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Situation Calculus

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15.a

Situations

USING LOGIC TO PLAN

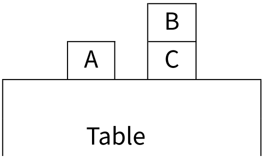
We need ways of representing

- the world
- the goal
- how actions change the world

We haven't said much about changing the world.

Difficulty After an action, new things are true, and some previously true facts are no longer true.

EXAMPLE · BLOCKS WORLD



SITUATIONS

SITUATIONS

Extend the concept of a **state** by additional logical terms.
Consist of initial situation (usually called S_0) and all situations generated by applying an action to a situation.

Providing **facts about situations**

By relating predicates to situations.
e.g. instead of saying just $\text{On}(A, \text{B})$, say $\text{On}(A, \text{B})$ in situation S_0

ACTIONS

Are performed in a situation.
Produce new situations with new facts.
e.g. Forward and $\text{Turn}(\text{Right})$

REPRESENTING PREDICATES RELATIVE TO A SITUATION

Can **add an argument** for a situation to each predicate that can change.

e.g. instead of $\mathsf{On}(A, B)$, write $\mathsf{On}(A, B, S_0)$

Alternatively, **introduce a predicate** Holds

On etc. become functions

e.g. $\mathsf{Holds}(\mathsf{On}(A, B), S_0)$

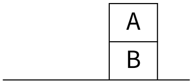
What do things like $\mathsf{On}(A, B)$ *mean now?*

A set of situations in which A is **on** B.

HOW THIS WILL WORK

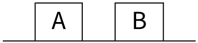
Before some action, we might have in our KB

On(A, B, S_0)
On$(\mathrm{B}, \text{Table}, S_0)$



After an action that moves A to the table, say, we add

Clear(B, S_1)
On(A, Table, S_1)

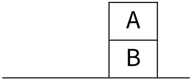


All these propositions are true. We have dealt with the issue of change, by keeping track of what is true when.

SAME THING, SLIGHTLY DIFFERENT NOTATION

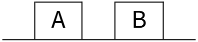
Before

$\text{Holds}(\text{On}(A, \mathrm{B}), S_0)$
$\text{Holds}(\text{On}(\mathrm{B}, \text{Table}), S_0)$



After, add

$\text{Holds}(\text{Clear}(\mathrm{B}), S_1)$
$\text{Holds}(\text{On}(A, \text{Table}), S_1)$



REPRESENTING ACTIONS

We need to represent

- results of doing an action
- conditions that need to be in place to perform an action

For convenience, we will define **functions** to abbreviate actions

- e.g. $\mathsf{Move}(A, \mathrm{B})$ denotes the **action type** of moving A onto B
- These are action types, because actions themselves are specific to time, etc.

Now, introduce a **function** Result, designating “the situation resulting from doing an action type in some situation”.

- e.g. $\mathsf{Result}(\mathsf{Move}(A, \mathrm{B}), S_0)$ means “the situation resulting from doing an action of type $\mathsf{Move}(A, \mathrm{B})$ in situation S_0”.

HOW DOES IT WORK?

Keep in mind that things like

$$\text{Result}(\text{Move}(A, \mathrm{B}), S_0)$$

are terms and denote **situations**.

They can appear anywhere we would expect a situation.

So we can say things like

$$S_1 = \text{Result}(\text{Move}(A, \mathrm{B}), S_0)$$
$$\text{On}(A, \mathrm{B}, \text{Result}(\text{Move}(A, \mathrm{B}), S_0)) \equiv \text{On}(A, \mathrm{B}, S_1)$$

Alternatively,

$$\text{Holds}(\text{On}(A, \mathrm{B}), \text{Result}(\text{Move}(A, \mathrm{B}), S_0))$$

AXIOMATISING ACTIONS

We can describe the results of actions, together with their **preconditions**.

e.g. "If nothing is on x and y, then one can move x to on top of y, in which case x will then be on y."

$\forall x, y, s. \mathsf{Clear}(x, s) \wedge \mathsf{Clear}(y, s) \rightarrow \mathsf{On}(x, y, \mathsf{Result}(\mathsf{Move}(x, y), s))$

Alternatively,

$$\forall x, y, s. \mathsf{Holds}(\mathsf{Clear}(x), s) \wedge \mathsf{Holds}(\mathsf{Clear}(y), s) \\ \rightarrow \mathsf{Holds}(\mathsf{On}(x, y), \mathsf{Result}(\mathsf{Move}(x, y), s))$$

This is an **effect axiom**.
It includes a precondition as well.

