Relational Algebra

3. Relational Algebra

- *Relational Algebra* is a procedural DML.

- It specifies operations on relations to define new relations:

  Select, Project, Union, Intersection, Difference, Cartesian Product, Join, Divide.

3.1 SELECT

- Selects a subset of the tuples of a relation r, satisfying some condition.

$$\sigma_B(r) = \{t \in r : B(t)\}$$

- B is the selection condition, composed of selection clauses combined using AND, OR and NOT.

- A selection clause has the form

  <attribute name> <op> <constant>
  or
  <attribute name> <op> <attribute name>

  (*join*, introduce later)

where <op> is one of =, <, ≤, >, ≥ or ≠.

RESEARCHER	
Person#	Name
1	Dr.C.C.Chen
2	Dr.R.G.Wilkinson

STUDENT	
Person#	Name
1	Dr.C.C.Chen
3	Ms.K.Juliff
4	Ms.J.Gledill
5	Ms.B.K.Lee

COURSE	
Department	Name
Psychology	Ph.D.
Comp.Sci.	Ph.D.
Comp.Sci.	M.Sc.
Psychology	M.Sc.

ENROLMENT				
Enrolment#	Supervisee	Supervisor	Department	Name
1	1	2	Psychology	Ph.D.
2	3	1	Comp.Sci.	Ph.D.
3	4	1	Comp.Sci.	M.Sc.
4	5	1	Comp.Sci.	M.Sc.

- *Example*: Select the enrolment records for the students of person 1.

$$\sigma_{(Supervisor=1)}(ENROLMENT)$$

ENROLMENT				
Enrolment#	Supervisee	Supervisor	Department	Name
1	1	2	Psychology	Ph.D.
2	3	1	Comp.Sci.	Ph.D.
3	4	1	Comp.Sci.	M.Sc.
4	5	1	Comp.Sci.	M.Sc.

Enrolment#	Supervisee	Supervisor	Department	Name
2	3	1	Comp.Sci	Ph.D.
3	4	1	Comp.Sci	M.Sc.
4	5	1	Comp.Sci	M.Sc.

- Example: Select the enrolment records for person 1's non-Ph.D. students:

$$\sigma_{(Supervisor=1)\text{AND NOT}(Name\neq"PH.D")}(ENROLMENT)$$

ENROLMENT				
Enrolment#	Supervisee	Supervisor	Department	Name
1	1	2	Psychology	Ph.D.
2	3	1	Comp.Sci.	Ph.D.
3	4	1	Comp.Sci.	M.Sc.
4	5	1	Comp.Sci.	M.Sc.

Enrolment#	Supervisee	Supervisor	Department	Name
3	4	1	Comp.Sci	M.Sc
4	5	1	Comp.Sci	M.Sc

Properties:

- Commutative:

$$\sigma_{<cond1>} (\sigma_{<cond2>} (R)) =$$
$$\sigma_{<cond2>} (\sigma_{<cond1>} (R))$$

- Consecutive selects can be combined:

$$\sigma_{<cond1>} (\sigma_{<cond2>} (R)) =$$
$$\sigma_{<cond1>} \text{ AND } _{<cond2>} (R))$$

3.2 PROJECT

- Projects onto a subset X of the attributes of a relation.

$$\pi_X(r) = \{t[X]: t \in r\}$$

- *Remember* that a tuple, t is a mapping from attributes to elements of their domains. t[X] is the restriction of that mapping to the set of attributes X.

- Example: Which courses are students enrolled in?

$$\pi_{Department,Name}(ENROLMENT) =$$

ENROLMENT				
Enrolment#	Supervisee	Supervisor	Department	Name
1	1	2	Psychology	Ph.D.
2	3	1	Comp.Sci.	Ph.D.
3	4	1	Comp.Sci.	M.Sc.
4	5	1	Comp.Sci.	M.Sc.

Department	Name
Psych.	Ph.D.
Comp.Sci	Ph.D.
Comp.Sci	M.Sc.

Properties:

- if <list2> contains all the attributes in <list1> then

$$\pi_{<list1>}(\pi_{<list2>}(R)) = \pi_{<list1>}(R)$$

else

The operation is not well defined.

