Effective Propositional Inference

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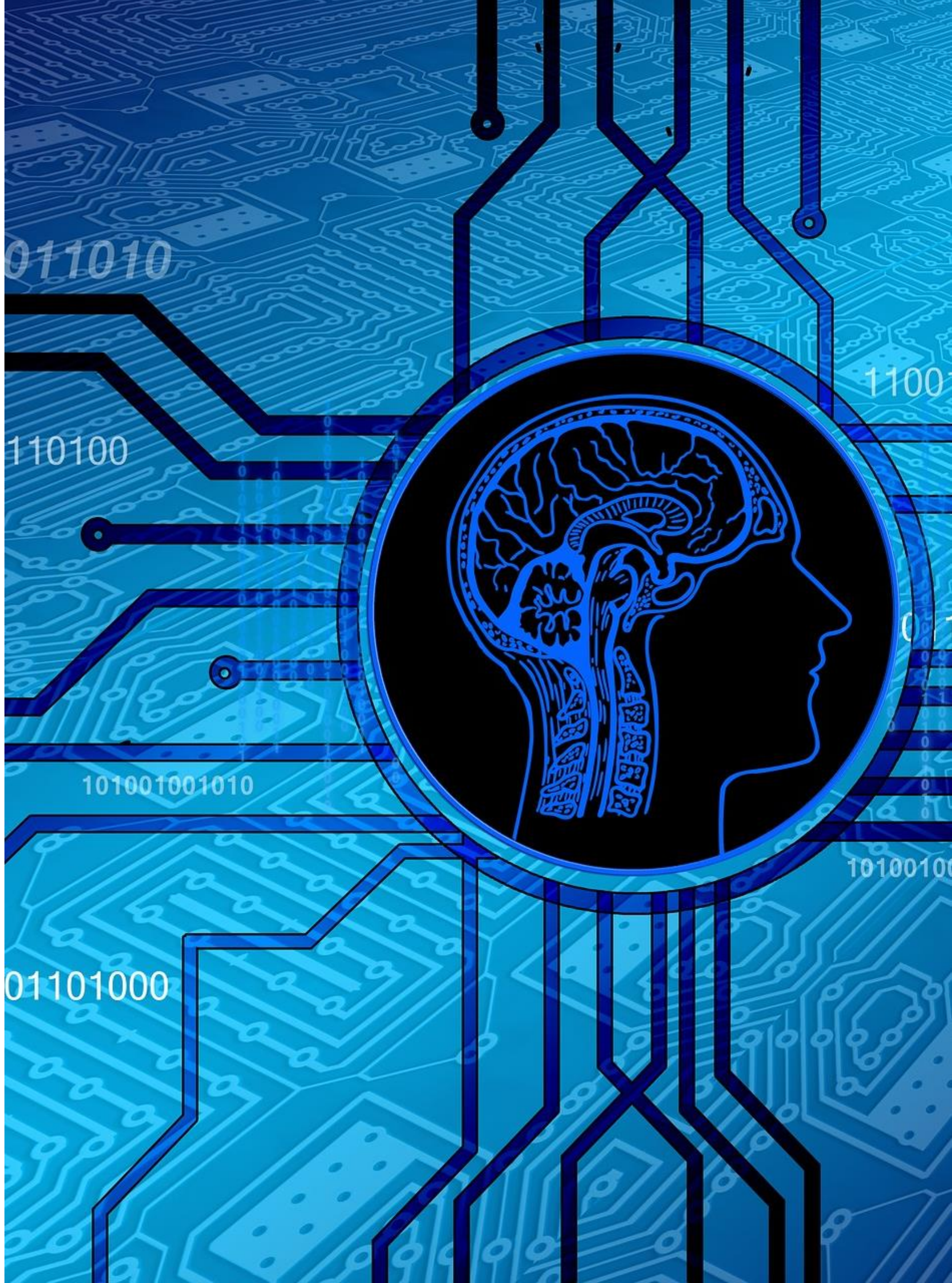
Informatics 2D: Reasoning and Agents
Lecture 10



Outline

Two families of efficient algorithms for propositional inference:

Complete backtracking search algorithms

- DPLL algorithm (Davis, Putnam, Logemann, Loveland)

Incomplete local search algorithms

- WalkSAT algorithm

Clausal Form (CNF)

DPLL and WalkSAT manipulate formulae in conjunctive normal form (CNF).

