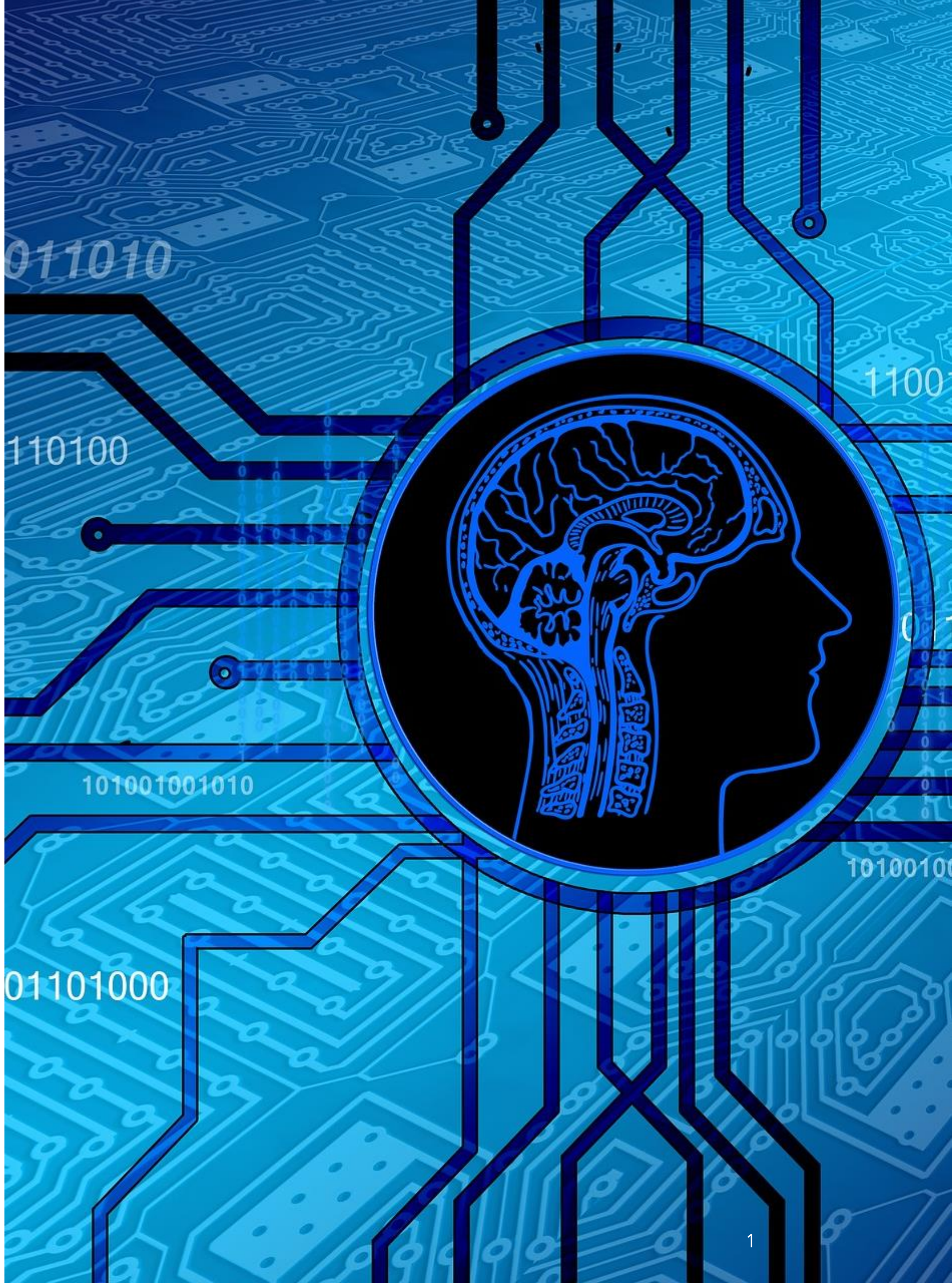
Informed Search

Petros Papapanagiotou

Informatics 2D: Reasoning and Agents
Lecture 4



Review: Tree search

function TREE-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
  loop do
    if the frontier is empty **then return** failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state **then return** the corresponding solution
    expand the chosen node, adding the resulting nodes to the frontier

*A search strategy is defined by picking the order of node expansion from the **frontier.***

Review: Tree search

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What if we **order** the nodes in the frontier by decreasing *desirability*?

A search strategy is defined by picking the order of node expansion from the ***frontier.***

Best-first search

An instance of general TREE-SEARCH or GRAPH-SEARCH

→ Use an evaluation function *f(n)* for each node *n*

- estimate of "desirability"

→ Expand most desirable unexpanded node, usually the node with the lowest evaluation

Heuristics

From the Greek «ευρίσκω» (/eˈvɾi.sko/), meaning to find, to discover.

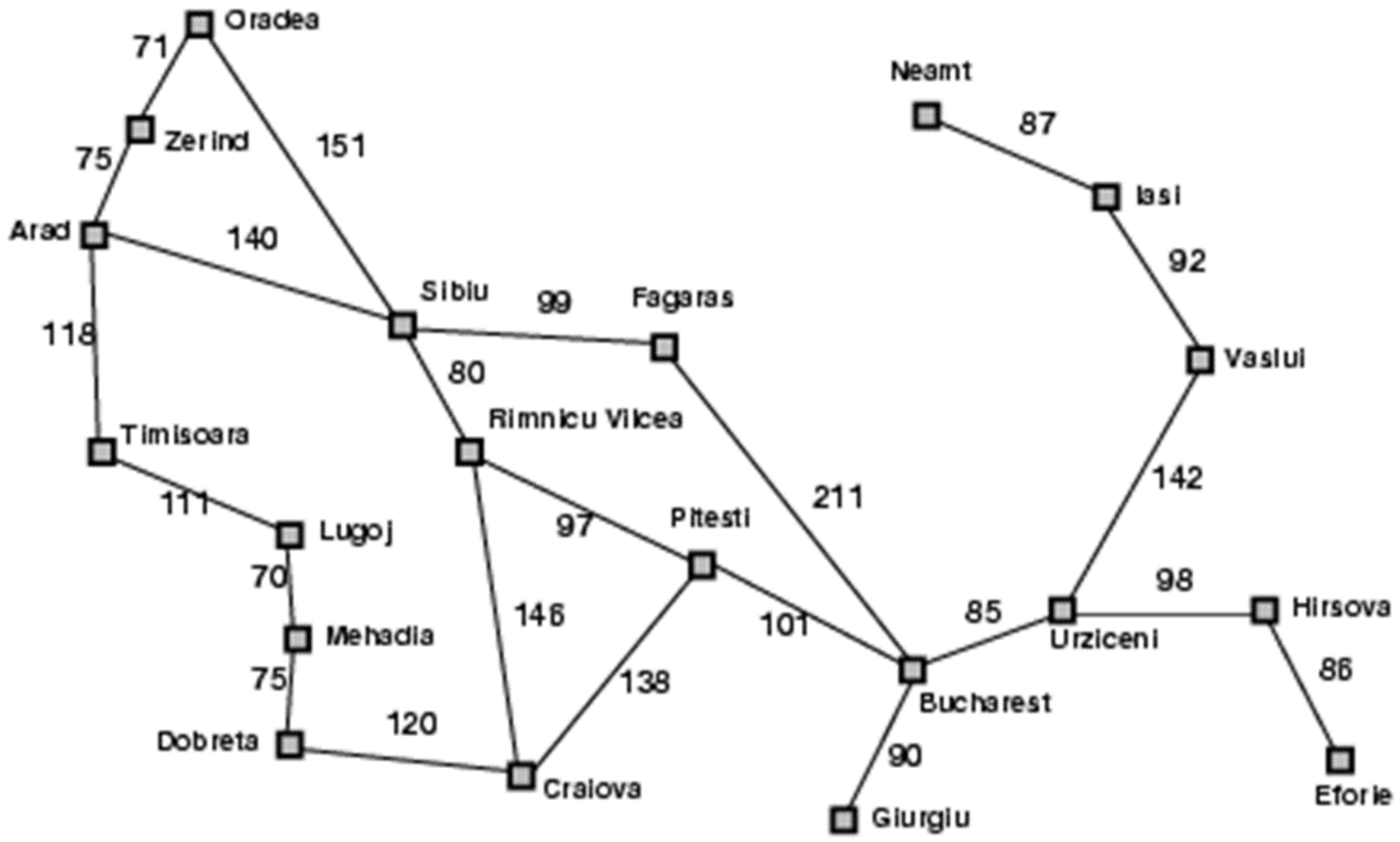
Any method that is believed or practically proven to be useful for the solution of a given problem.

