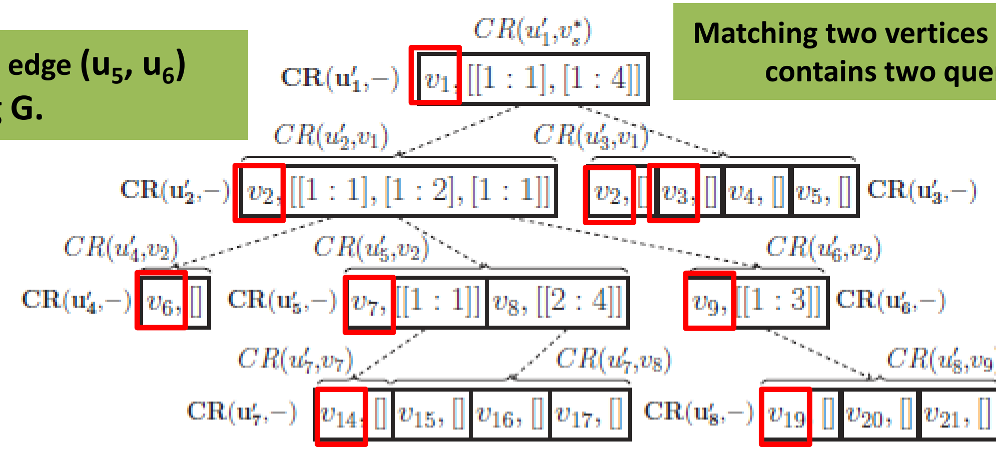
- Example

Validate non-tree edge (u_5, u_6) here using G.

Matching two vertices because u_3' contains two query nodes.



NEC tree q'



With matching order
(u_1', u_2', u_4', u_5', u_7', u_3', u_6', u_8',)

Combine / Permute to get all embeddings based on each NEC node.
In this example, as NEC node u_3' contains two query nodes, this embedding corresponds to 2 embeddings by permuting v_2 and v_3.

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CFL-Match

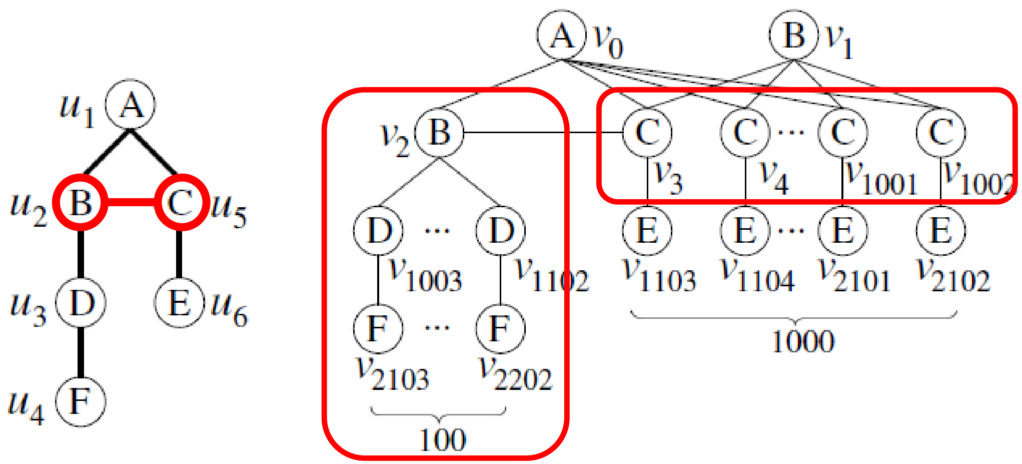
- Overview of CFL-Match

  - A New Search Framework
    - Core-Forest-Leaf based query decomposition
    - Postpone Cartesian product with search order of core, forest and leaf

  - Comact Auxiliary Data Structure
    - Linear to the size of the data graph G
    - In contrast to the exponential sized data structure in TurboISO

Challenges of Subgraph Matching

Challenge I: Redundant Cartesian Products by Dissimilar Vertices.



(a) Query q (b) Data graph G

10^5 - 100 partial mappings are redundant.