SITUATION CALCULUS

This approach is called the **situation calculus**.

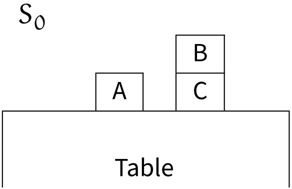
We axiomatise all our actions, then use a general theorem prover to prove that a situation exists in which our goal is true.

The actions in the proof would comprise our plan.

EXAMPLE

KB

On$(A,$ Table$, S_0)$
On$(\mathrm{B}, \mathrm{C}, S_0)$
On$(\mathrm{C},$ Table$, S_0)$
Clear(A, S_0)
Clear(B, S_0)
+ axioms about actions



Goal

$\exists s'.$On(A, B, s')

EXAMPLE

We want to prove $\text{On}(A, B, s')$ for some s'.

1. Find axiom
$\forall x, y, s.\text{Clear}(x, s) \wedge \text{Clear}(y, s) \rightarrow \text{On}(x, y, \text{Result}(\text{Move}(x, y), s))$

2. Goal would be true if we could prove
$\text{Clear}(A, s) \wedge \text{Clear}(B, s)$ by backward chaining.

3. But both are true in S_0, so we can conclude
$\text{On}(A, B, \text{Result}(\text{Move}(A, B), S_0))$

We are done!
We look at the proof and see only one action, $\text{Move}(A, B)$, which is executed in situation S_0, so this is our **plan**.

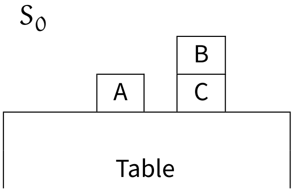
15.b

Frame problem

EXAMPLE · HARDER GOAL[1]

KB

On$(A$, Table, $S_0)$
On(B, C, S_0)
On$(C$, Table, $S_0)$
Clear(A, S_0)
Clear(B, S_0)
+ axioms about actions



Goal

$\exists s'.$On$(A, B, s') \wedge$ On(B, C, s')

[1] It's not really harder, B is already on C, and we just showed how to put A on B.

WITH NEW GOAL

Suppose we try to prove the first subgoal, $\mathsf{On}(A, B, s')$.

1. We use the same axiom
$\forall x, y, s.\mathsf{Clear}(x, s) \wedge \mathsf{Clear}(y, s) \rightarrow \mathsf{On}(x, y, \mathsf{Result}(\mathsf{Move}(x, y), s))$

2. Again, by chaining, we can conclude
$\mathsf{On}(A, B, \mathsf{Result}(\mathsf{Move}(A, B), S_0))$
Abbreviating $\mathsf{Result}(\mathsf{Move}(A, B), S_0)$ as S_1, we have $\mathsf{On}(A, B, S_1)$.

3. Substituting S_1 for s' in our other subgoal makes that $\mathsf{On}(B, C, S_1)$.
If this were true, we are done.

But we have **no reason to believe this is true**!
Sure, $\mathsf{On}(B, C, S_0)$, but how does the planner know this is still true, i.e., $\mathsf{On}(B, C, S_1)$?
In fact, it doesn't, so it fails to find an answer!

THE FRAME PROBLEM

We have failed to express the fact that everything that **isn't changed by an action should really stay the same**.

Can fix by adding **frame axioms**.

$$\forall x, s.\mathsf{Clear}(x, s) \rightarrow \mathsf{Clear}(x, \mathsf{Result}(\mathsf{Paint}(x), s))$$

$$\ldots$$

There are lots of these!

Is this a big problem?

BETTER FRAME AXIOMS

Can fix with neater formulation:

$$\begin{aligned}\forall x, y, s, a. &\mathsf{On}(x, y, s) \wedge (\forall z. a = \mathsf{Move}(x, z) \rightarrow y = z) \\ &\rightarrow \mathsf{On}(x, y, \mathsf{Result}(a, s))\end{aligned}$$

Can combine with effect axioms to get **successor-state axioms:**

$$\begin{aligned}\forall x, y, s, a. &\mathsf{On}(x, y, \mathsf{Result}(a, s)) \leftrightarrow \\ &\mathsf{On}(x, y, s) \wedge (\forall z. a = \mathsf{Move}(x, z) \rightarrow y = z) \\ &\vee(\mathsf{Clear}(x, s) \wedge \mathsf{Clear}(y, s) \wedge a = \mathsf{Move}(x, y))\end{aligned}$$

HOW DOES THIS HELP OUR EXAMPLE?