- No guarantee that it will always work or lead to an optimal solution!

We use heuristics to guide tree search.

- This may not change the worst case complexity of the algorithm, but can help in the average case.

We introduce conditions (admissibility, consistency) in order to identify good heuristics, i.e. those which actually lead to an improvement over uninformed search.



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
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Romania

Greedy best-first search

Greedy best-first search

Evaluation function $f(n) = h(n)$ (heuristic)

- estimated cost of cheapest path from state at node n to a *goal* state
- e.g., $h_{SLD}(n)$ = straight-line distance from n to Bucharest

Greedy best-first search expands the node that appears to be closest to goal

Arad							

Best-first search

Straight–line distance
to Bucharest

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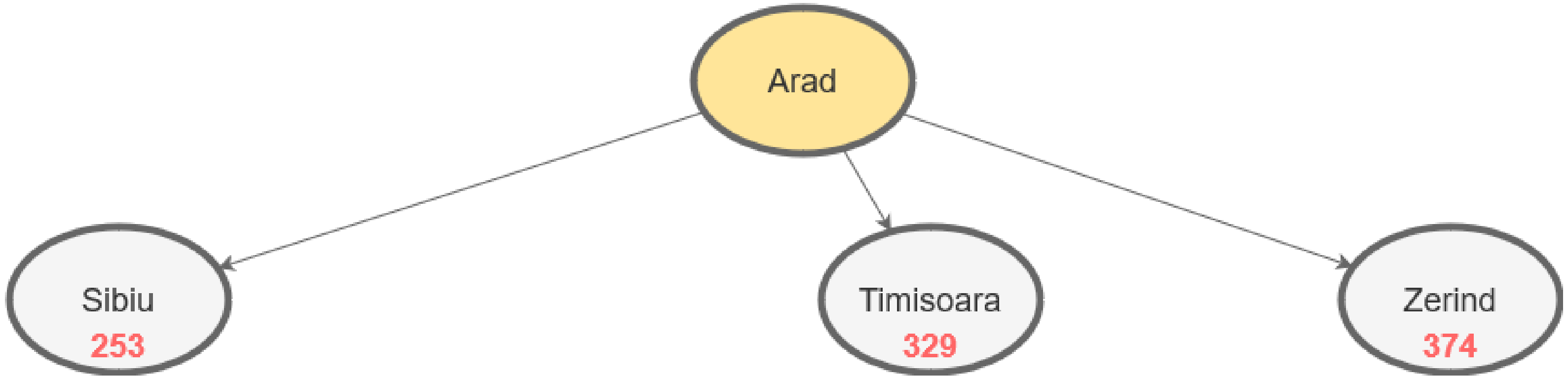
Sibiu	Timiso-ara	Zerind					

Best-first search



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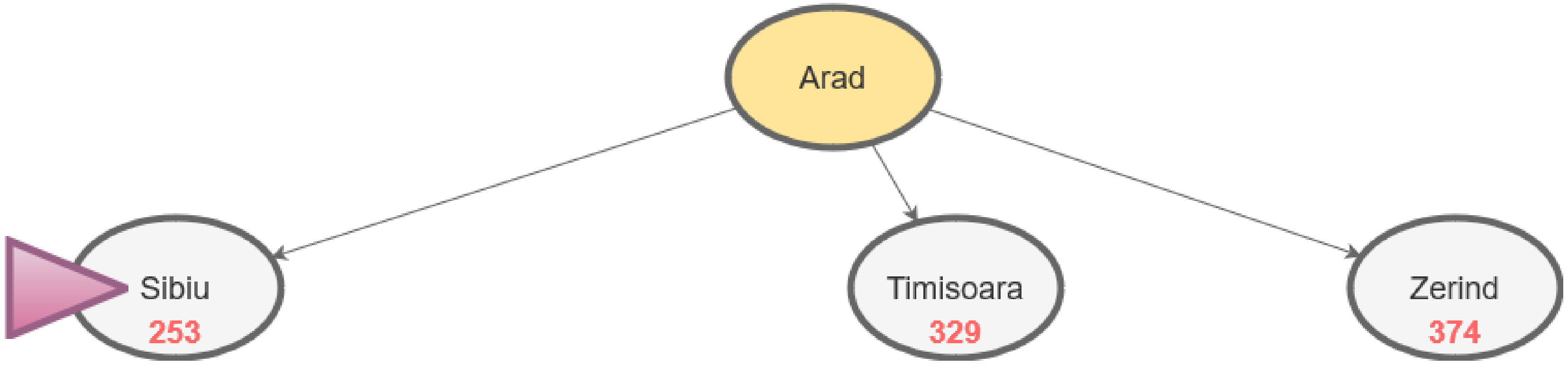

Timiso-ara	Zerind	Arad	Fagaras	Oradea	RV		

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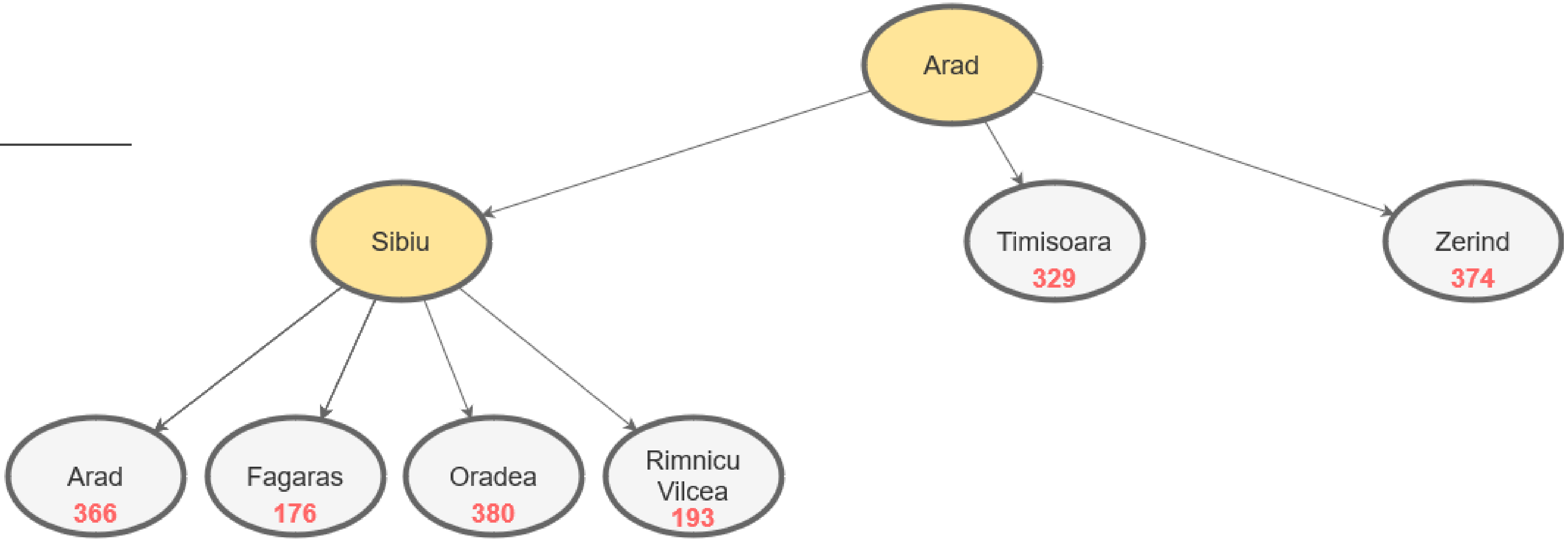