Matching order of QuickSI and Turbo$_{ISO}$: $(u_1, u_2, u_3, u_4 \boxed{, u_5}, u_6)$.

$(u_1, u_2, u_5, u_3, u_4, u_6)$

Cartesian products:

- 100 mappings $(v_0, v_2, v_{1000+i}, v_{2100+i})$ $(3 \le i \le 102)$ of (u_1, u_2, u_3, u_4)
- 1000 mappings (v_0, v_j) $(3 \le j \le 1002)$ of (u_1, u_5)

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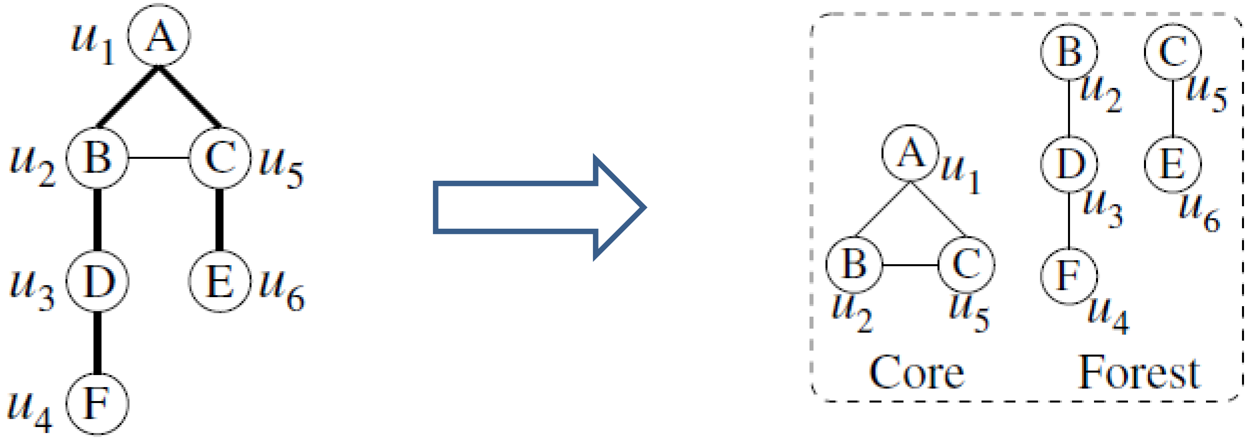
Challenges of Subgraph Matching

Challenge I: Redundant Cartesian Products by Dissimilar Vertices.

Our Solution : Postpone Cartesian products.

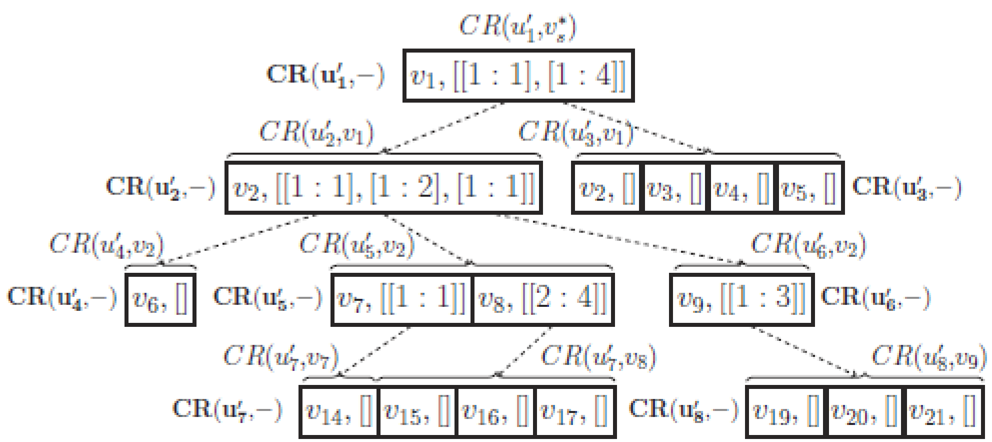
- Decompose ***q*** into **a dense subgraph** and **a forest**, and process the dense subgraph first.