Sentence	• Formula whose satisfiability is to be determined • Conjunction of clauses
Clause	• Disjunction of literals
Literal	• Proposition symbol or negated proposition symbol

e.g. $(A, \neg B), (B, \neg C)$ represents $(A \vee \neg B) \wedge (B \vee \neg C)$

Conversion to CNF

$$(B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1})$$

Eliminate $\Leftrightarrow$: replace $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$

- $\left(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})\right) \wedge \left((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}\right)$

Eliminate $\Rightarrow$: replace $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$

- $(\neg B_{1,1} \vee (P_{1,2} \vee P_{2,1})) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

Move $\neg$ inwards : use de Morgan's rules and double negation $\neg\neg\alpha = \alpha$

- $(\neg B_{1,1} \vee (P_{1,2} \vee P_{2,1})) \wedge \left((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1}\right)$

Create clauses: apply distributivity law ($\vee$ over $\wedge$) and flatten

- $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

DPLL

The DPLL algorithm

Determine if an input propositional logic sentence (in CNF) is satisfiable.

Improvements over truth table enumeration:

- Early termination
- Pure symbol heuristic
- Unit clause heuristic

Early termination

A *clause* is true if **one** of its literals is true,

- e.g. if A is true then $(A \lor \neg B)$ is true.

A *sentence* is false if **any** of its clauses is false,

- e.g. if A is false and B is true then
- $(A \lor \neg B)$ is false, so any sentence containing it is false.

Pure symbol heuristic

Pure symbol: **always** appears with the same "*sign*" or *polarity* in **all** clauses.

- e.g., In the three clauses $(A \lor \neg B)$, $(\neg B \lor \neg C)$, $(C \lor A)$:
  - A and B are pure, C is impure.

Make literal containing a pure symbol true.

- e.g. Let A and $\neg B$ both be true.

Unit clause heuristic

Unit clause: only one literal in the clause

- e.g. (A)

The only literal in a unit clause must be true.

- e.g. A must be true.

Also includes clauses where **all but one** literal is false,

- e.g. (A,B,C) where B and C are false since it is equivalent to (A, false, false) i.e. (A).

The DPLL algorithm

function DPLL-SATISFIABLE?(s) **returns** *true* or *false*
  inputs: s, a sentence in propositional logic

  $clauses \leftarrow$ the set of clauses in the CNF representation of s
  $symbols \leftarrow$ a list of the proposition symbols in s
  return DPLL($clauses, symbols, \{\,\}$)

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function DPLL($clauses, symbols, model$) **returns** *true* or *false*

  if every clause in $clauses$ is true in $model$ **then return** *true*
  if some clause in $clauses$ is false in $model$ **then return** *false*
  $P, value \leftarrow$ FIND-PURE-SYMBOL($symbols, clauses, model$)
  if P is non-null **then return** DPLL($clauses, symbols - P, model \cup \{P{=}value\}$)
  $P, value \leftarrow$ FIND-UNIT-CLAUSE($clauses, model$)
  if P is non-null **then return** DPLL($clauses, symbols - P, model \cup \{P{=}value\}$)
  $P \leftarrow$ FIRST($symbols$); $rest \leftarrow$ REST($symbols$)
  return DPLL($clauses, rest, model \cup \{P{=}true\}$) **or**
      DPLL($clauses, rest, model \cup \{P{=}false\}$))

Tautology Deletion (Optional)

Tautology: both a proposition and its negation in a clause.