- commutes with selection:

$$\pi_X(\sigma_B(R)) = \sigma_B(\pi_X(R))$$

Exercise: Verify the above with:

$\pi_{\{Department\}}(\sigma_{(Department=\text{“}Psychology\text{”})}(ENROLMENT))$.

Properties:

- if <list2> contains all the attributes in <list1> then

$$\pi_{<list1>}(\pi_{<list2>}(R)) = \pi_{<list1>}(R)$$

else

The operation is not well defined.

- commutes with selection: B cannot be specified outside of X

$$\pi_X(\sigma_B(R)) = \sigma_B(\pi_X(R))$$

Exercise: Verify the above with:

$\pi_{\{Department\}}(\sigma_{(Department=\text{“}Psychology\text{”})}(ENROLMENT))$.

Questions

1) $\pi \ (R \ U \ S)) = \pi \ (R) \ U \ \pi \ (\mathrm{S})$?

2) $\pi \ (R \cap S)) = \pi \ (R) \cap \pi \ (\mathrm{S})$?

Answer:

2) $\pi\ (R \cap S)) \neq \pi\ (R) \cap \pi\ (S)$

Example:

$R = (Animal, Cat), S = (Animal, Dog)$

π: project on the first column

$\pi\ (R \cap S)) = \{\}$

$\pi\ (R) \cap \pi\ (S) = \{\text{Animal}\}$

3.3 UNION

- Is the set theoretic union of the tuples of two relations.

$$r \cup s = \{t : t \in r \; or \; t \in s\}$$

- Note: Requires R and S to be union compatible: that there is a 1-1 correspondence between their attributes, in which corresponding attributes are over the same domain.

- Example:

$$R1 \leftarrow \sigma_{(Supervisor=2)}(ENROLMENT)$$

$$R2 \leftarrow \sigma_{(Name=\text{“}M.Sc\text{”})}(ENROLMENT)$$

$$R1 \cup R2 =$$

Enrolment#	Supervisee	Supervisor	Department	Name
1	1	2	Psych.	Ph.D.
3	4	1	Comp.Sci	M.Sc
4	5	1	Comp.Sci	M.Sc

- Example: $STUDENT \cup RESEARCHER =$

Person#	Name
1	Dr C.C.Chen
3	Ms K.Juliff
4	Ms J.Gledhill
5	Ms B.K.Lee
2	Dr R.G.Wilkinson

3.4 INTERSECTION

- Is the set theoretic intersection of the tuples of two relations.

$$r \cap s = \{t\colon t \in r \text{ or } t \in s\}.$$

- Example:

$$R_1 \leftarrow \sigma_{(\text{Supervisor}=1)}(ENROLMENT)$$

$$R_2 \leftarrow \sigma_{(\text{Name}=\text{“Ph.D.”})}(ENROLMENT)$$

$$R_1 \cap R_2 =$$

Enrolment#	Supervisee	Supervisor	Department	Name
2	3	1	Comp.Sci.	Ph.D.

- Example: STUDENT ∩ RESEARCHER =

STUDENT	
Person#	Name
1	Dr.C.C.Chen
3	Ms.K.Juliff
4	Ms.J.Gledill
5	Ms.B.K.Lee

RESEARCHER	
Person#	Name
1	Dr.C.C.Chen
2	Dr.R.G.Wilkinson

Person#	Name
1	Dr C.C. Chen

3.5 DIFFERENCE

- Is the set difference of the tuples of two relations.

$$r - s = \{t : t \in r \text{ and } t \notin s\}$$

- Example: STUDENT − RESEARCHER =

Person#	Name
3	Ms K. Juliff
4	Ms J. Gledhill
5	Ms B.K. Lee

3.6 CARTESIAN PRODUCT

$$r \times s = \{t_1 || t_2 : t_1 \in r \text{ and } t_2 \in s\}$$

Where $t_1 || t_2$ indicates the concatenation of tuples.

