# Lecture 1 — Notation, Functions, and Limits

Henry Watson

Georgetown University

8/14/23

# Today's Agenda

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1. Introductions

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2. Math Camp Logistics and schedule

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3. Variables and measurement

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6. Series, sequences, and limits

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- Lecture 4: Calculus 2 - Integrals and multivariate calculus
- Lecture 5: Probability

# Schedule

| | Monday 14th | Tuesday 15th | Wednesday 16th | Thursday 17th | Friday 18th |
|---|---|---|---|---|---|
| 9:30 - 12:00 | Notation, functions, and limits | Linear algebra | Calculus 1 | Calculus 2 | Probability |
| 12:00 - 12:10 | Break | Break | Break | Break | Break |
| 12:10 - 1:00 | Problem Set | Problem Set | Problem Set | Lunch Break | LaTeX |
| 1:00 - 2:00 | Lunch Break | Lunch Break | Lunch Break | Problem Set | Lunch Break |
| 2:00 - 2:30 | Review PS | Review PS | Review PS | Review PS | Q&A with Prof Klasnja |
| 2:30 - 3:30 | Software Installation | R - Day 1 | R - Day 2 | R - Day 3 | Q&A Continued |

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- The goal of a "causal claim" is rarely achieved in full, but we can make compelling arguments
- Qualitative methods can answer "how" and "why" questions

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- **Constant:** A concept or measure that has a single value for a given set.

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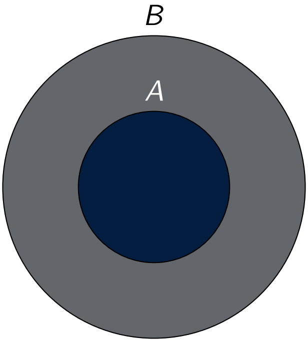
- **Mutually exclusive sets:** the intersection is the empty set.

# $A \subset B$ (Proper Subset)



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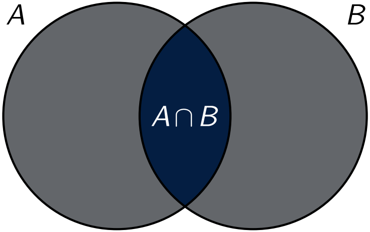

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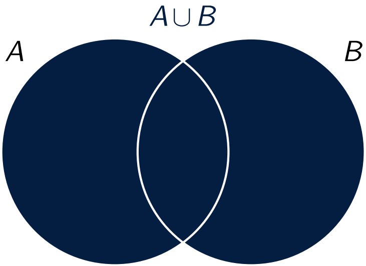
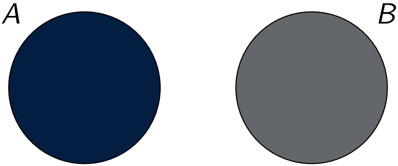


# $A \cap B = \emptyset$ (Mutually Exclusive)



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    - Ratio: meaningful zero (re-scaled thermometer)

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- Visualizaton
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  - Numerical (especially Continuous): histograms, box plots
- Analysis
  - Numerical data can use summary statistics (mean, median, standard deviation)
  - Categorical - Categorical : cross-tabulation
  - Categorical - Continuous : difference in means
  - Continuous - Continuous : scatter plots, regression

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- Natural log: log base $e$
  - $e \approx 2.7183$
  - Useful properties in calculus
  - Taking the natural log of a variable allows us to interpret relationships in terms of percentage changes

# Exponent, logarithms and root rules

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# Exponent, logarithms and root rules

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- $\sqrt{x} = x^{\frac{1}{2}}$
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  for $n > 1$
- $\frac{\sqrt[n]{x}}{\sqrt[n]{z}} = \sqrt[n]{\frac{x}{z}}$
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- To be a function, there must only be one unique value in its range ($y$) for each value in its domain ($x$)

# Graph examples

In which of these graphs can we observe a function?

