**USING WIND DATA TO ESTIMATE NEARSHORE DEPOSITIONAL CONDITIONS UNDER SHOALING WAVES**

## Background

In nearshore areas of large lakes, oscillatory currents under waves will be the principal source of energy available to influence the deposition and resuspension of sediment with unidirectional currents and bed slope effects driving net sediment transport (Grant and Madsen 1979, Rowan et al. 1992). Wave-induced shear stress from near-bed oscillatory flows results from the orbital motions of water under waves which can be described in terms of the orbital diameter and velocity of motion. This diminishes with distance from the surface and becomes increasingly elliptical as waves move from deep water where depth < 1/2 the wave length into shallower water where water depth = 1/2 to 1/25 the wave length (Figure 1).

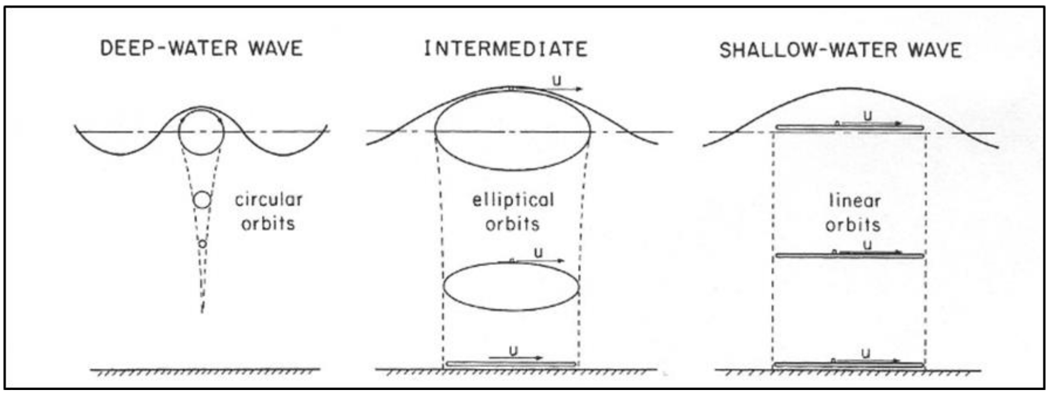


**Figure 1: Illustration of water particle orbital velocities under waves** ***(from: http://www.coastalwiki.org/wiki/Shallow-water_wave_theory)***

This calculator has been developed to assess wave energy at the lakebed in the nearshore zone of large lakes during wave shoaling from deep to shallow water by estimating near-bed orbital diameters and velocities under waves. These can then be used to estimate the corresponding particle size potentially entrained by a given near-bed orbital velocity or compared with the near-bed orbital velocity required for entrainment of a 23 µm sediment particle as a surrogate for the depositional environment boundary or mud deposition boundary depth (Stanley et al. 1983, Rowan et al. 1992). Calculator output will allow the approximate delineation of those zones where wave effects can be considered negligible, and where they could have a significant effect on net sedimentation (i.e. depositional conditions). This information will be useful when considering the potential effects of marine construction, dredging, or if there is interest in determining the rate at which recent, “clean” sediment is burying historically contaminated sediment.

Although for most applications and locations there will be no direct wave measurements to use in these calculations, they can be estimated from long-term wind records which are generally more readily available. Wave hindcasting from wind records is a common coastal engineering practice (CERC 1984) and the use of empirical models in wave hindcasting is widespread and well established with the SMB (Sverdrup–Munk–Bretschneider) method historically being one of the most popular. Although the SMB “significant wave” [1] ($H_s$) method is theoretically inferior to techniques yielding wave spectrum characteristics, it has been widely adopted for engineering design purposes because of its simplicity and suitability for local scale predictions (Kang et al. 1982, CERC 1984, Aisjah et al. 2016, Re et al. 2016).

The development of a fully developed wave field is dependent on wind velocity, duration and fetch (see Figure 2 excerpted from CERC 1984). For most lake coastal regions (i.e. as opposed to ocean coasts) the wave field will be fetch and/or duration limited. As illustrated in Figure 2, for fetches of less than 60 km, fully developed wave conditions will be fetch limited and it will be possible to generate estimates of $H_s$ based solely on wind speed and fetch.

