Fatigue Crack Growth: number of cycles to failure

Paris law

$$\frac{da}{dN} = C(\Delta K)^m$$

$$K = Y_{(a)}\,\sigma\sqrt{\pi a}$$

$$\Delta K = Y\Delta\sigma\sqrt{\pi a}$$

$$\Delta K = K_{max} - K_{min}$$

$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

$$\frac{da}{dN} = C(Y\,\Delta\sigma\sqrt{\pi a})^m \qquad \frac{da}{C(Y\,\Delta\sigma\sqrt{\pi a})^m} = dN \qquad \frac{1}{C(Y\,\Delta\sigma\sqrt{\pi})^m}\frac{da}{\sqrt{a}^m} = dN$$

$$\frac{1}{C(Y\,\Delta\sigma\sqrt{\pi})^m}\int_{a_o}^{a_f}\frac{da}{\sqrt{a}^m} = \int_0^{N_f} dN$$

$$\int \frac{da}{\sqrt{a}^m} = \int a^{-\frac{m}{2}}da = \frac{1}{-\frac{m}{2}+1}a^{-\frac{m}{2}+1}\Big|_{a_o}^{a_f}$$

$$= \frac{1}{1-\frac{m}{2}}\left[a_f^{1-\frac{m}{2}} - a_o^{1-\frac{m}{2}}\right]$$

$$\mathrm{N} = \frac{1}{C(Y\,\Delta\sigma\sqrt{\pi})^m}\frac{1}{1-\frac{m}{2}}\left[a_f^{\frac{2-m}{2}} - a_o^{\frac{2-m}{2}}\right]$$

$$\mathrm{N} = \frac{2}{(m-2)CY^m(\Delta\sigma)^m\,\pi^{\frac{m}{2}}}\left[\frac{1}{a_o^{\frac{m-2}{2}}} - \frac{1}{a_f^{\frac{m-2}{2}}}\right]$$

For cases where Y depends on crack length, the integrations generally will be performed numerically.