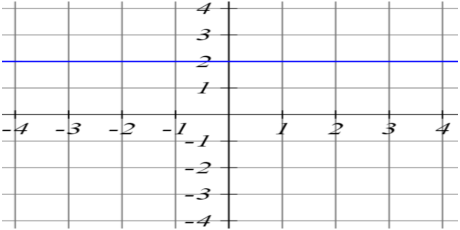



Figure: a)

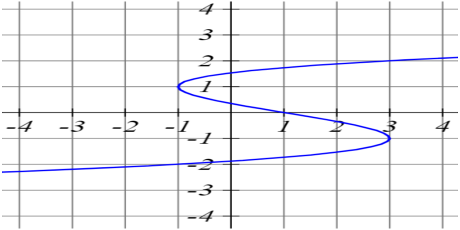
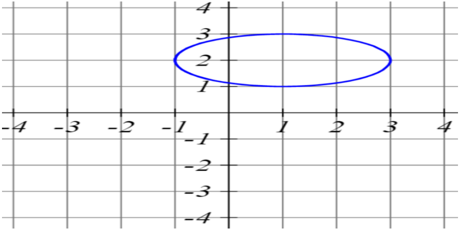
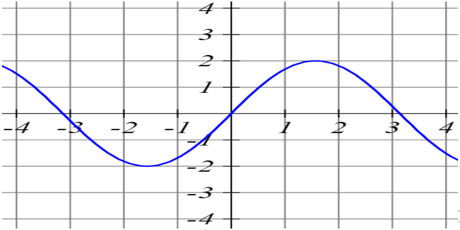



Figure: b)



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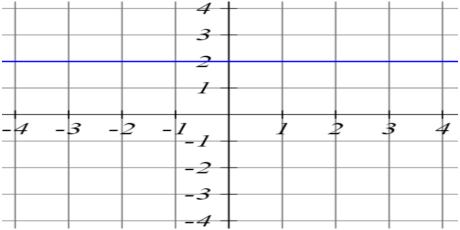


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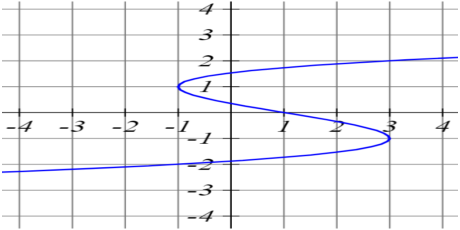


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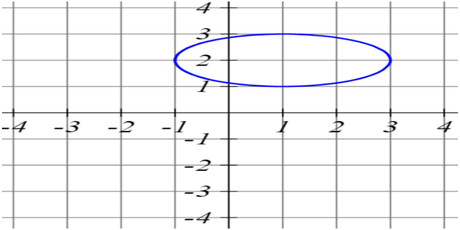


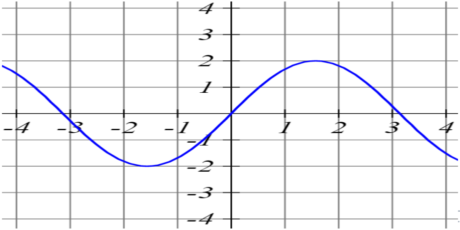

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# Examples of functions of one variable - linear equation

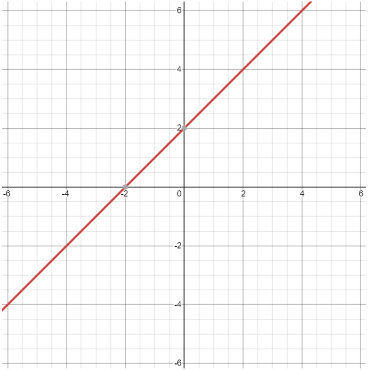


Figure: y=2+x

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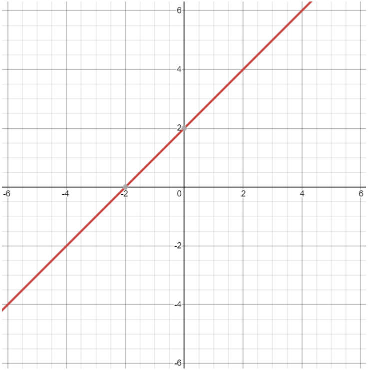


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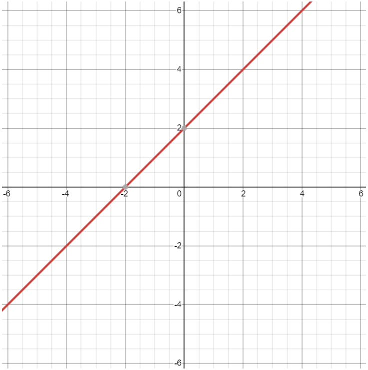


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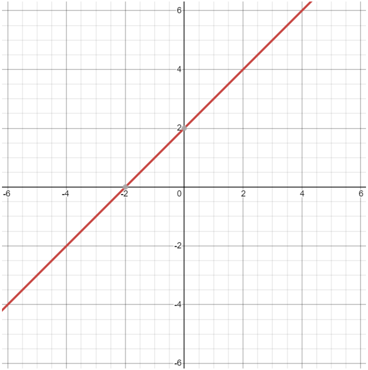