- e.g. (A, B, ¬A)

Clause bound to be true.

- e.g. whether A is true or false.
- Therefore, can be deleted.

Mid-Lecture Exercise

Apply DPLL heuristics to the following sentence:

$$(S_{2,1}),\ (\neg S_{1,1}),\ (\neg S_{1,2}),$$

$$(\neg S_{2,1}, W_{2,2}),\ (\neg S_{1,1}, W_{2,2}),\ (\neg S_{1,2}, W_{2,2}),$$

$$(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$$

Use case splits if model not found by the heuristics.

Symbols: $S_{1,1}$, $S_{1,2}$, $S_{2,1}$, $W_{2,2}$

Solution

Pure symbol heuristic:

$(S_{2,1})$

$(\neg S_{1,1})$

$(\neg S_{1,2})$

$(\neg S_{2,1}, W_{2,2})$

$(\neg S_{1,1}, W_{2,2})$

$(\neg S_{1,2}, W_{2,2})$

$(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$

Solution

Pure symbol heuristic:

- No literal is pure.

Unit clause heuristic:

$(S_{2,1})$

$(\neg S_{1,1})$

$(\neg S_{1,2})$

$(\neg S_{2,1}, W_{2,2})$

$(\neg S_{1,1}, W_{2,2})$

$(\neg S_{1,2}, W_{2,2})$

$(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$

Solution

Pure symbol heuristic:

- No literal is pure.

Unit clause heuristic:

- $S_{2,1}$ is true

$\mathbf{T}$

$(\neg S_{1,1})$

$(\neg S_{1,2})$

$(\mathbf{F}, W_{2,2})$

$(\neg S_{1,1}, W_{2,2})$

$(\neg S_{1,2}, W_{2,2})$

$(\neg W_{2,2}, \mathbf{T}, S_{1,1}, S_{1,2})$

Solution

Pure symbol heuristic:

- No literal is pure.

Unit clause heuristic:

- $S_{2,1}$ is true

Early termination heuristic:

- $(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$ is true

T

$(\neg S_{1,1})$

$(\neg S_{1,2})$

$(\mathbf{F}, W_{2,2})$

$(\neg S_{1,1}, W_{2,2})$

$(\neg S_{1,2}, W_{2,2})$

T

Solution

Pure symbol heuristic:

- No literal is pure.

Unit clause heuristic:

- $S_{2,1}$ is true
- $S_{1,1}$ is false

Early termination heuristic:

- $(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$ is true

T

T

$(\neg S_{1,2})$

$(\mathbf{F}, W_{2,2})$

$(\mathbf{T}, W_{2,2})$

$(\neg S_{1,2}, W_{2,2})$

T

Solution

Pure symbol heuristic:

- No literal is pure.

Unit clause heuristic:

- $S_{2,1}$ is true
- $S_{1,1}$ is false
- $S_{1,2}$ is false

Early termination heuristic:

- $(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$ is true
- $(\neg S_{1,1}, W_{2,2})$ is true

T

T

T

(**F**, $W_{2,2}$)

T

(**T**, $W_{2,2}$)

T

Solution

Pure symbol heuristic:

- No literal is pure.

Unit clause heuristic:

- $S_{2,1}$ is true
- $S_{1,1}$ is false
- $S_{1,2}$ is false

Early termination heuristic:

- $(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$ is true
- $(\neg S_{1,1}, W_{2,2})$ is true
- $(\neg S_{2,1}, W_{2,2})$ is true

T

T

T

(**F**, $W_{2,2}$)

T

T

T

Solution

Pure symbol heuristic:

- No literal is pure.

