Introduction to Algorithms and Data Structures

Lecture 13: QuickSort

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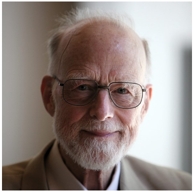
QuickSort

Invented by British computer scientist Tony Hoare in 1960 while studying in Moscow, published in 1961.



Divide-and-Conquer algorithm:

1. If the input array has $<$ two elements, do nothing.

   Otherwise, call Partition: Pick a pivot key and use it to divide the array into two:

$\leq$ pivot	pivot	$\geq$ pivot

2. Sort the two subarrays recursively.

QuickSort

Algorithm QuickSort(A, p, r)

1. **if** $p < r$ **then**
2. $\quad$ $split \leftarrow \text{Partition}(A, p, r)$
3. $\quad$ $\text{QuickSort}(A, p, split - 1)$
4. $\quad$ $\text{QuickSort}(A, split + 1, r)$

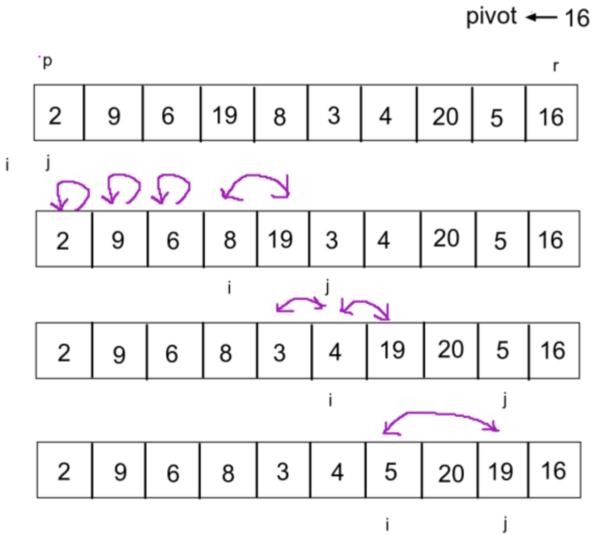
Partition

Algorithm Partition(A, p, r)

$pivot \leftarrow A[r].key$
$i \leftarrow p - 1$
for $j \leftarrow p$ **to** $r - 1$ **do**
    if $A[j] \leq pivot$
        $i \leftarrow i + 1$
        exchange $A[i]$ and $A[j]$
exchange $A[i+1]$ and $A[r]$
return $i + 1$

Invariant: i is 1 less than the leftmost $>$ *pivot* value in the range $p \ldots j$ (or is $j - 1$ if no $>$ *pivot* is there).

Partition example (done on video)



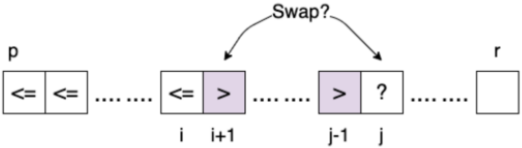
Finally we will swap A[i+1] (20) with A[r] (16) and return (i+1)

Correctness of Partition

Assume we are part-way through Partition and *for this j* (and this i), we have the Invariant (all $A[i+1 \ldots j-1]$ are $>$ *pivot*). Induction step:

- If $A[j].key > pivot$, the algorithm changes nothing.

  And the range of $>$ *pivot* cells expands by 1 (as j gets incremented).
- If $A[j].key \leq pivot$, we will swap $A[j]$ with $A[i+1]$. $A[i+1].key$ *either* ...
  - was $\leq$ *pivot* (and $i = j-1$), in which case $i' = i+1$ is j and we swap $A[j]$ with itself, and then have the same pattern after j increments, or
  - was $>$ *pivot*, in which case ...



- After loop, Invariant implies that the first $>$ *pivot* (if any) is in $A[i+1]$.

  Hence swapping $A[r]$ and $A[i+1]$ gives a “partition” of the desired form.

Results from Partition

- We might get a fairly balanced partition, with "pivot" lying near the middle ("*split*" roughly halfway between p and r).
  - When this happens, the Divide-and-Conquer balance is like MergeSort (and also, we have $\Theta(n)$ work at the "top level")
- Alternatively, we could get a very unbalanced partition, with one side empty or very small
  - Then we are doing linear work to reduce the size of the problem to be solved only a tiny bit. More like BubbleSort.
- Or anything in between, depends on the original arrangement of keys (and what "rank" *pivot* has among the keys in the array).

Running Time of QuickSort

Partition

$$T_{\text{partition}}(n) = \Theta(n)$$

QuickSort

$$\begin{aligned} T_{\text{quickSort}}(n) \quad &\in \quad \max_{1 \leq s \leq n-1} \big(T_{\text{quickSort}}(s) + T_{\text{quickSort}}(n-s-1) \big) \\ &\quad + \quad T_{\text{partition}}(n) + \Theta(1) \\ &= \quad \max_{1 \leq s \leq n-1} \big(T_{\text{quickSort}}(s) + T_{\text{quickSort}}(n-s-1) \big) + \Theta(n). \end{aligned}$$

Implies

$$\boxed{T_{\text{quickSort}}(n) \in \Theta(n^2)}$$

To show $\Omega(n^2)$ you need a specific structured input (not too hard).

QuickSort

The average-case *running-time of a sorting algorithm is the average number of computational steps (comparisons) carried out on a uniform random permutation of the keys* $\{1, \ldots, n\}$.

- We don't say "amortized" as we have a single computation, and are comparing input of exactly the same size.
- For sorting-algorithms, typically the running-time can be captured by the number of "comparisons" (these measures tend to be asymptotically equivalent).
- *Uniform random permutation* means all permutations arise with same probability.

The average-case running time of QuickSort is $\Theta(n \lg(n))$.

QuickSort

- QuickSort can be very fast in practice.
- But performs badly — $\Theta(n^2)$ — on sorted and almost sorted arrays.

Practical Improvements

- Different choice of pivot (key of middle item, random)
- Refined partitioning routine
- Use InsertionSort for small arrays (similar to “TimSort”)

RandomQuickSort

- The $\Theta(n \lg(n))$ result for average-case can be shown to carry over characterize (expected) running-time for RandomQuickSort (choose the pivot randomly).

Sorting in Python

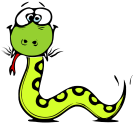
The default sorting algorithm in python is "Timsort", an optimized version of MergeSort developed by Tim Peters, a major contributor to the development of CPython.

- Does a pre-processing step looking for "runs" of strictly decreasing or (non-strict) increasing items.
- We understand how to handle "runs" (without sorting).
- Then merge the sorted runs, using InsertSort for short subarrays, and MergeSort for bigger subarrays.

Reading Material

Personal Reading:

- QuickSort and its analysis, sections 7.1, 7.2 and 7.4 of [CLRS]
- "QuickShort" interview with Tony Hoare can be viewed at `http://anothercasualcoder.blogspot.com/2015/03/my-quickshort-interview-with-sir-tony.html`
- To read about "Timsort" ... read the `listsort.txt` file in the `Objects` directory from the `python` download.



Source code (eg version 3.7.4) can be downloaded `https://www.python.org/downloads/source/`