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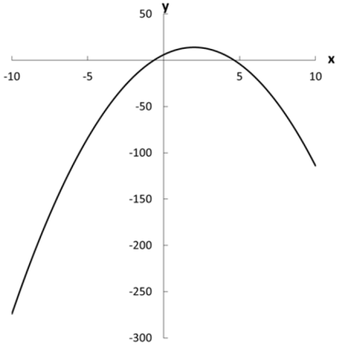


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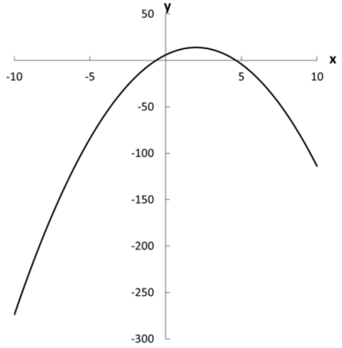


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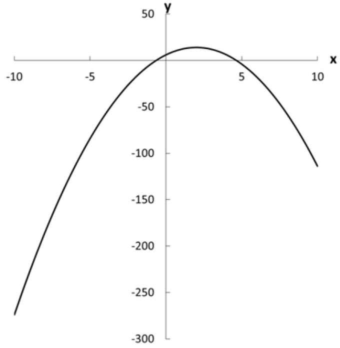


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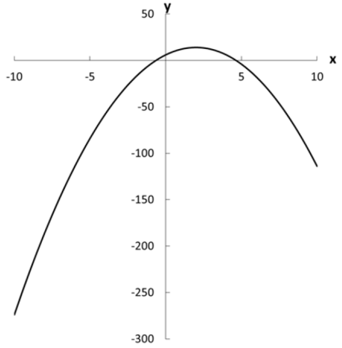


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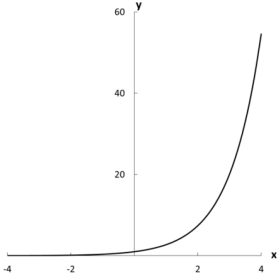
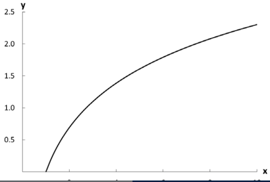
# Exponent, logarithms and roots - Graphs

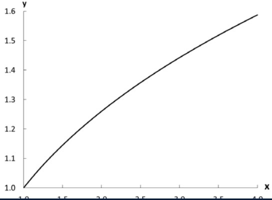
Exponential function




Logarithmic function




Root function



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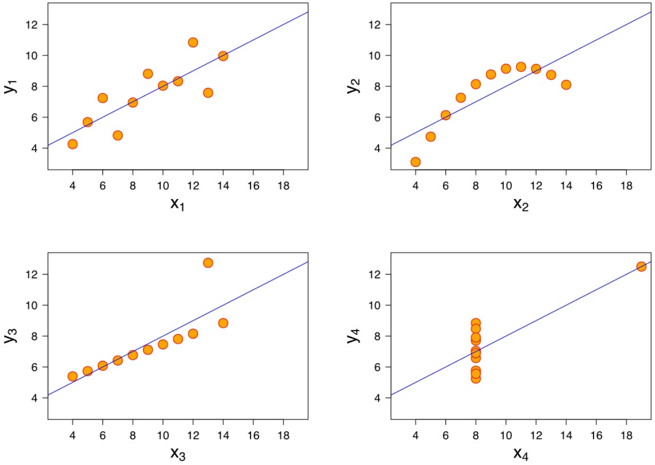
- Statistical methods will *estimate* the most appropriate parameters; "a line of best fit"

# Anscombe's Quartet



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$$\frac{x(x-1)}{2} \tag{5}$$

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$$x=1 \text{ OR } x=7$$

# Why two solutions?

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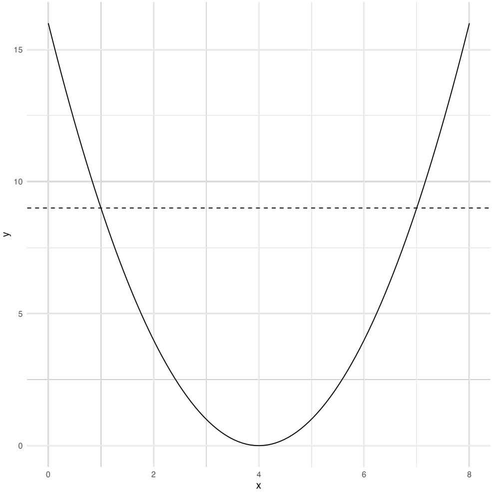