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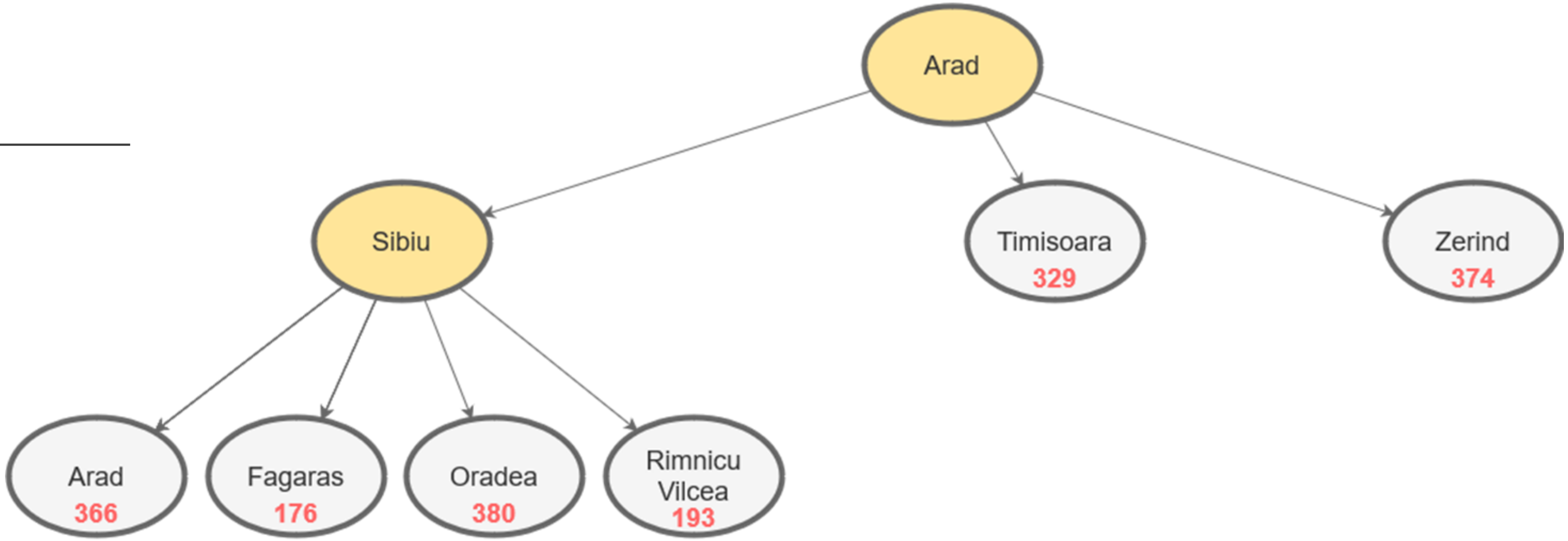

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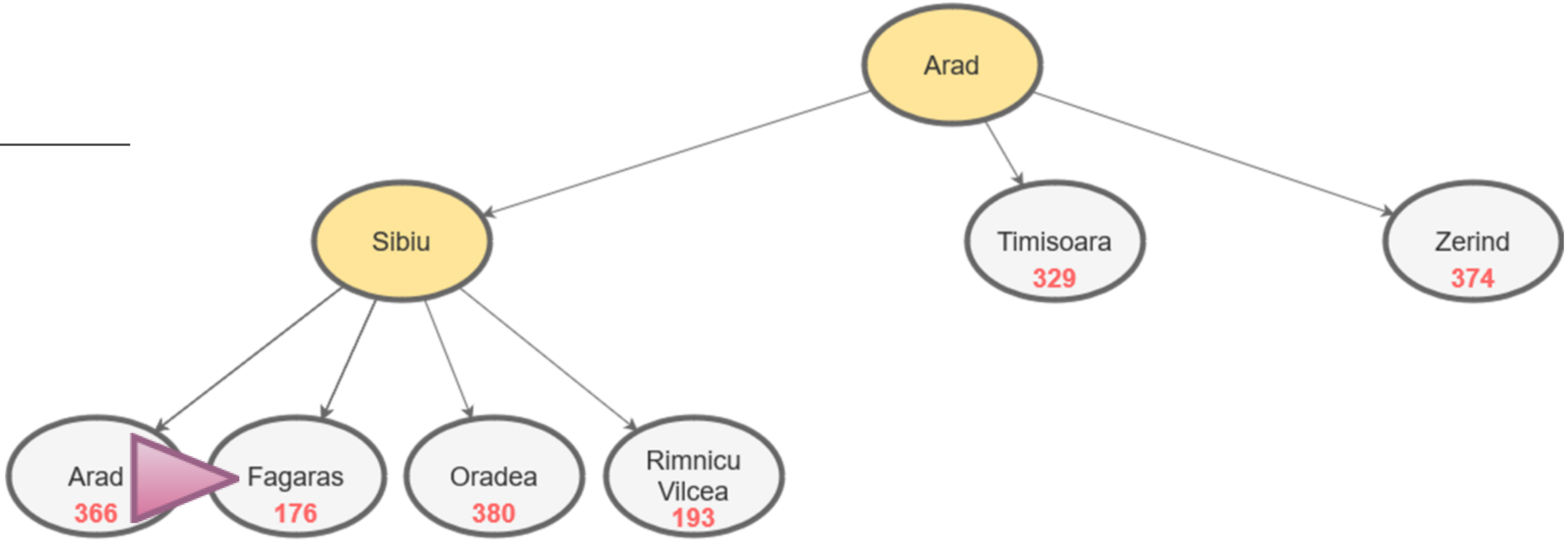