Unit clause heuristic:

- $S_{2,1}$ is true
- $S_{1,1}$ is false
- $S_{1,2}$ is false

Early termination heuristic:

- $(\neg W_{2,2}, S_{2,1}, S_{1,1}, S_{1,2})$ is true
- $(\neg S_{1,1}, W_{2,2})$ is true
- $(\neg S_{2,1}, W_{2,2})$ is true

Unit clause heuristic:

- $W_{2,2}$ is true

T

T

T

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T

T

T

WalkSAT

The WalkSAT algorithm

Incomplete, local search algorithm

Evaluation function:

- The min-conflict heuristic of minimizing the number of unsatisfied clauses

Algorithm checks for satisfiability by randomly flipping the values of variables

Balance between greediness and randomness

function WALKSAT($clauses, p, max\_flips$) **returns** a satisfying model or *failure*
inputs: *clauses*, a set of clauses in propositional logic
p, the probability of choosing to do a "random walk" move, typically around 0.5
max_flips, number of flips allowed before giving up

```
model ← a random assignment of true/false to the symbols in clauses
for i = 1 to max_flips do
    if model satisfies clauses then return model
    clause ← a randomly selected clause from clauses that is false in model
    with probability p flip the value in model of a randomly selected symbol from clause
    else flip whichever symbol in clause maximizes the number of satisfied clauses
return failure
```

The WalkSAT algorithm

Hard satisfiability problems

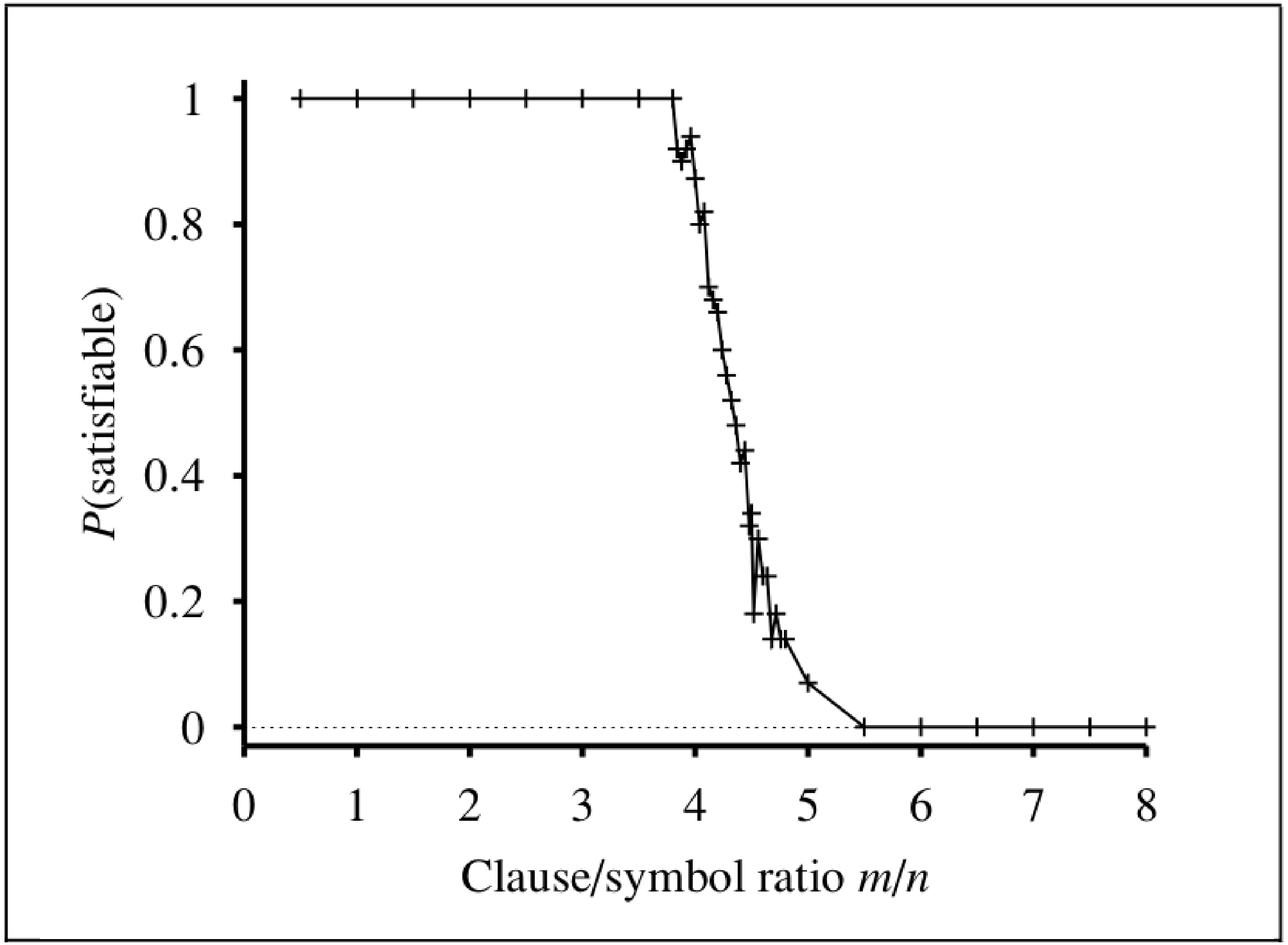
Consider random 3-CNF sentences.