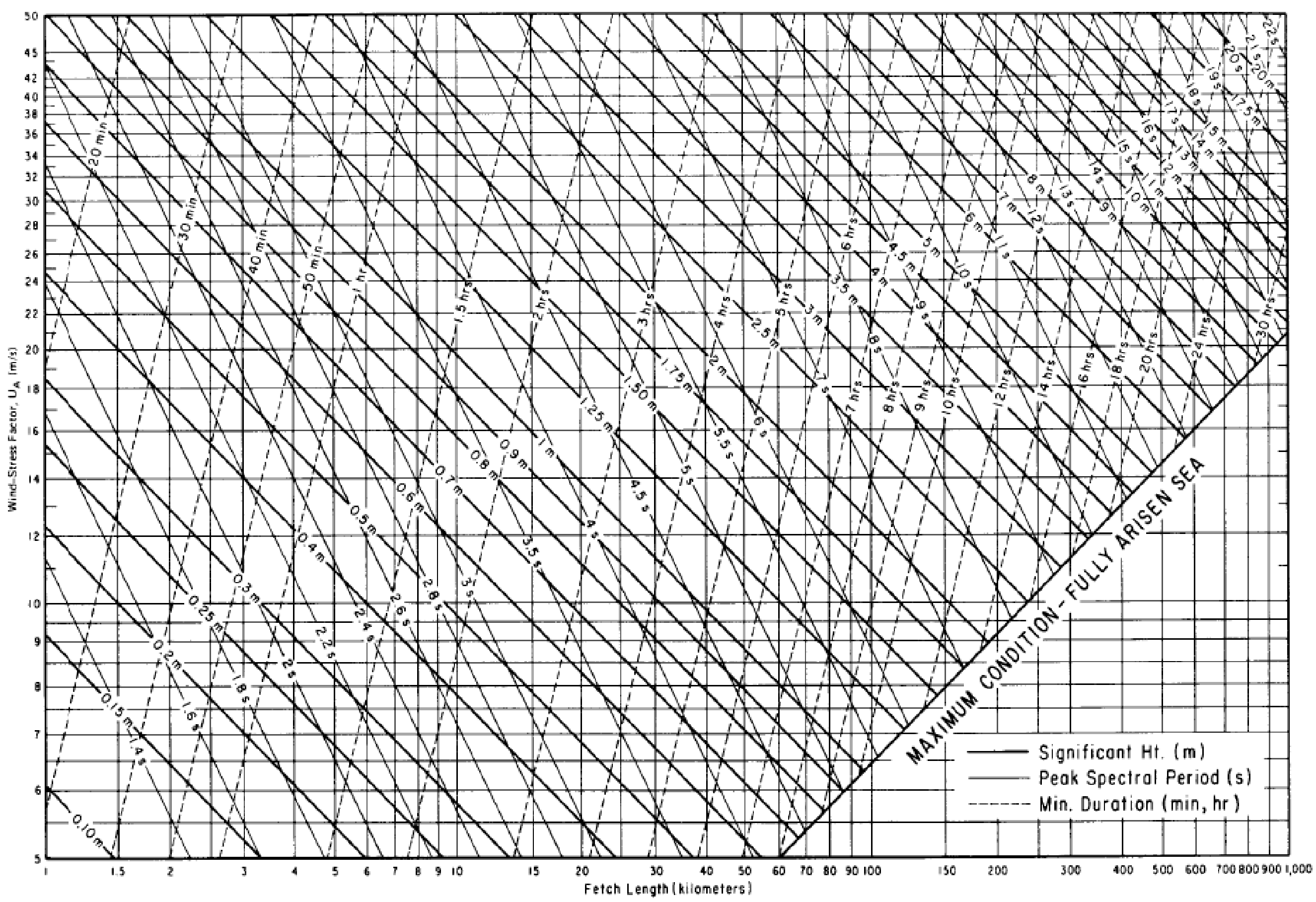


**Figure 2: Nomograms of deep water wave prediction curves as functions of windspeed (m s$^{-1}$), fetch (km), and wind duration (hours) (from CERC 1984)**

[1] Significant wave height, *Hs*, is the mean of the highest third of the waves. It provides a good representation of the average height of the highest waves in a wave group which will generally be of greatest interest when estimating the effects of wave conditions.