Example:

$ENROLMENT \times RESEARCHER$

E'ment#	S'ee	S'or	D'ment	E'ment. Name	Person#	R'cher. Name
1	1	2	Psych.	Ph.D.	1	Dr C.C. Chen
1	1	2	Psych.	Ph.D.	2	Dr R.G.Wilkinson
2	3	1	Comp.Sci	Ph.D.	1	Dr C.C. Chen
2	3	1	Comp.Sci	Ph.D.	2	Dr R.G.Wilkinson
3	4	1	Comp.Sci	M.Sc.	1	Dr C.C. Chen
3	4	1	Comp.Sci	M.Sc.	2	Dr R.G.Wilkinson
4	5	1	Comp.Sci	M.Sc.	1	Dr C.C. Chen
4	5	1	Comp.Sci	M.Sc.	2	Dr R.G.Wilkinson

More useful is:

$R1 \leftarrow ENROLMENT \times RESEARCHER$

$\sigma_{(Supervisor=Person\#)}(R1) =$

E'ment#	S'ee	S'or	D'ment	E'ment. Name	Person#	R'cher. Name
1	1	2	Psych.	Ph.D.	2	Dr R.G.Wilkinson
2	3	1	Comp.Sci.	Ph.D.	1	Dr C.C. Chen
3	4	1	Comp.Sci.	M.Sc.	1	Dr C.C. Chen
4	5	1	Comp.Sci.	M.Sc.	1	Dr C.C. Chen

- or even better:

$$R1 \leftarrow ENROLMENT \times RESEARCHER$$

$$R2 \leftarrow \sigma_{(Supervisor=Person\#)}(R1)$$

$$\pi_{\{E'ment\#,S'ee,S'or,R'cher.Name,D'ment,E'ment.Name\}}(R2) =$$

E’ment#	S’ee	S’or	R’cher. Name	D’ment	E’ment. Name
1	1	2	Dr R.G.Wilkinson	Psych.	Ph.D.
2	3	1	Dr C.C. Chen	Comp.Sci.	Ph.D.
3	4	1	Dr C.C. Chen	Comp.Sci.	M.Sc.
4	5	1	Dr C.C. Chen	Comp.Sci.	M.Sc.

- The last of these is also known as natural join, the next to last is equi-join.

3.7 JOIN

Is used to combine related tuples from two relations.

- 3.7.1 Theta-join
  $r \bowtie_B s = \{t_1 || t_2 : t_1 \in r \text{ and } t_2 \in s \text{ and } B\}$

B is composed of conditions (combined with AND) of the form $A_i \theta\ B_j$ where A_i is an attribute of R, B_j is an attribute of S, and θ is a comparison operator.

- 3.7.2 Equi-join

  Is a theta-join where each comparison operator is “=”.

Example:

$$ENROLMENT \bowtie_{(Supervisor=Person\#)} RESEARCHER$$

- 3.7.3 Natural join

Is an equi-join where only one attribute from each comparison is retained.

Example:

$$ENROLMENT \bowtie_{(Supervisor),(Person\#)} RESEARCHER$$

- Question: If two relations have no join attributes,

how do you define the join result? Why?

- 3.7.3 Natural join

Is an equi-join where only one attribute from each comparison is retained.

Example:

$$ENROLMENT \bowtie_{(Supervisor),(Person\#)} RESEARCHER$$

- Question: If two relations have no join attributes,

how do you define the join result? Why?

$$R\,(A, B) \bowtie S\,(B, C) \bowtie T\,(C, D)$$

- *Notes:*

1. In a natural join, there may be several pairs of join attributes.

Example:

COURSE		
Department	Name	By
Comp.Sci	Ph.D.	Research
Comp.Sci.	M.Sc.	Research
Psychology	M.Sc.	Coursework

Calculate

$$ENROLMENT \bowtie_{(Department,Name),(Department,Name)} COURSE$$

- 2. If the pairs of joining attributes are exactly those that are identically named, we can write

$$ENROLMENT \bowtie COURSE$$

3.8 DIVIDE

Suppose R is a relation over Z, S over X with $X \subseteq Z$. Let $Y = Z - X$. Then $R \div S$ is a relation over Y ,

$$R \div S = \{t : t \times S \subseteq \mathrm{R}\}$$

Example:

P	
A	B
a_1	b_1
a_1	b_2
a_2	b_1
a_3	b_2
a_4	b_1
a_5	b_1
a_5	b_2

Q
B
b_1
b_2

$$P \div Q =$$

A
a_1
a_5

Typical use: Which courses are offered by all departments?

$$COURSE \div (\pi_{\{Department\}} COURSE)$$

Note: $\{\sigma, \pi, \cup, -, \times\}$ are sufficient to define all these operations: this is a relationally complete set of operators.