Properties of Energy and Power Signals

An energy signal $x(t)$ has zero power

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \underbrace{\int_{-T}^{T} |x(t)|^2 dt}_{\to E_x<\infty} = 0$$

A power signal has infinite energy

$$E_x = \lim_{T\to\infty} (2T) \underbrace{\frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt}_{\to P_x>0}$$

Signal Energy and Power

Let $i(t)$ be the current through a resistor, then the energy dissipated in the resistor is

$$E_R = \lim_{T\to\infty} \int_{-T}^{T} i^2(t) R dt$$

The signal energy for a (possibly complex) signal $x(t)$ is

$$E_x = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

The average of the signal energy over time is the signal power.

Complex Signals, Complex Numbers

$z(t) = x(t) + jy(t), j = \sqrt{-1}$

- $x = \mathrm{Re}\{z\}$
- $y = \mathrm{Im}\{z\}$
- x and y are also called the in-phase and quadrature components of z

Complex number in polar form: $z = re^{j\phi}$

- r: modulus/magnitude of z
- ϕ: angle/phase of z
- $\exp(j\phi)$: $\cos(\phi) + j\sin(\phi)$

$$e^z = e^{x+jy} = e^x e^{jy} = e^x(\cos(y) + j\sin(y))$$

Causal Signals

- **Causal signals** are non-zero only for $t \geq 0$ (starts at $t = 0$, or later).
- **Non-Causal signals** are non-zero for $t < 0$.
- **Anti-Causal signals** are non-zero only for $t \leq 0$ (goes backward in time from $t = 0$).

Periodic Extension

- Periodic signals can be generated by periodic extension of any segment of length one period T_0 (or a multiple of the period).
- A signal defined only over an interval T_0 can use periodic extension to make a periodic signal.

Periodic Signals

A continuous-time signal is periodic if and only if there exists a $T_0 > 0$ such that

$$x(t + T_0) = x(t) \quad \forall t, T_0 \text{ is the period of } x(t).$$

A discrete-time signal is periodic if and only if there exists an integer $N_0 > 0$ such that

$$x[n + N_0] = x[n] \quad \forall n, N_0 \text{ is the period of } x[n].$$

In sum: For every period/sample, these signals are identical.

- The smallest T_0 or N_0 is the **fundamental period** of the periodic signal.

Discrete Amplitude Signals

- Discrete amplitude signals take on only a countable set of values.
- **Example**: Quantized signal (binary, fixed point, floating point).
- A digital signal is a quantized discrete-time signal.
- Requires treatment as a random process (not part of this course).

Even and Odd Symmetry

An even signal is symmetric about the origin:

$$x(t) = x(-t)$$

An odd signal is anti-symmetric about the origin:

$$x(t) = -x(-t)$$

Any signal can be decomposed into even and odd components:

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

Check that:

$$x_e(t) = x_e(-t)$$

$$x_o(t) = -x_o(-t)$$

$$x_e(t) + x_o(t) = x(t)$$

The decomposition depends on the location of the origin. Shifting the signal changes the decomposition.

Combinations of Operations

Scaling, shifting, and reversals can be performed in any order, but care is required.
Example: $x(2(t-1))$:

- Compress by 2, advance by 1 $\leftrightarrow$ Advance by 1, compress by 2.
- $x(2(t-1))$: Shift first (magnitude: 1), then compress (magnitude: 2).
- $x(2t-2)$: Shift first (magnitude: 2), then compress (magnitude: 2) – this is incorrect.

Time Shift

For a continuous-time signal $x(t)$ and a time $t_1 > 0$:

- **Advance:** $x(\tau = t) \rightarrow \tau = t - t_1, x(t - t_1)$
- **Delay:** $x(\tau = t) \rightarrow \tau = t + t_1, x(t + t_1)$

For a discrete-time signal $x[n]$, and an integer $n_1 > 0$:

- **Advance:** $x[\tau = n] \rightarrow \tau = n - n_1, x[n - n_1]$
- **Delay:** $x[\tau = n] \rightarrow \tau = n + n_1, x[n + n_1]$

Time Reversal

$$x(-t) \text{ vs. } x(t), \quad x[-n] \text{ vs. } x[n]$$

Time Scaling, Discrete Time

- **Compress:** $x[nk]$ – extracts every kth sample of $x[n]$ (intermediate samples are lost, sequence is shorter).

- **Expand:** $x[n/m]$ – specifies every mth sample (intermediate samples must be synthesized, sequence is longer).

Time Scaling, Continuous Time

- **Compress:** $x(bt), b > 1$
- **Expand:** $x(bt), 0 < b < 1$

Amplitude Scaling

- Continuous: $ax(t)$
- Discrete: $ax[n]$

Signal Characteristics and Models

- Operations on the time dependence of a signal:
  - Time scaling
  - Time reversal
  - Time shift
  - Combinations
- Signal characteristics:
  - Periodic signals
  - Complex signals
  - Signal sizes
  - Signal Energy and Power

Summary

- A signal is a collection of data.
- Systems act on signals (inputs and outputs).
- Mathematically, they are similar:
  - A signal can be represented by a function.
  - A system can be represented by a function (the domain is the space of input signals).
- Course emphasis: 1D signal, non-random systems.