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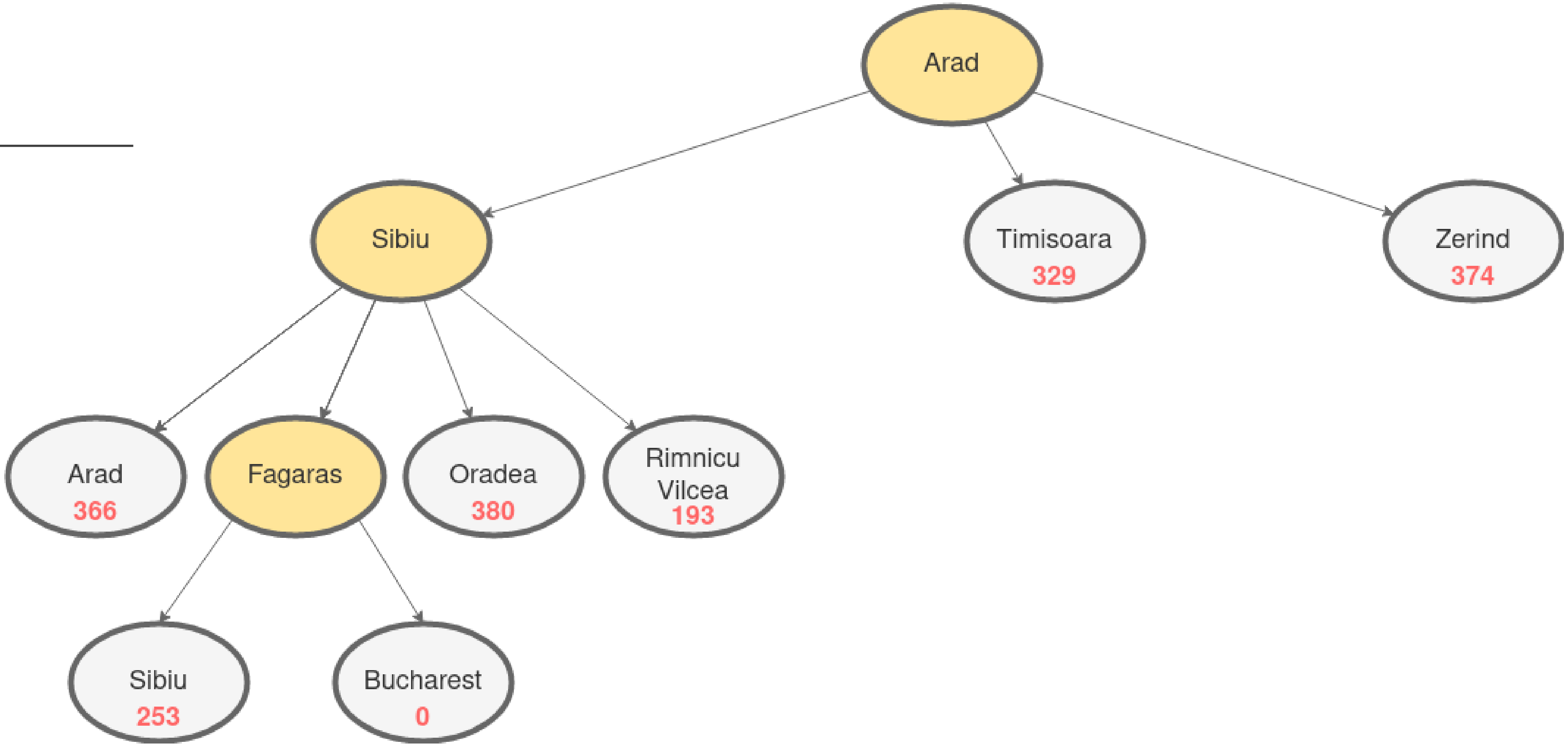


Properties of best-first search

Complete?

Time complexity?

Space complexity?

Optimal?

Properties of best-first search

Complete?
No! Can get stuck in loops.

Time complexity?

Space complexity?

Optimal?

Properties of best-first search

Complete?
No! Can get stuck in loops.

GRAPH-SEARCH is complete in finite spaces.

Time complexity?

Space complexity?

Optimal?

Properties of best-first search

Complete?
No! Can get stuck in loops.

Time complexity?
$O(b^m)$ for tree version

Space complexity?

Optimal?

Properties of best-first search

Complete?
No! Can get stuck in loops.

Time complexity?
$O(b^m)$ for tree version

A good heuristic can lead to dramatic improvement!

Space complexity?

Optimal?

Properties of best-first search

Complete?
No! Can get stuck in loops.

Time complexity?
$O(b^m)$ for tree version

Space complexity?
$O(b^m)$ – keeps all nodes in memory

Optimal?

Properties of best-first search

Complete?
No! Can get stuck in loops.

Time complexity?
$O(b^m)$ for tree version

Space complexity?
$O(b^m)$ – keeps all nodes in memory

Optimal?
No

A* Search

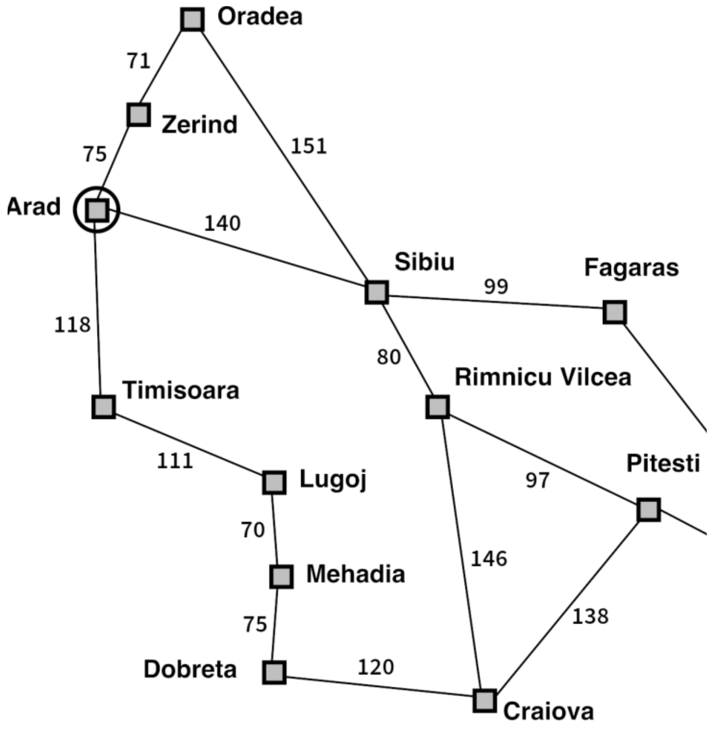
A* search

Evaluation function $f(n) = g(n) + h(n)$

- $g(n)$ = cost so far to reach n
- $h(n)$ = estimated cost from n to goal
- $f(n)$ = estimated total cost of path through n to goal

Avoid expanding paths that are already expensive

Best-first search



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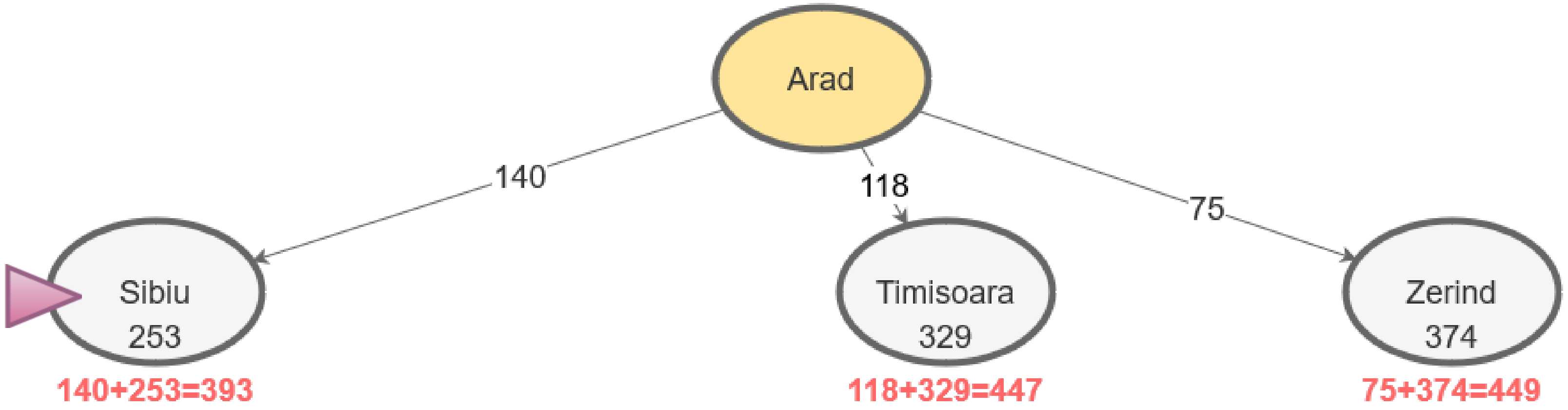
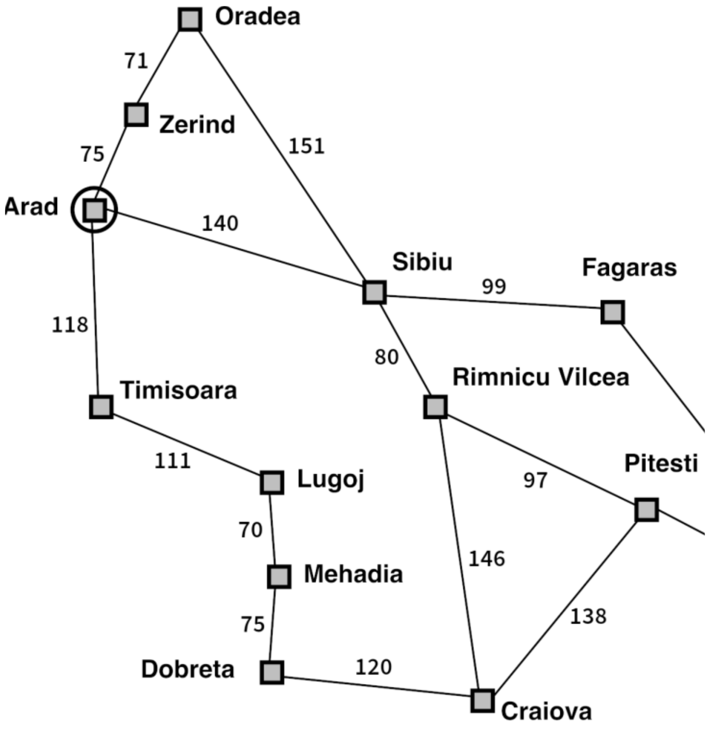
Best-first search