- Example:

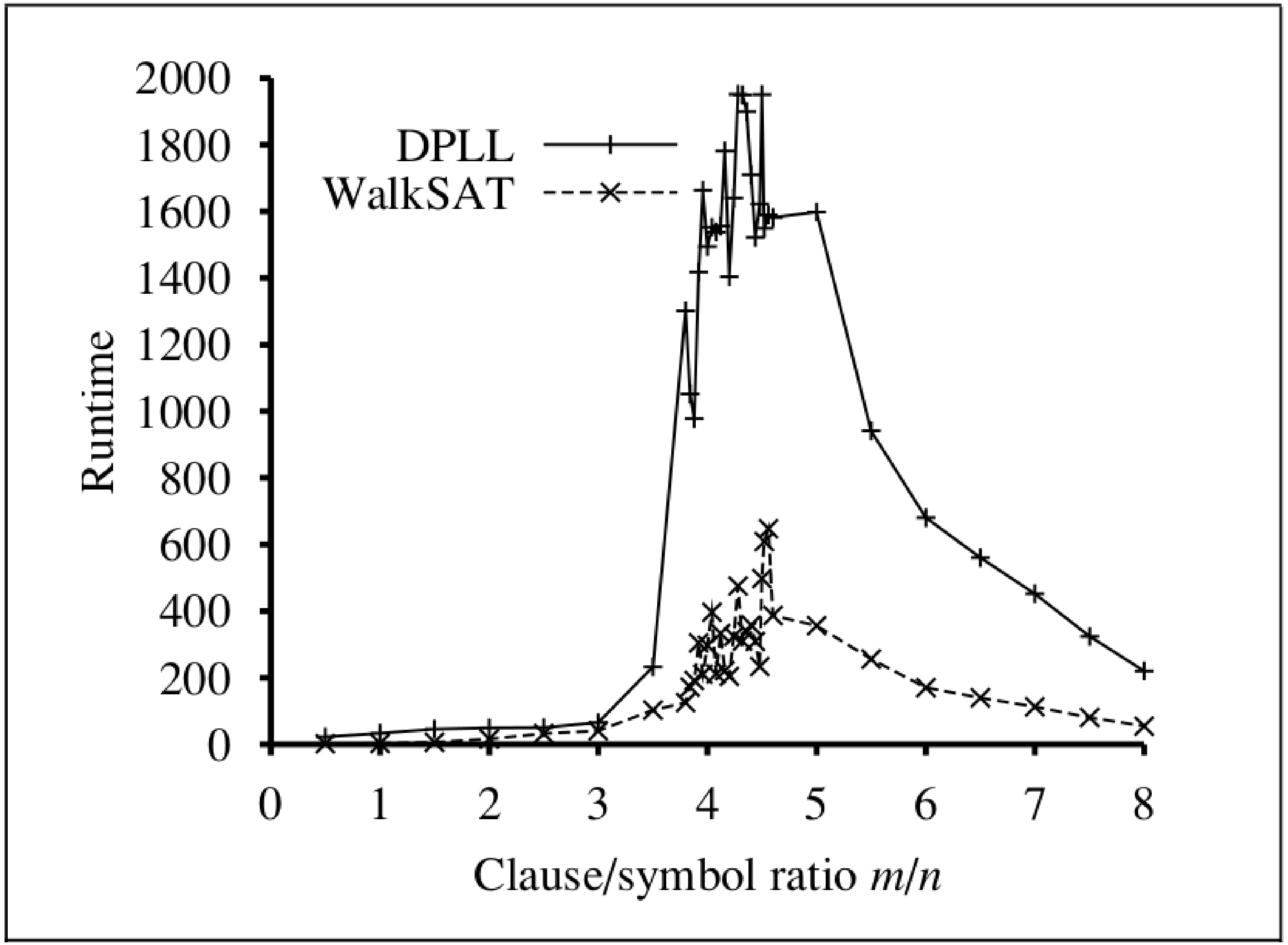
$$(\neg D \lor \neg B \lor C) \land (B \lor \neg A \lor \neg C) \land (\neg C \lor \neg B \lor E) \land (E \lor \neg D \lor B) \land (B \lor E \lor \neg C)$$