Challenges of Subgraph Matching

Challenge II: Exponential size of the path-based data structure in TurboISO.

- TurboISO builds a data structure that materializes all embeddings of query paths in a data graph
- Worst-case space complexity: **$O(|V(G)|^{|V(q)-1|})$**.



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Challenges of Subgraph Matching

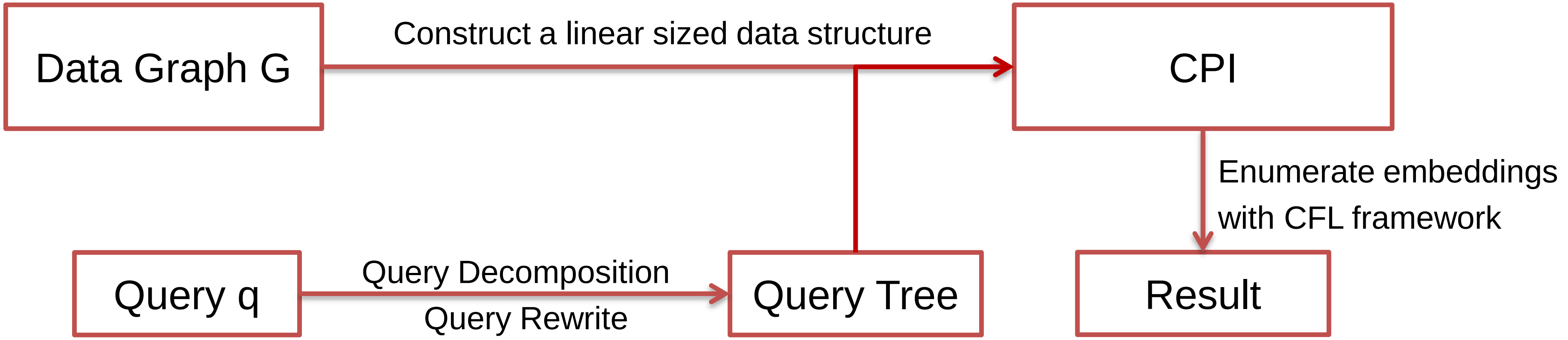
Challenge II: Exponential size of the path-based data structure in TurboISO.

Our Solution:

Polynomial-size data structure, compact path-index (CPI) .

Our Approach

- CFL-Match
  - A Core-Forest-Leaf decomposition based Framework
  - Compact Path-Index (CPI) based Matching

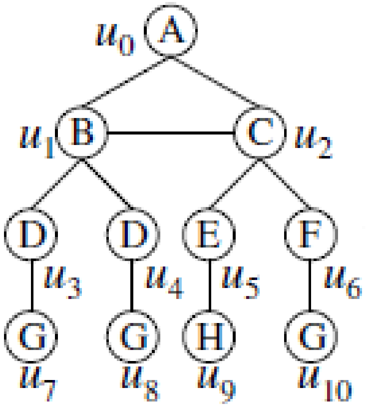


Core-Forest-Leaf Decomposition

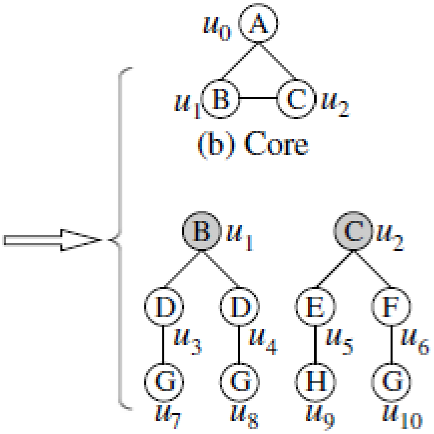
➢ Core-Forest Decomposition

Compute the **minimal connected** subgraph **c** containing **all non-tree edges** of ***q*** regarding any spanning tree.

- The subgraph **c** is the core-structure of ***q*** , denoted as $\mathbf{V_C}$.
- The subgraph of ***q*** consisting of all other edges not in the **c**, is called the forest-structure of ***q***, denoted as $\mathbf{V_T}$.



