4.2.2 MINIMIZATION PROCEDURE

We have seen that it is possible to implement a given logic function with various circuits. These circuits may have different structures and different costs. When designing a logic circuit, there are usually certain criteria that must be met. One such criterion is likely to be the cost of the circuit, which we considered in the previous discussion. In general, the larger the circuit, the more important the cost issue becomes. In this section we will assume that the main objective is to obtain a minimum-cost circuit.

Having said that cost is the primary concern, we should note that other optimization criteria may be more appropriate in some cases. For instance, in Chapter 3 we described several types of programmable-logic devices (PLDs) that have a predefined basic structure and can be programmed to realize a variety of different circuits. For such devices the main objective is to design a particular circuit so that it will fit into the target device. Whether or not this circuit has the minimum cost is not important if it can be realized successfully on the device. A CAD tool intended for design with a specific device in mind will automatically perform optimizations that are suitable for that device. We will show in section 4.6 that the way in which a circuit should be optimized may be different for different types of devices.

In the previous subsection we concluded that the lowest-cost implementation is achieved when the cover of a given function consists of prime implicants. The question then is how to determine the minimum-cost subset of prime implicants that will cover the function. Some prime implicants may have to be included in the cover, while for others there may be a choice. If a prime implicant includes a minterm for which $f = 1$ that is not included in any other prime implicant, then it must be included in the cover and is called an *essential prime implicant*. In the example in Figure 4.9, both prime implicants are essential. The term x_2x_3 is the only prime implicant that covers the minterm m_7, and $\overline{x}_1$ is the only one that covers the minterms m_0, m_1, and m_2. Notice that the minterm m_3 is covered by both of these prime implicants. The minimum-cost realization of the function is

$$f = \overline{x}_1 + x_2x_3$$

We will now present several examples in which there is a choice as to which prime implicants to include in the final cover. Consider the four-variable function in Figure 4.10. There are five prime implicants: $\overline{x}_1x_3$, $\overline{x}_2x_3$, $x_3\overline{x}_4$, $\overline{x}_1x_2x_4$, and $x_2\overline{x}_3x_4$. The essential ones (highlighted in blue) are $\overline{x}_2x_3$ (because of m_{11}), $x_3\overline{x}_4$ (because of m_{14}), and $x_2\overline{x}_3x_4$ (because of m_{13}). They must be included in the cover. These three prime implicants cover all minterms for which $f = 1$ except m_7. It is clear that m_7 can be covered by either $\overline{x}_1x_3$ or $\overline{x}_1x_2x_4$. Because $\overline{x}_1x_3$ has a lower cost, it is chosen for the cover. Therefore, the minimum-cost realization is

$$f = \overline{x}_2x_3 + x_3\overline{x}_4 + x_2\overline{x}_3x_4 + \overline{x}_1x_3$$

From the preceding discussion, the process of finding a minimum-cost circuit involves the following steps:

1. Generate all prime implicants for the given function f.
2. Find the set of essential prime implicants.