Discrete-Time Signals

- Samples: $x[n]$, where n is an integer over some (possibly infinite) interval.
- $x[n] = x(nT)$, where n is an integer and T is the sampling period.

Continuous-Time Signals

- Function of a time variable: t, τ, t_1.
- Signal is denoted as $v, v(.)$, or $v(t)$, where t is a dummy variable.

Types of Systems

- **Continuous-time system**: Has continuous-time inputs and outputs.
  - Examples: AM or FM radio, conventional (all mechanical) car.
- **Discrete-time system**: Has discrete-time inputs and outputs.
  - Examples: PC computer game, MATLAB, your mortgage.
- **Hybrid systems**: Combine continuous and discrete systems.
  - Examples: A/D, D/A converters, modern cars with ECUs.

Continuous and Discrete-Time Signals

- CT: Values for all points in time in some (possibly infinite) interval $(f(t))$.
- DT: Values for only discrete points in time $(f[n])$.

Signals can also be functions of space (images) or of space and time (video), and may be continuous or discrete in each dimension.

Systems

A system takes a signal and converts it into another signal.

- Internally, a system may contain many different types of signals.
- The systems perspective considers all these together.

Signals

Any physical or abstract quantity that can be measured and conveys information can be thought of as a signal.

Course Overview

Idea 3: Frequency Domain Lets You Control Linear Systems

Example: To control a car's speed, applying more gas causes the car to speed up.

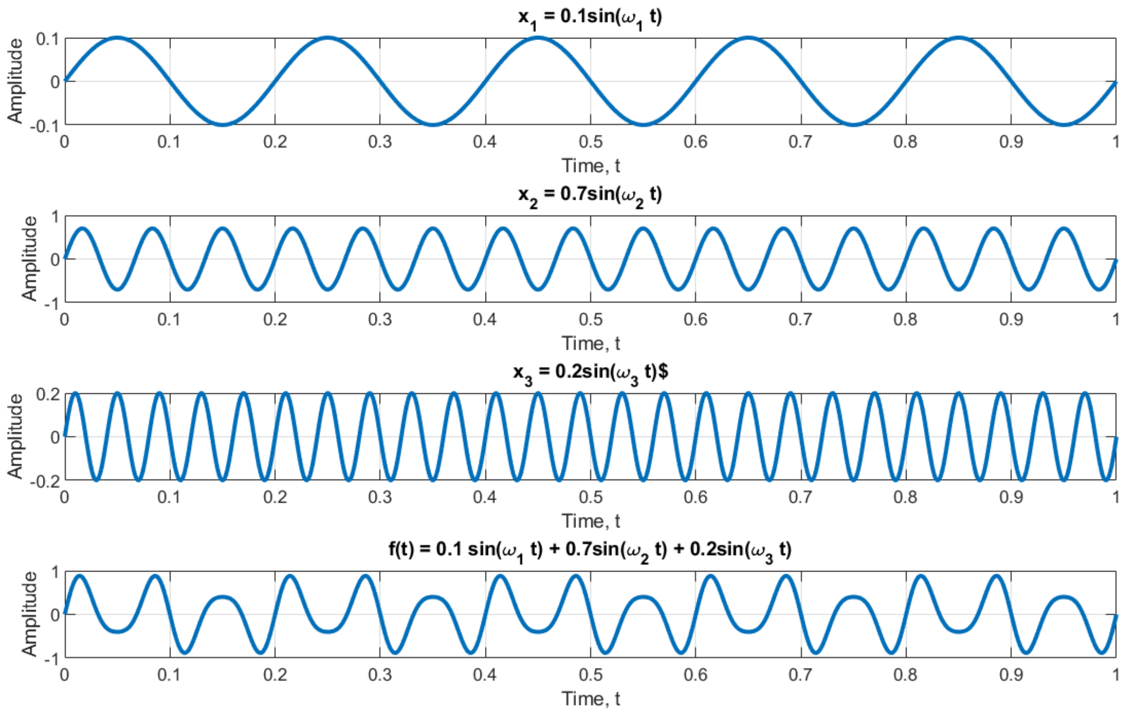
- Frequency domain analysis explains why and provides design insights.

Idea 2: Linear Systems are Easy to Analyze for Sinusoids

- Transforming inputs into the frequency domain simplifies solving.

Idea 1: Frequency Domain Representation of Signals

- Represents signals as combinations of sinusoids.



Why Use Frequency Domain Representation?

- Simpler for many types of signals (e.g., AM radio signal).
- Easier system analysis (Linear Systems).
- Reveals fundamental characteristics of a system.

Demo: Piano Chord

The plot of amplitude vs. frequency is much easier to understand than a waveform of stacked chords.