(a) Query q



(b) Core

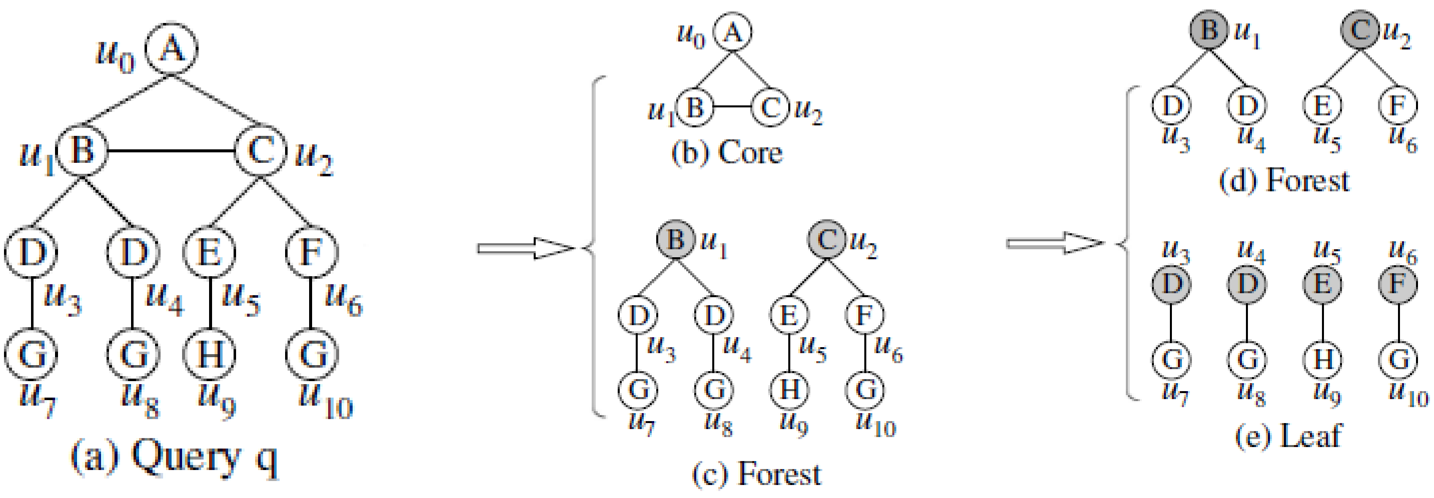
(c) Forest

Core-Forest-Leaf Decomposition

➢ Forest-Leaf Decomposition

Compute the set $\mathbf{V_l}$ of leaf vertices (degree-one vertices) by rooting each tree at its connection vertex.

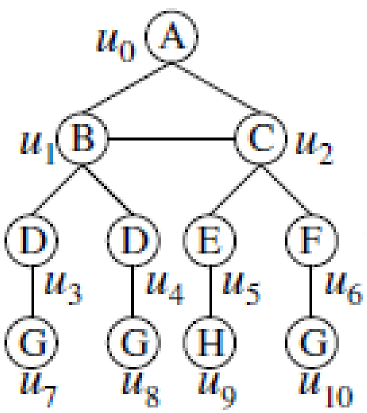
- The set $\mathbf{V_l}$ is called the leaf set.
- The set of vertices not in $\mathbf{V_C \cup V_l}$ is called the forest set.



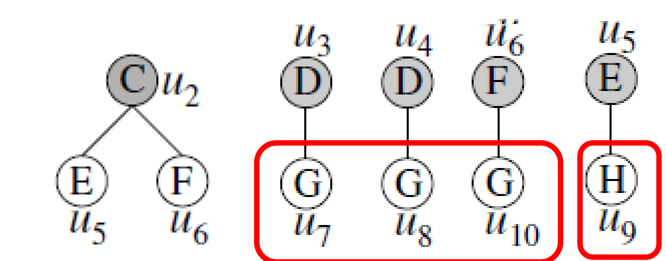
CFL-Match

- **A Core-Forest Leaf based Framework**
  1) Core-Forest-Leaf Decomposition
  2) Construct CPI (will be explained later)
  3) Mapping Extraction
     i. Core-Match
     ii. Forest-Match
     iii. Leaf-Match