We want to prove $\text{On}(B, C, \text{Result}(\text{Move}(A, B), S_0))$ given $\text{On}(B, C, S_0)$.

Axiom says $\forall x, y, s, a.\text{On}(x, y, \text{Result}(a, s)) \leftrightarrow$
$\text{On}(x, y, s) \land (\forall z. a = \text{Move}(x, z) \rightarrow y = z)$
$\lor(\text{Clear}(x, s) \land \text{Clear}(y, s) \rightarrow a = \text{Move}(x, y))$

So we need to show
$\text{On}(B, C, S_0) \land (\forall z.\text{Move}(A, B) = \text{Move}(B, z) \rightarrow C = z)$ is true:

- The first conjunct is in the KB.
- The second one is true since actions are the same iff they have the same name and involve the exact same objects*:
  $A(x_1, \ldots, x_m) = A(y_1, \ldots, y_m)$ iff $x_1 = y_1, \ldots, x_m = y_m$.
  So $\text{Move}(A, B) = \text{Move}(B, z)$ is false.

*Another assumption in KB: $A(x_1, \ldots, x_m) \neq B(y_1, \ldots, y_n)$.

These are known as **Unique Action Axioms**.

Suppose $\forall x.\exists y.G(x, y)$ is the goal in resolution refutation.

We need to **negate** the goal:

$\neg\forall x.\exists y.G(x, y) \equiv \exists x.\forall y.\neg G(x, y)$

Then skolemise (i.e drop the existential quantifier):

$\neg G(X_0, y)$

INTUITION

y is to be unified to construct **witness**.

X_0 must **not** be instantiated.

KB AND AXIOMS AS CLAUSES

Variables a, x, y, z, s

Constants A, B, C, S_0

Initial State

$\mathsf{On}(A, \mathsf{Table}, S_0)$
$\mathsf{On}(B, C, S_0)$
$\mathsf{On}(C, \mathsf{Table}, S_0)$
$\mathsf{Clear}(A, S_0)$
$\mathsf{Clear}(B, S_0)$

(neg.) Goal

$\neg\mathsf{On}(A, B, s') \vee \neg\mathsf{On}(B, C, s')$

KB AND AXIOMS AS CLAUSES

Effect Axiom

$\neg\text{Clear}(x, s) \vee \neg\text{Clear}(y, s) \vee \text{On}(x, y, \text{Result}(\text{Move}(x, y), s))$

Skolem function

Frame Axioms

$\neg\text{On}(x, y, s) \vee a = \text{Move}(x, Z(x, y, s, a)) \vee \text{On}(x, y, \text{Result}(a, s))$

$\neg\text{On}(x, y, s) \vee \neg y = Z(x, y, z, s, a) \vee \text{On}(x, y, \text{Result}(a, s))$

Unique Action Axioms

$\neg\text{Move}(A, B) = \text{Move}(B, z)$

Unique Name Axiom

$\neg C_i = C_j$ for every pair of distinct constants C_i and C_j in KB.

RESOLUTION REFUTATION

¬On(A,B,s') ∨ ¬On(B,C,s')

¬Clear(x,s) ∨ ¬Clear(y,s) ∨
On(x,y, Result(Move(x,y),s))

Clear(A,S0)

¬Clear(A,s) ∨ ¬Clear(B,s) ∨
¬On(B,C, Result(Move(A,B),s))

Clear(B,S0)

¬On(x,y,s) ∨
a=Move(x,Z(x,y,z,s,a)) ∨
On(x,y,Result(a,s))

¬Clear(B,S0) ∨
¬On(B,C, Result(Move(A,B),S0))

¬On(B,C, Result(Move(A,B),S0))

¬On(B,C,S0) ∨
Move(A,B)=Move(B,Z(x,y,z,s,a))

¬Move(A,x) = Move(B,z)

¬On(B,C,S0) → □ ← On(B,C,S0)

FRAME PROBLEM PARTIALLY SOLVED

This solves the representational part of the frame problem.

Still have to compute that everything that was true and wasn't changed is still true.

Inefficient (as is general theorem proving).

SOLUTION

Special purpose representations and special purpose algorithms, called **planners**.