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Either x + 3 = 0 OR x + 4 = 0
$x = -3$ OR $x = -4$

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$$x = \frac{-8 \pm \sqrt{8^2 - (4 * 3 * 13)}}{2 * 3} \tag{17}$$

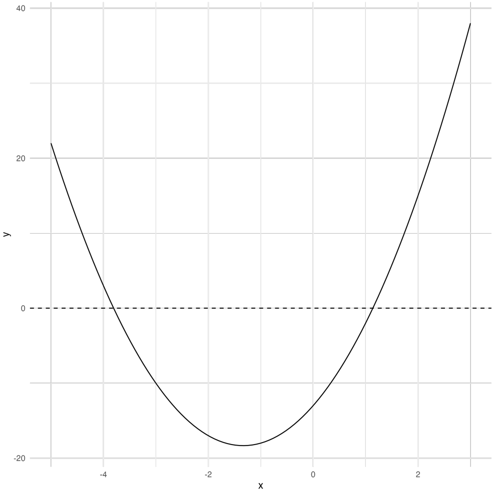
$$x = \frac{-8 + \sqrt{220}}{6} \tag{18}$$

$$x \approx 1.14 \tag{19}$$

$$x = \frac{-8 - \sqrt{220}}{6} \tag{20}$$

$$x \approx -3.81 \tag{21}$$

# Checking our work graphically



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- Piecewise function: the function (relationship between x and y) is different for different parts of the domain

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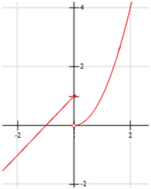
Example: Estimate the value of the following limits: $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$ for the following function:

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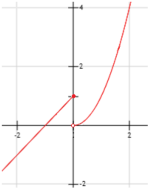
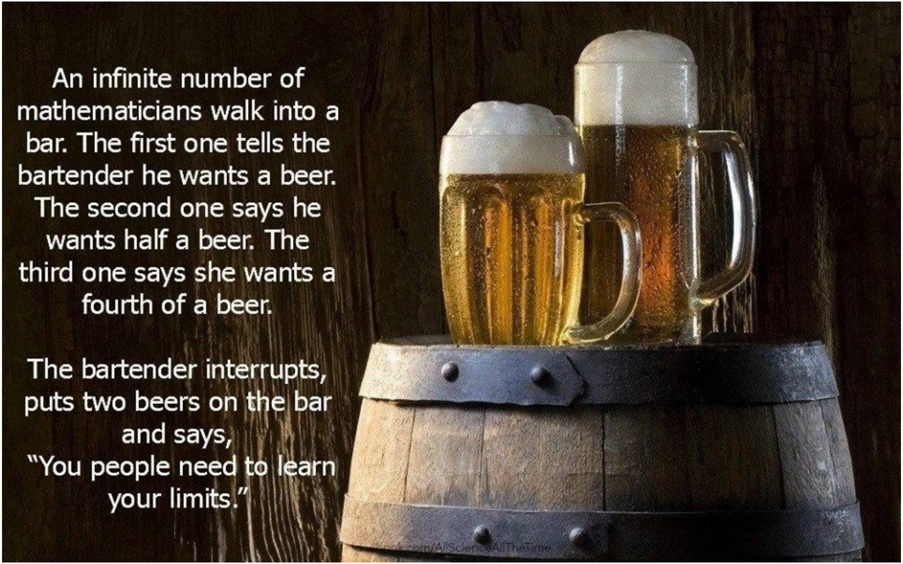


$$f(x) = \begin{cases} x+1 & x \leq 0 \\ x^2 & x > 0 \end{cases}$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x + 1 = 1$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2 = 0$$


An infinite number of mathematicians walk into a bar. The first one tells the bartender he wants a beer. The second one says he wants half a beer. The third one says she wants a fourth of a beer.
The bartender interrupts, puts two beers on the bar and says,
"You people need to learn your limits."

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- Functions are **continuous** if they do not have sudden breaks, and **discontinuous** otherwise

# Math Camp Exercises - Day 1

1. M&S Chapter 1: Exercises 3, 5
2. M&S Chapter 2: Exercises 16, 17, 19, 20, 21, 22, 26, 29
3. M&S Chapter 3: Exercises 2, 3
4. M&S Chapter 4: Exercises 3, 4, 5, 6, 9