- m = number of clauses
- n = number of symbols

Hard problems seem to cluster near m/n = 4.3 (critical point)

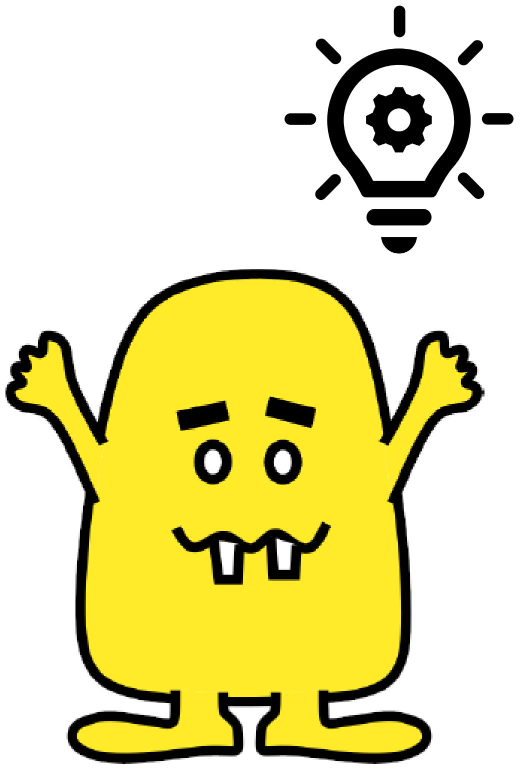


Hard satisfiability problems



Hard satisfiability problems

Median runtime for 100 satisfiable random 3-CNF sentences, n = 50



Inference in the Wumpus World

Inference-based agents in the wumpus world

A wumpus-world agent using propositional logic:

- $\neg P_{1,1}$
- $\neg W_{1,1}$
- $B_{x,y} \Leftrightarrow (P_{x,y+1} \lor P_{x,y-1} \lor P_{x+1,y} \lor P_{x-1,y})$
- $S_{x,y} \Leftrightarrow (W_{x,y+1} \lor W_{x,y-1} \lor W_{x+1,y} \lor W_{x-1,y})$
- $W_{1,1} \lor W_{1,2} \lor \ldots \lor W_{4,4}$
- $\neg W_{1,1} \lor \neg W_{1,2}$
- $\neg W_{1,1} \lor \neg W_{1,3}$
- …