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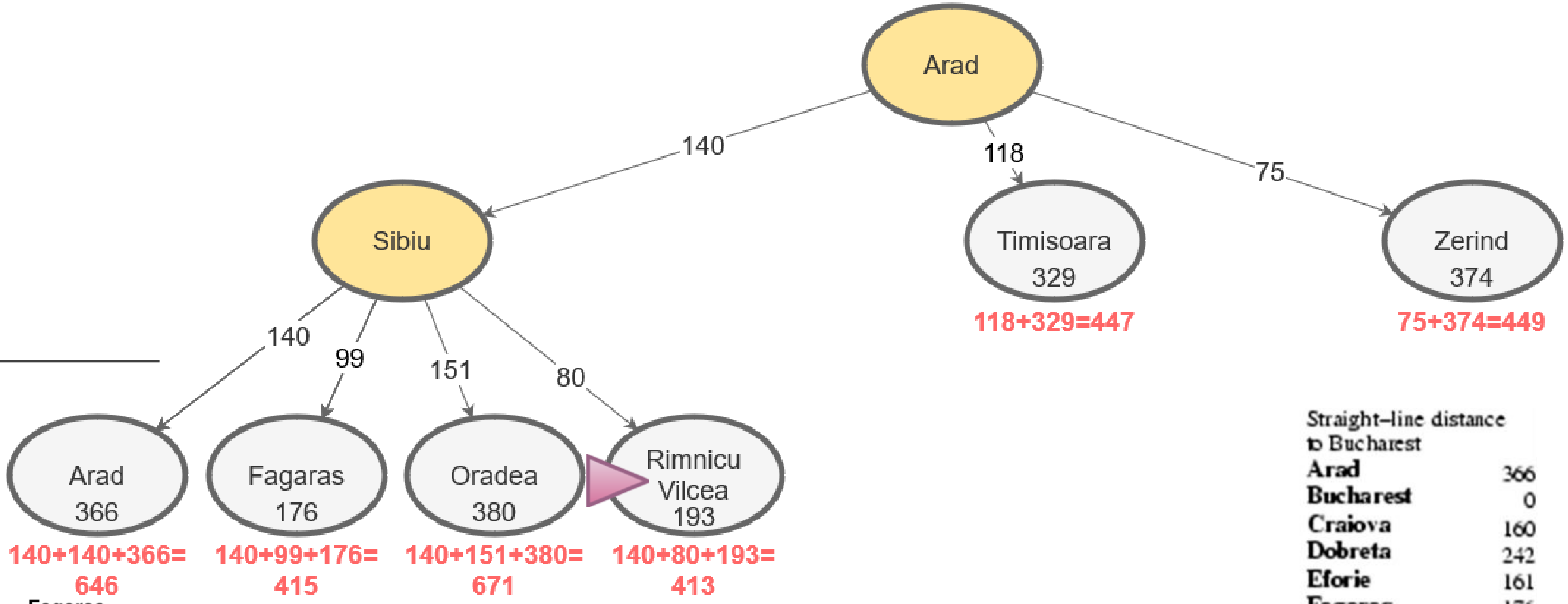

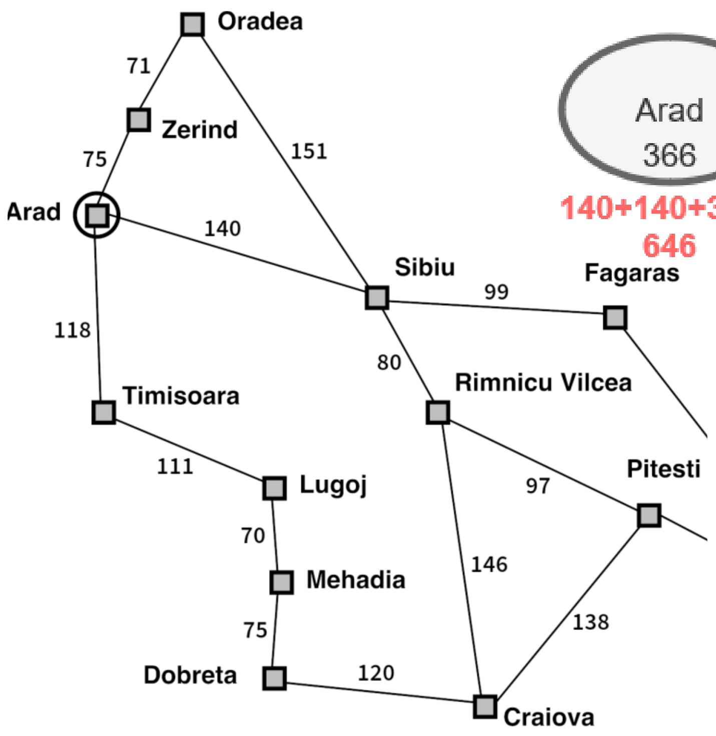

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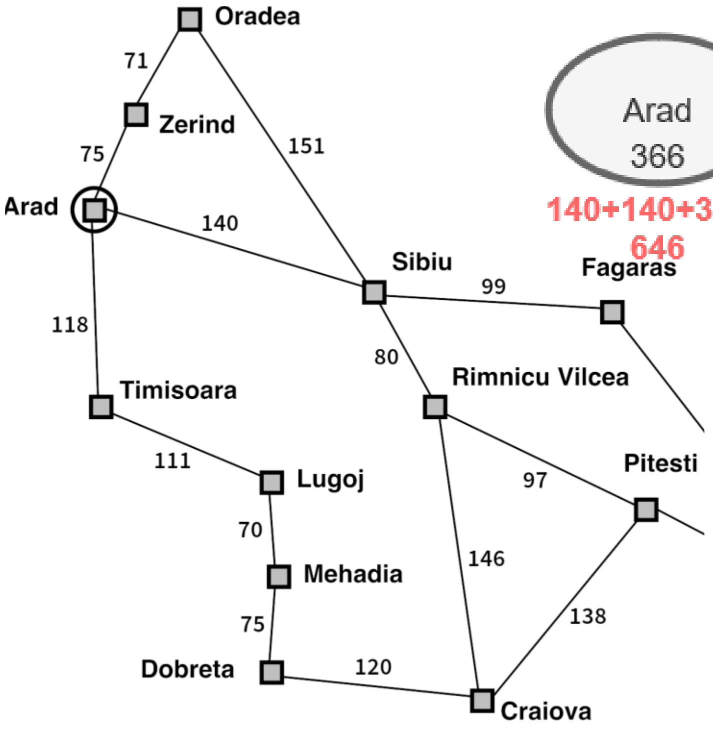