**Equations**

The key wave properties needed for these calculations are: (a) wave height *H* - the vertical distance in metres from a wave crest to the adjacent wave trough; (b) wave period *T* - the time in seconds for two successive wave crests to pass a given point; (c) wave length *L* - the horizontal distance in metres between corresponding points (e.g. wave crests) on two successive waves; and (d) mean (still) water depth *h* in metres (CERC 1984).

The following empirical equations from the Shore Protection Manual (CERC 1984) summarize the estimation of fetch-limited deep water (i.e. $h > 0.5$ L) wave height and period using wind speed and fetch as inputs:

(1) $H_s = 0.01616\ U_w F^{0.5}$ and

(2) $T_s = 0.6238\ (U_w F)^{\ 0.33}$

where

$H_s$ = significant wave height (m),
$T$ = significant wave period (s)
$U_w$ = wind speed (m $s^{-1}$) at a height of 10m and
$F$ = fetch (km)

Since these equations require wind speeds at 10m it may be necessary for data to be adjusted to account for the power law wind profile using the following (Hsu et al. 1994):

(3) $u_2 = u_1\ (z_2/z_1)^{\ 0.1}$ (for winds measured over open water), or

(4) $u_2 = u_1\ (z_2/z_1)^{\ 0.14}$ (for winds measured over land)

where

$u_2$ = wind speed at the desired reference height of $z_2$ (10 m), and
$u_1$ = wind speed measured at height $z_1$.

Linear (Airy) wave equations have been widely applied for waveforms in deep water while Stokes wave equations provide a superior description of the water surface for shoaling shallow-water waves by adding additional terms to the basic linear wave equation that yield waveforms with steeper peaks separated by flatter troughs (CERC 1984).

**Wave Length**

From linear wave theory (Weigel 1964; Komar 1976; Komar and Miller 1973), the deep water wave length *L* (in metres) can be expressed as:

(5) $L = \frac{gT^2}{2\pi}$

where

g = acceleration due to gravity (assumed to be 9.8 m $s^{-2}$), and
T = wave period (s)

**Shoaling Wave Length**
The shoaling wave length $L_s$ can then be expressed in terms of water depth as:

(6) $L_s = L \tanh\left(\frac{2\pi}{L_s}\right)$

where

$L_s$ is initialized as $L$, and
tanh = hyperbolic tangent

**Shoaling Wave Height**
The deep water wave height $H$ will be transformed as waves travel into progressively shallower water both as wave length decreases and as waves are refracted (if they approach the shoreline at an angle).

Shoaling wave height can be expressed in terms of depth as (CERC 1984):

(7) $H_s = H\left(\frac{1}{tanh(kh)\left[1+\frac{2kh}{sinh(2kh)}\right]}\right)^{0.5}$

where

$H$ = deep water wave height (m)
$k$ = wave number ( $2\pi/L_s$ )
$h$ = depth (m)
tanh = hyperbolic tangent
sinh = hyperbolic sine

**Refraction Correction Coefficient**
For refracting waves, $H_s$ must also be multiplied by the refraction correction coefficient $k_r$ which can be calculated using Snell's Law by first computing the shoaling wave angle $\alpha_s$:

(8) $\alpha_s = arcsin\left[\left(\frac{L_s}{L_d}\right) sin\,\alpha_d\right]$

where

$L_s$ = shoaling wave length,
$L_d$ = deep water wave length, and
$\alpha_d$ = deep water incident angle

The refraction correction coefficient $k_r$ can then be expressed as:

(9) $$k_r = \left(\frac{\cos\alpha_d}{\cos\alpha_s}\right)^{0.5}$$

where

$\alpha_d$ = deep water incident angle
$\alpha_s$ = shallow water incident angle

**Orbital Velocity**

Although orbital velocities and diameters under waves vary continuously, consideration of the <u>maximum horizontal</u> orbital velocity $U_{max}$ (m $s^{-1}$) near the lake bed can be expressed through simplification of Stokes wave equations as (CERC 1984):