$\Rightarrow$ 64 distinct proposition symbols, 155 sentences

function HYBRID-WUMPUS-AGENT(*percept*) **returns** an *action*
  inputs: *percept*, a list, [*stench*,*breeze*,*glitter*,*bump*,*scream*]
  persistent: *KB*, a knowledge base, initially the atemporal "wumpus physics"
    t, a counter, initially 0, indicating time
    plan, an action sequence, initially empty

  TELL(*KB*, MAKE-PERCEPT-SENTENCE(*percept*, t))
  TELL the *KB* the temporal "physics" sentences for time t
  safe ← $\{[x, y] \ : \ \text{ASK}(KB, OK^t_{x,y}) = \mathit{true}\}$
  if ASK(*KB*, $Glitter^t$) = *true* **then**
    plan ← [*Grab*] + PLAN-ROUTE(*current*, {[1,1]}, *safe*) + [*Climb*]
  if *plan* is empty **then**
    unvisited ← $\{[x, y] \ : \ \text{ASK}(KB, L^{t'}_{x,y}) = \mathit{false}$ for all $t' \leq t\}$
    plan ← PLAN-ROUTE(*current*, *unvisited* ∩ *safe*, *safe*)
  if *plan* is empty and ASK(*KB*, $HaveArrow^t$) = *true* **then**
    possible_wumpus ← $\{[x, y] \ : \ \text{ASK}(KB, \neg W_{x,y}) = \mathit{false}\}$
    plan ← PLAN-SHOT(*current*, *possible_wumpus*, *safe*)
  if *plan* is empty **then** // no choice but to take a risk
    not_unsafe ← $\{[x, y] \ : \ \text{ASK}(KB, \neg OK^t_{x,y}) = \mathit{false}\}$
    plan ← PLAN-ROUTE(*current*, *unvisited* ∩ *not_unsafe*, *safe*)
  if *plan* is empty **then**
    plan ← PLAN-ROUTE(*current*, {[1, 1]}, *safe*) + [*Climb*]
  action ← POP(*plan*)
  TELL(*KB*, MAKE-ACTION-SENTENCE(*action*, t))
  $t \leftarrow t + 1$
  return *action*

The Wumpus Agent

The Wumpus Agent

```
function PLAN-ROUTE(current, goals, allowed) returns an action sequence
   inputs: current, the agent's current position
           goals, a set of squares; try to plan a route to one of them
           allowed, a set of squares that can form part of the route

   problem ← ROUTE-PROBLEM(current, goals, allowed)
   return A*-GRAPH-SEARCH(problem)
```

We need more!

Effect axioms

$$L_{1,1}^0 \wedge FacingEast^0 \wedge Forward^0 \Rightarrow L_{2,1}^1 \wedge \neg L_{1,1}^1$$

We need extra axioms about the world.

Frame problem! – representational & inferential

Frame axioms:

$$Forward^t \Rightarrow (HaveArrow^t \Leftrightarrow HaveArrow^{t+1})$$
$$Forward^t \Rightarrow (WumpusAlive^t \Leftrightarrow WumpusAlive^{t+1})$$

Successor-state axioms:

$$HaveArrow^{t+1} \Leftrightarrow (HaveArrow^t \wedge \neg Shoot^t)$$

Expressiveness limitation of propositional logic

KB contains "physics" sentences for every single square.

For every time t and every location [x,y],

$$L^{t}_{x,y} \wedge FacingRight^{t} \wedge Forward^{t} \Rightarrow L^{t+1}_{x+1,y}$$

Rapid proliferation of clauses!

Why?

Fundamentals behind *SAT/SMT solvers*.

Highly specialised and optimised tools.

- Capable of solving problems with thousands of propositions and millions of constraints, despite NP-completeness and exponential algorithms!

Close relation to CSPs and optimization problems.

Very large array of applications, e.g.:

- Circuit routing and testing, automatic test generation, formal verification, planning & scheduling, configuration/customisation, etc.