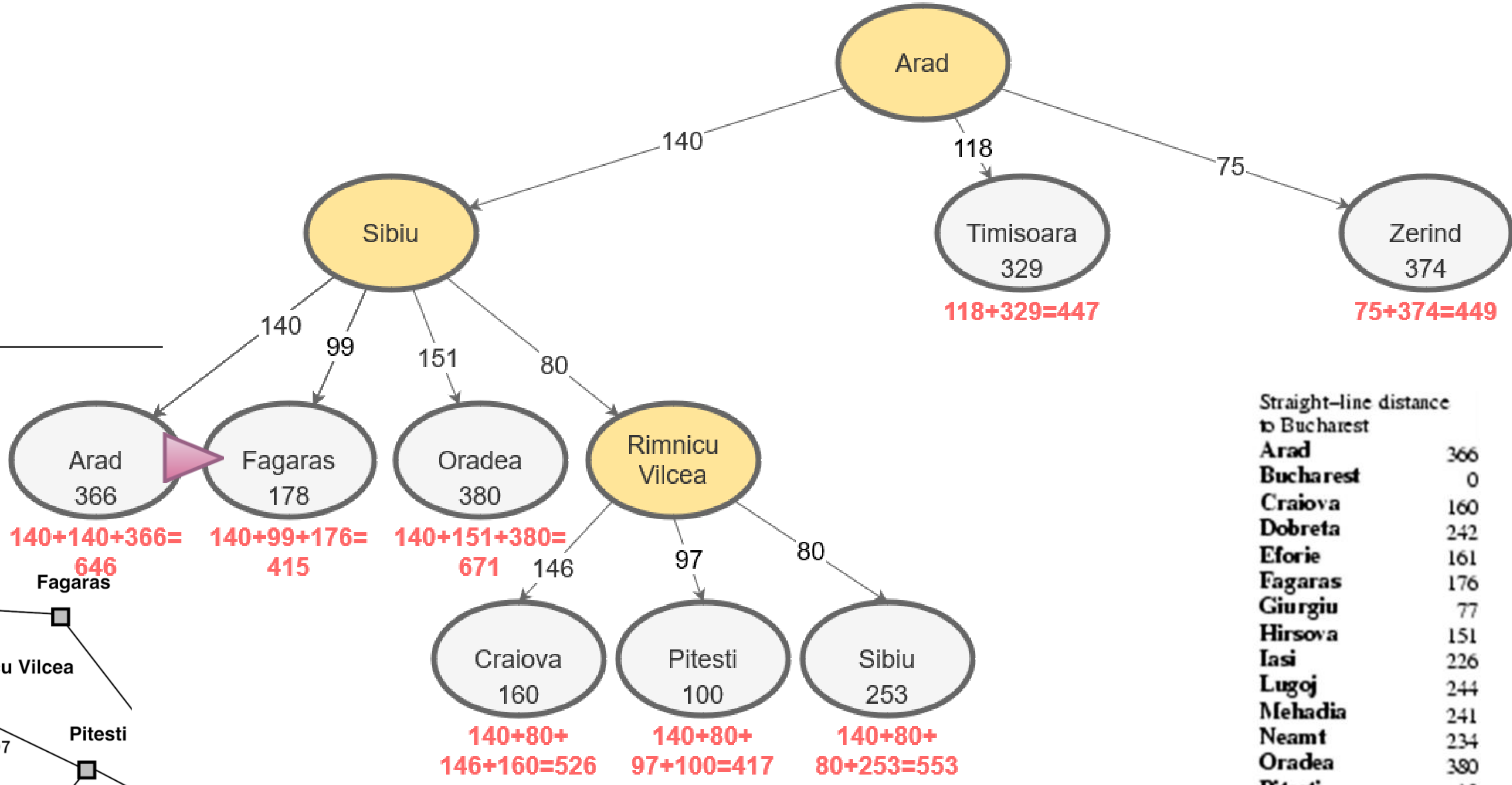

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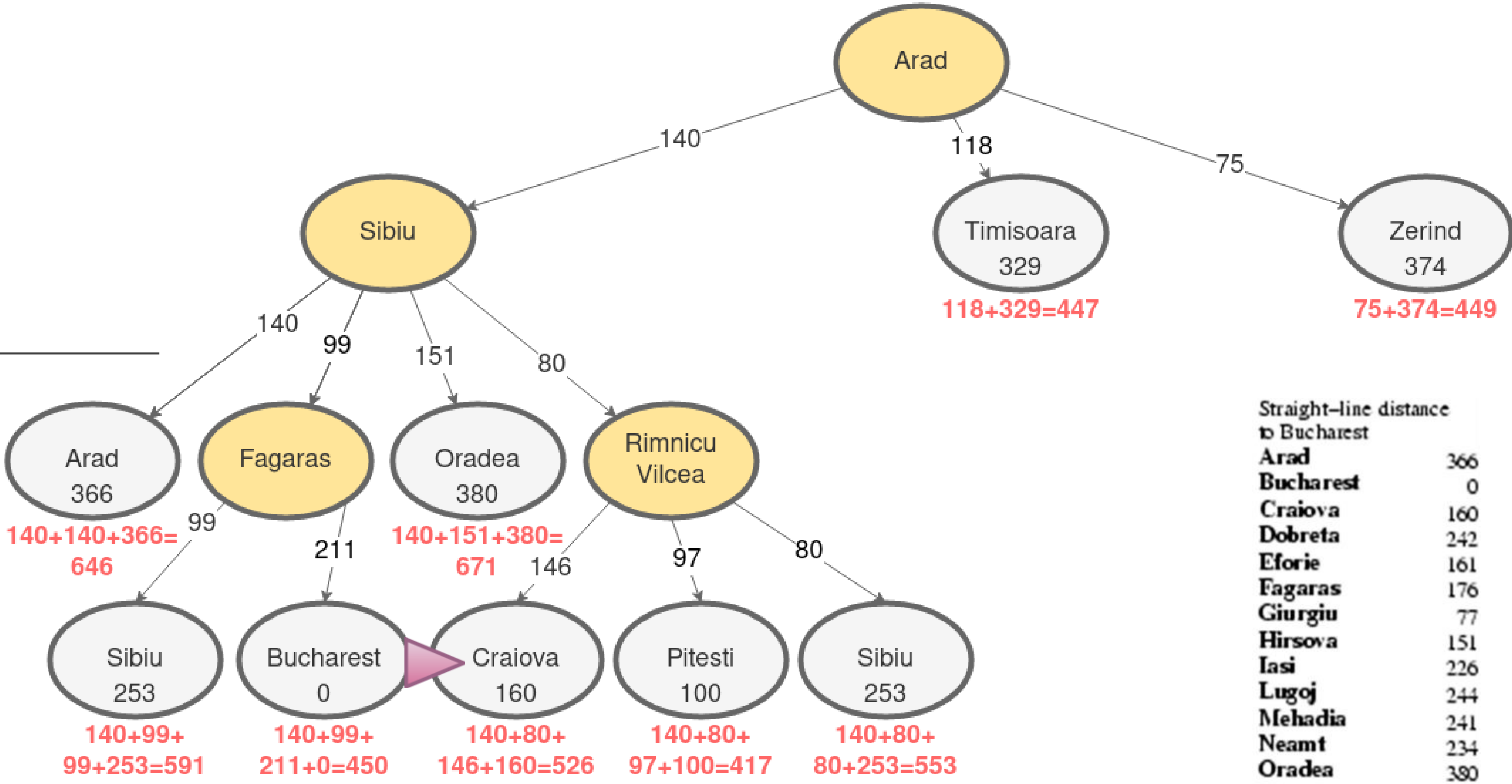