(10) $$U_{max} = \frac{\pi H_s}{T\, sinh\left(\frac{2\pi h}{L_s}\right)}$$

where

$H_s$ = shoaling wave height,
T = wave period,
sinh = hyperbolic sine,
h = depth, and
$L_s$ = shoaling wave length

**Orbital Diameter**

The maximum horizontal orbital diameter $d_o$ (m) near the lake bed can be expressed as:

(11) $$do = \frac{H_s}{sinh\left(\frac{2\pi h}{L_s}\right)}$$

where

$H_s$ = shoaling wave height,
sinh = hyperbolic sine,
h = depth, and
$L_s$ = shoaling wave length

### Entrained Sediment Particle Size

The empirical Komar and Miller (1973) small particle formula (for laminar boundary layer at the bed) provides a simplified means of estimating the threshold velocity for particle motion over various combinations of wave conditions and depths:

(12) $$\rho \frac{U_0^2}{(\rho_s - \rho)gD} = 0.21\ (d_0/D)^{0.5}$$

where

$\rho$ = fluid density,
$\rho_s$ = particle density,
$g$ = 9.8 m $s^{-2}$ ,
$d_o$ = orbital diameter (m), and
$D$ = particle diameter (m)

Assuming $\rho$ = 1,000 kg $m^{-3}$ and $\rho_s$ = 2,650 kg $m^{-3}$ (i.e. quartz sand) this can be expressed as:

(13) $$D = (U_o{}^2/3.3957\ d_o{}^{0.5})^2$$

For particle sizes greater than 0.5 mm, Komar and Miller (1973) developed a large particle formula similar to equation 8 incorporating a turbulent boundary layer:

(14) $$\rho \frac{U_0^2}{(\rho_s - \rho)gD} = 0.46\pi\ (d_0/D)^{0.25}$$

Making the same assumptions as above allows this to be expressed as:

(15) $$D = (U_o^2 / 23.3678\ d_o^{0.25})^{1.3333}$$

## REFERENCES

Aisjah, A.S., S. Arifin and W.L. Danistha 2016. Sverdruv Munk Bretschneider Modification (SMB) for Significant Wave Height Prediction in Java Sea, *British Journal of Applied Science & Technology* 16(2): 1-8.

CERC 1984. *Shore Protection Manual, Volume 1*. U.S. Army Corps of Engineers, , Vicksburg Mississippi.

Grant, W.D. and O.S. Madsen 1979. Combined wave and current interaction with a rough bottom. *Journal of Geophysical Research*, Vol. 84, No. C4, 1797-1808.

Hsu S. A., E.A. Meindl and D.B. Gilhousen 1994, “Determining the Power-Law Wind-Profile Exponent under Near-Neutral Stability Conditions at Sea.” *J. Appl. Meteor.*, Vol. 33, pp.757-765.

Kang, S.W., Y. P. Sheng, and W. Lick 1982. Wave Action and Bottom Shear Stresses in Lake Erie, *Journal of Great Lakes Research* 8(3):482-494.

Komar, P.D. 1976. *Beach Processes and Sedimentation*. Prentice-Hall Inc., Englewood Cliffs, New Jersey. ISBN: 100130725951

Komar, P.D. and M.C. Miller 1973. The threshold of sediment movement under oscillatory water waves. *Journal of Sedimentary Research*, 43 (4): 1101-1110.

Re, C.L., M. Cannarozzo and G.B. Ferreri 2016. Present-day use of an empirical wave prediction method, *Maritime Engineering*, Volume 169 Issue MA1.

Rowan, D. J., J. Kalff and J.B. Rasmussen 1992. Estimating the mud deposition boundary depth in lakes from wave theory. *Canadian Journal of Fisheries and Aquatic Sciences*, 49: 2490-2497.

Stanley, D.J**.,** S. K. Addy and E**.** W. Behrens 1983. The mudline: variability of its position relative to shelfbreak, p. 279-298. In D. S. Stanley and T. Moore [ed.] *The shelfbreak: critical interface on continental margins*, SEPM Volume 33. doi:10.2110/pec.83.06.

Wiegel, R.L. 1964. *Oceanographical Engineering*. Englewood Cliffs, N.J.: Prentice-Hall, 532p.