CALCULATING NEARSHORE DEPOSITIONAL CONDITIONS UNDER SHOALING WAVES

In nearshore areas of large lakes, oscillatory currents under waves will be the principal source of energy available to influence the deposition and resuspension of sediment with unidirectional currents and bed slope effects driving net sediment transport (Grant and Madsen 1979, Rowan et al. 1992). Wave-induced shear stress from near-bed oscillatory flows results from the orbital motions of water under waves which can be described in terms of the orbital diameter and velocity of motion. This diminishes with distance from the surface and becomes increasingly elliptical as waves move from deep water where depth < 1/2 the wave length into shallower water where water depth = 1/2 to 1/25 the wave length (Figure 1).

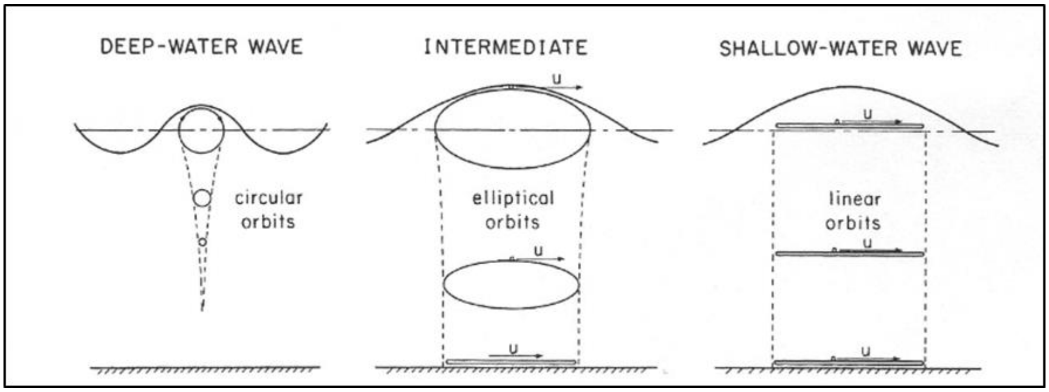

Figure 1: Illustration of water particle orbital velocities under waves ***(from: http://www.coastalwiki.org/wiki/Shallow-water_wave_theory)***

This calculator has been developed to assess wave energy at the lakebed in the nearshore zone of large lakes during wave shoaling from deep to shallow water by estimating near-bed orbital diameters and velocities under waves. These can then be used to estimate the corresponding particle size potentially entrained by a given near-bed orbital velocity or compared with the near-bed orbital velocity required for entrainment of a 23 μm sediment particle as a surrogate for the depositional environment boundary or mud deposition boundary depth (Stanley et al. 1983, Rowan et al. 1992). Calculator output will allow the approximate delineation of those zones where wave effects can be considered negligible, and where they could have a significant effect on net sedimentation. This information will be useful when considering the potential effects of marine construction, dredging, or if there is interest in determining the rate at which recent, "clean" sediment is burying historically contaminated sediment.

EQUATIONS

Linear (Airy) wave equations have been widely applied for waveforms in deep water while Stokes wave equations provide a superior description of the water surface for shoaling shallow-water waves by adding additional terms to the basic linear wave equation that yield waveforms with steeper peaks separated by flatter troughs (CERC 1984).

The key wave properties needed for these calculations are: (a) wave height H - the vertical distance in metres from a wave crest to the adjacent wave trough; (b) wave period T - the time in seconds for two successive wave crests to pass a given point; (c) wave length L - the horizontal distance in metres between corresponding points (e.g. wave crests) on two successive waves; and (d) mean (still) water depth h in metres (CERC 1984).

Wave Length

From linear wave theory (Weigel 1964; Komar 1976; Komar and Miller 1973), the deep water wave length L (in metres) can be expressed as:

$$(1)\ L = \frac{gT^2}{2\pi}$$

where

g = acceleration due to gravity (assumed to be 9.8 m s^{-2}), and
T = wave period (s)

Shoaling Wave Length

The shoaling wave length L_s can then be expressed in terms of water depth as:

$$(2)\ L_s = L \tanh\left(\frac{2\pi}{Ls}\right)$$

where

L_s is initialized as L, and
tanh = hyperbolic tangent

Shoaling Wave Height

The deep water wave height H will be transformed as waves travel into progressively shallower water both as wave length decreases and as waves are refracted (if they approach the shoreline at an angle).

Shoaling wave height can be expressed in terms of depth as (CERC 1984):

$$(3)\ H_s = H\left(\frac{1}{tanh(kh)\left[1+\frac{2kh}{sinh(2kh)}\right]}\right)^{0.5}$$

where

H = deep water wave height (m)
k = wave number ($2\pi/L_s$)
h = depth (m)
tanh = hyperbolic tangent
sinh = hyperbolic sine

Refraction Correction Coefficient
For refracting waves, H_s must also be multiplied by the refraction correction coefficient k_r which can be calculated using Snell's Law by first computing the shoaling wave angle α_s:

$$(4)\ \alpha_s = arcsin\left[\left(\frac{L_s}{L_d}\right) sin\, \alpha_d\right]$$

where

L_s = shoaling wave length,
L_d = deep water wave length, and
α_d = deep water incident angle

The refraction correction coefficient k_r can then be expressed as:

$$(5)\ k_r = \left(\frac{cos\, \alpha_d}{cos\, \alpha_s}\right)^{0.5}$$

where

α_d = deep water incident angle
α_s = shallow water incident angle

Orbital Velocity
Although orbital velocities and diameters under waves vary continuously, consideration of the maximum horizontal orbital velocity U_{max} (m s^{-1}) near the lake bed can be expressed through simplification of Stokes wave equations as (CERC 1984):

$$(6)\ U_{max} = \frac{\pi H_s}{T\, sinh\left(\frac{2\pi h}{L_s}\right)}$$

where

H_s = shoaling wave height,
T = wave period,
sinh = hyperbolic sine,
h = depth, and
L_s = shoaling wave length
Orbital Diameter

The maximum horizontal orbital diameter d_o (m) near the lake bed can be expressed as:

$$(7)\ do = \frac{H_S}{sinh\left(\frac{2\pi h}{L_S}\right)}$$

where

H_s = shoaling wave height,
sinh = hyperbolic sine,
h = depth, and
L_s = shoaling wave length

Entrained Sediment Particle Size

The empirical Komar and Miller (1973) small particle formula (for laminar boundary layer at the bed) provides a simplified means of estimating the threshold velocity for particle motion over various combinations of wave conditions and depths:

$$(8)\ \rho \frac{U_0^2}{(\rho_S - \rho)gD} = 0.21\ (d_0/D)^{0.5}$$

where

ρ = fluid density,
ρ_s = particle density,
g = 9.8 m s^{-2} ,
d_o = orbital diameter (m), and
D = particle diameter (m)

Assuming ρ = 1,000 kg m^{-3} and ρ_s = 2,650 kg m^{-3} (i.e. quartz sand) this can be expressed as:

$$(9)\quad D = (U_o^2 / 3.3957\ d_o^{0.5})^{2}$$

For particle sizes greater than 0.5 mm, Komar and Miller (1973) developed a
large particle formula similar to equation 8 incorporating a turbulent boundary layer:

$$(10)\qquad \rho \frac{U_0^2}{(\rho_S - \rho)gD} = 0.46\pi\ (d_0/D)^{0.25}$$

Making the same assumptions as above allows this to be expressed as:

$$(11)\qquad D = (U_o^2 / 23.3678\ d_o^{0.25})^{1.3333}$$

APPENDIX A: REFERENCES

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