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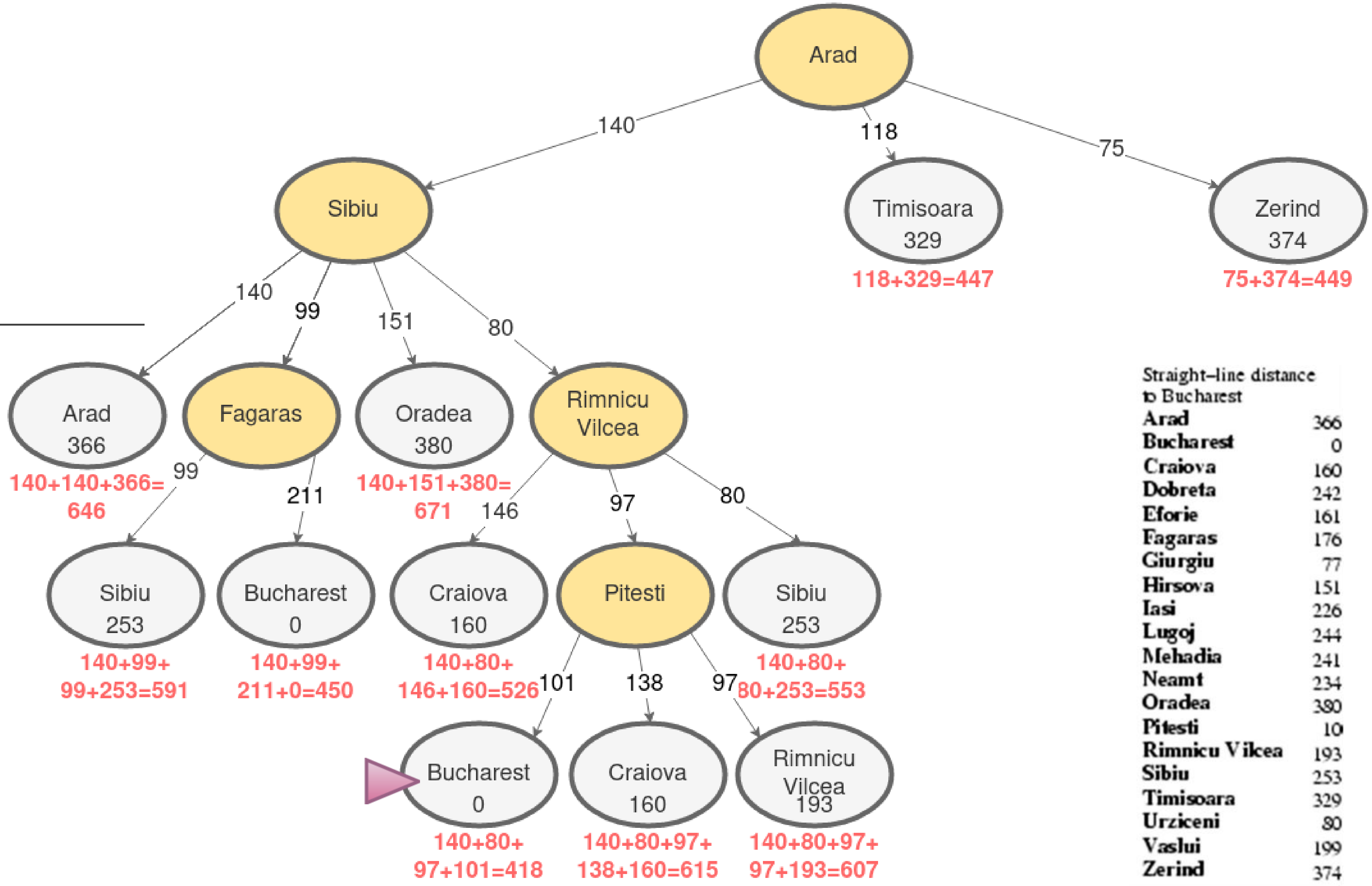
Best-first search



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Heuristics

Admissible heuristics

A heuristic *h(n)* is admissible if for every node *n:*

$$h(n) \leq \boldsymbol{h^*(n)}$$

where $\boldsymbol{h^*(n)}$ is the true cost to reach the goal state from *n*.

An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic

Example: $h_{SLD}(n)$ (never overestimates the actual road distance)

Admissible heuristic = optimal A*

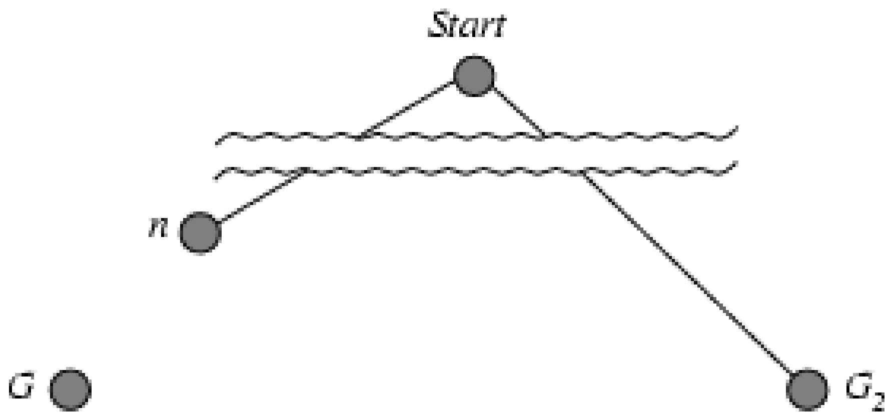
h(n) never overestimates the cost to reach the goal

Thus, $f(n) = g(n) + h(n)$ never overestimates the true cost of a solution

THEOREM

If $h(n)$ is admissible, A* using **TREE-SEARCH** is optimal

Proof: Optimality of A*

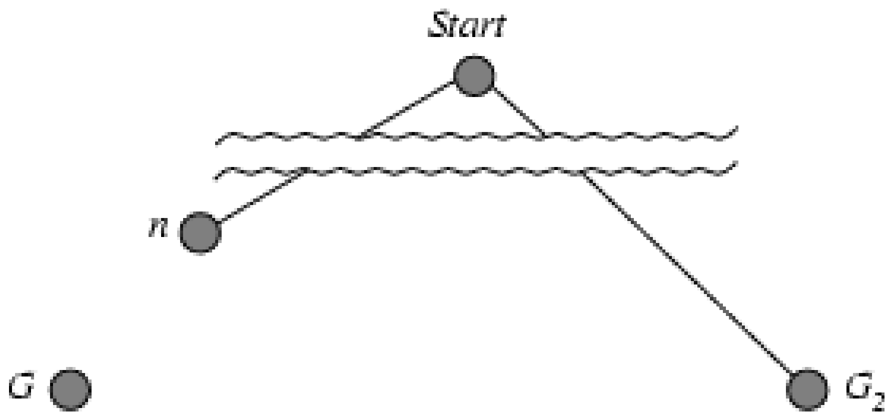


Suppose some suboptimal goal G_2 has been generated and is in the frontier.

Let n be an unexpanded node in the frontier such that n is on a shortest path to an optimal goal G.

$f(G_2) = g(G_2)$	since $h(G_2) = 0$
$g(G_2) > g(G)$	since G_2 is suboptimal
$f(G) = g(G)$	since $h(G) = 0$
$f(G_2) > f(G)$	from above

Proof: Optimality of A*



Suppose some suboptimal goal G_2 has been generated and is in the frontier.

Let n be an unexpanded node in the frontier such that n is on a shortest path to an optimal goal G.