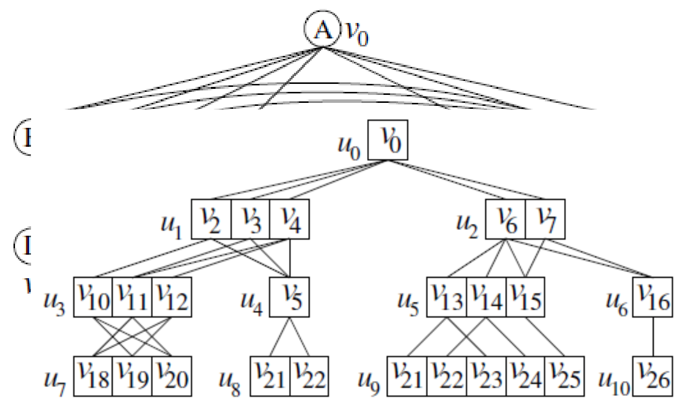
gorize leaf nodes according to labe
rm combination instead of enumer



(a) Query q



Forest (e) Leaf



(c) CPI

Auxiliary Data Structure

- **Compact Path-Index (CPI)**
  - Compactly store all candidate embeddings of the query tree.

- **CPI Structure**
  - **Candidate sets**

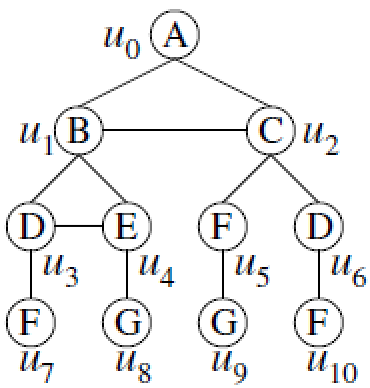
    Each query node ***u*** has a candidate set ***u.C***.
  - **Edge sets**

    This is an edge between $v \in u.C$ and $v' \in u'.C$ for adjacent query nodes ***u*** and ***u'*** in CPI if and only if ***(v, v')*** exists in ***G***.

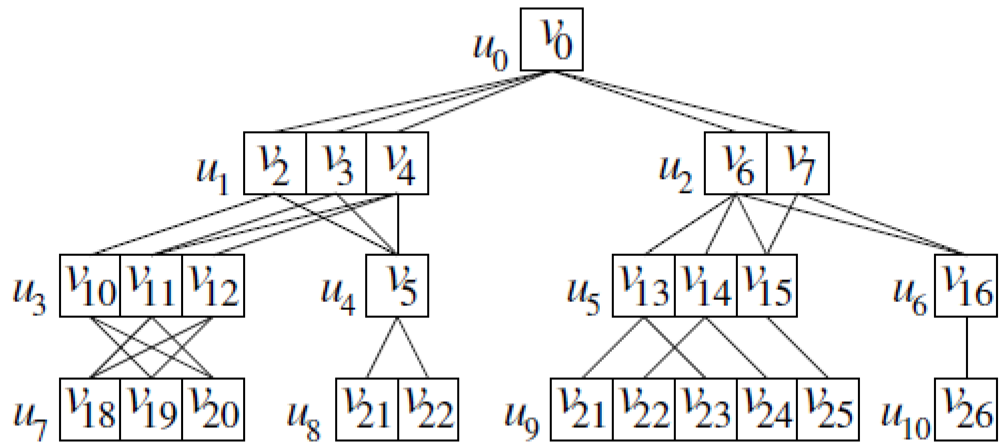
Auxiliary Data Structure

- **Compact Path-Index (CPI)**
  - Compactly store all candidate embeddings of the query tree.

- **CPI Structure**
  - **Example**



(a) Query q



(c) CPI

Auxiliary Data Structure

- **Soundness of CPI**

  For every query node ***u*** in CPI, if there is an embedding of ***q*** in ***G*** that maps ***u*** to ***v***, then ***v*** must be in ***u.C***.