$f(G) \quad < f(G_2)$ from above

$h(n) \quad \leq h^*(n)$ since h is admissible

$g(n) + h(n) \leq g(n) + h^*(n)$

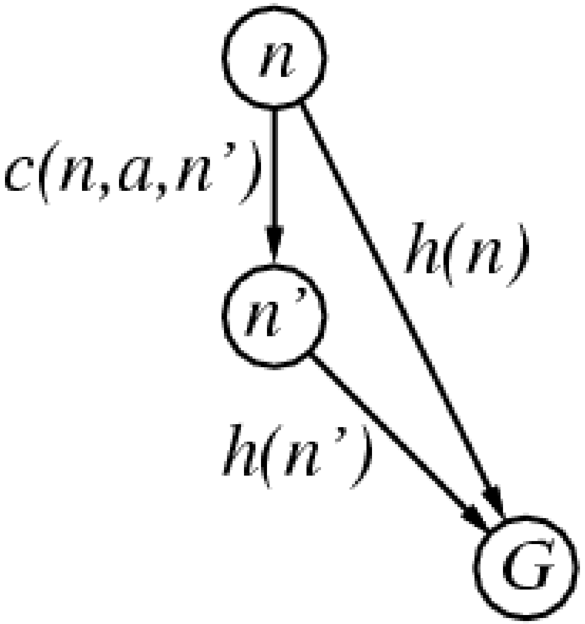
$f(n) \leq f(G)$

Hence $f(n) < f(G_2)$, and A will never select G2 for expansion*

Consistent heuristics

A heuristic *h(n)* is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*,

$$h(n) \leq c(n, a, n') + h(n')$$



If *h* is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &\geq f(n) \end{aligned}$$

i.e., *f*(*n*) is non-decreasing along any path.

THEOREM

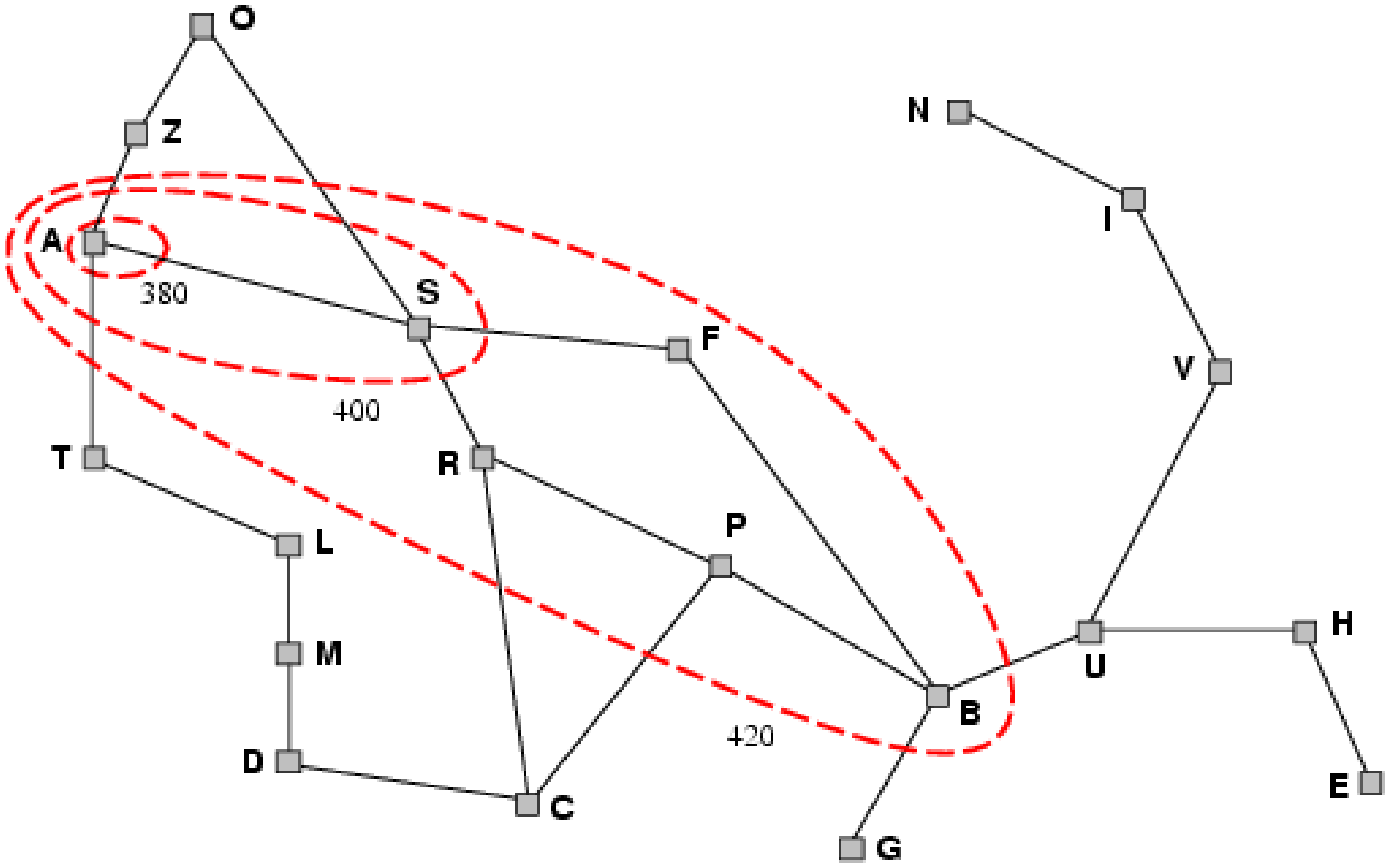
If *h*(*n*) is consistent, A* using **GRAPH-SEARCH** is optimal

Optimality of A^*

A^* expands nodes in order of increasing f value

Gradually adds "f-contours" of nodes

Contour i has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Properties of A*

Complete?

Time complexity?

Space complexity?

Optimal?

Properties of A*

Complete?
Yes (unless there are infinitely many nodes with $f \leq f(G)$)

Time complexity?

Space complexity?

Optimal?

Properties of A*

Complete?
Yes (unless there are infinitely many nodes with f ≤ *f*(*G*))

Time complexity?
Exponential

Space complexity?
Keeps all nodes in memory

Optimal?

Properties of A*

Complete?
Yes (unless there are infinitely many nodes with f ≤ *f*(*G*))

Time complexity?
Exponential

Space complexity?
Keeps all nodes in memory

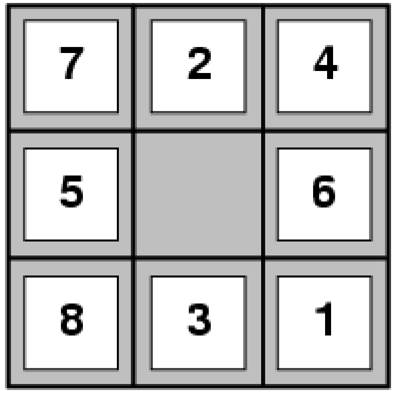
Optimal?
Yes

Admissible heuristics

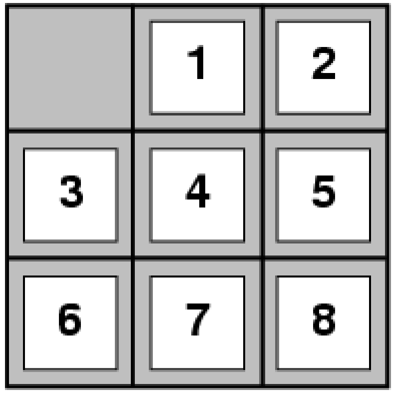
Example: 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



Start State



Goal State

$h_1(S) = ?$

$h_2(S) = ?$

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible) then

- h_2 dominates h1
- h_2 is better for search

Typical search costs (*average number of nodes expanded*):

- d=12 IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- d=24 IDS ≈ 54,000,000,000 nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Relaxed problems

A problem with fewer restrictions on the actions is called a *relaxed problem.*

The cost of an optimal solution to a relaxed problem is an *admissible heuristic* for the original problem.

If the rules of the 8-puzzle are relaxed so that a tile can move anywhere,

- then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to any adjacent square,

- then $h_2(n)$ gives the shortest solution

Use relaxation to automatically generate admissible heuristics!

Summary

Smart search based on heuristic scores.

- Best-first search
- Greedy best-first search
- A* search
- Admissible heuristics and optimality.

Why?

Informed search allows us to use domain knowledge to our advantage.

Optimality over some utility can often be the top priority.

A* is **very** popular!

- e.g. pathfinding

A* is simple, yet very efficient.

A* is *too* good sometimes (e.g. in games).