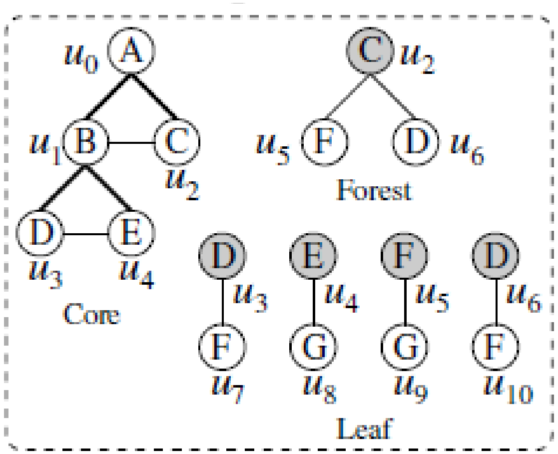
Theorem

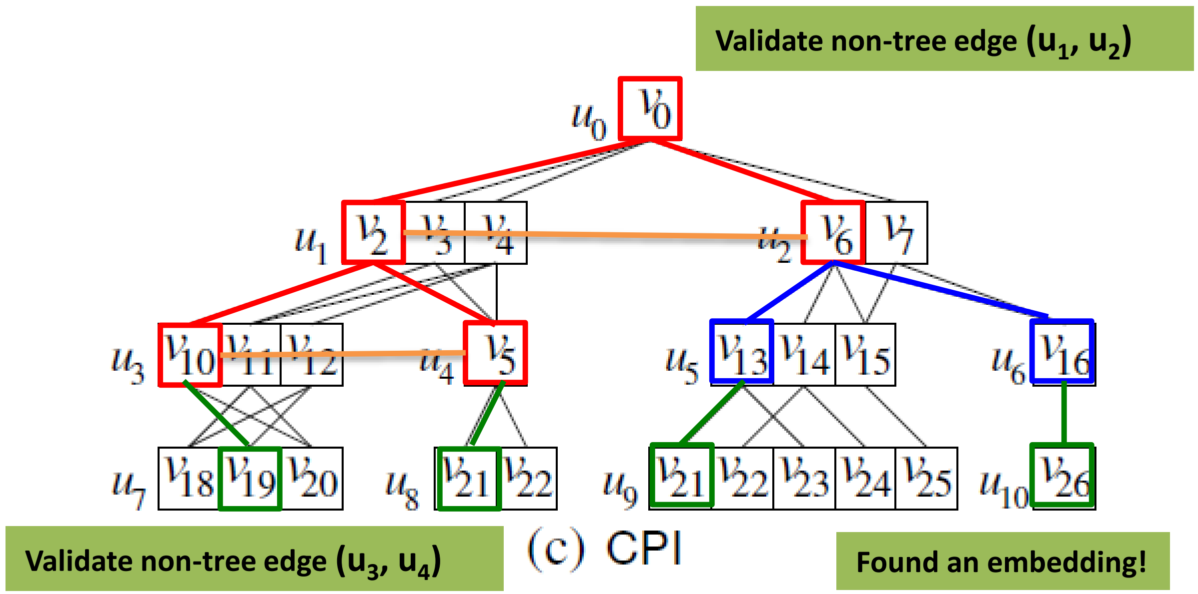
Given a sound CPI, all embeddings of ***q*** in ***G*** can be computed by **traversing only the CPI** while ***G*** is only probed for non-tree edge checkings.

- **The CPI in the previous example is a sound CPI.**

CPI-based Match

- **Traverse CPI to find mappings for query vertices**
  - Only probe ***G*** for non-tree edge validation





(c) CPI

Matching order (u_0 , u_1 , u_4 , u_3 , u_2, u_5, u_6, u_7, u_8, u_9, u_{10})

Tree **Forest** **Leaf**

Summary

- Graph Pattern Matching in Graph Database
  - Three index-based methods: G-Index, FG-Index, Swift-Index
  - Advanced subgraph isomorphism testing: QuickSI

- Graph Pattern Matching in Single Large Data Graph
  - TurboISO
    - Reduce query size using NEC
    - Candidate Region Exploration
  - CFL-Match
    - CFL-Framework based on query decomposition
    - Compact data structure CPI

Thank you!

Questions?