Ec143, Spring 2023

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Review Sheet 2

This review sheet is designed to assist you in your exam preparations. I suggest preparing written answers to each question. You may find it useful to study with your classmates. In the exam you may bring in a single 8.5 x 11 sheet of notes. No calculators or other aides will be permitted. Please bring blue books to the exam. The first midterm exam will occur in class on Thursday, March 16th.

[1] For $s \in \mathbb{S}$, a hypothetical years-of-schooling level, let an individual's potential earnings be given by $\log Y(s) = \alpha_0 + \beta_0 s + U$. Here U captures unobserved heterogeneity in labor market ability and other non-school determinants of earnings. Let the total cost of s years of schooling be given by $(\delta_0^* W + V^*) s + \frac{\kappa}{2} s^2$. Here W is an observable variable which shifts the marginal cost of schooling and V^* is unobserved heterogeneity. You may assume that both U and V^* are conditionally mean zero given W. Agents choose years of completed schooling to maximize expected utility

$$S = \arg\max_{s \in \mathbb{S}} \mathbb{E}\left[\log Y(s) - (\delta_0^* W + V^*) s - \frac{\kappa}{2} s^2 \middle| W, V\right].$$

[a] Show that observed schooling is given by

$$S = \gamma_0 + \delta_0 W + V, \quad \mathbb{E}[V| W] = 0$$

for $\gamma_0 = \beta_0/\kappa$, $\delta_0 = -\delta^*/\kappa$, and $V = -V^*/\kappa$.

[b] Assume that W measures commute time to the closest four year college from a respondent's home during adolescence. What sign do you expect δ_0 to have? Explain.

[c] Assume that $\mathbb{E}[U| W, V] = \mathbb{E}[U| V] = \lambda V$. Restate this assumption in words (HINT: Think about V as a latent variable/attribute). What sign do you expect λ to have? Briefly argue for and against this assumption?

[d] Let $\log Y = \log Y(S)$ denote actual earnings. Show that

$$\mathbb{E}^*[\log Y| S, V] = \alpha_0 + \beta_0 S + \lambda V. \quad (1)$$

[e] What determines variation in S conditional on $V = v$? What is the relationship between this variation and the unobserved determinants of log earnings? Use your answers

to provide an intuitive explanation (i.e., use words) for why the coefficient on schooling in (1) equals β_0.

[f] The random sample $\{(Y_i, S_i, W_i)\}_{i=1}^N$ is available. Suggest a procedure for consistently estimating β_0.

[g] Let

$$\mathbb{E}^* [\log Y | S] = a_0 + b_0 S.$$

From you analysis in part [f] you learn that $\lambda \approx 0$. Guess what value b_0 takes. Justify your answer.

[2] Let Y equal tons of banana's harvested in a given season for a randomly sampled Honduran banana farm. Output is produced using labor and land according to $Y = AL^{\alpha_0} D^{1-\alpha_0}$, where L is the number of employed workers and D is the size of the farm in acres and we assume that $0 < \alpha_0 < 1$. The price of a unit of output is P, while that of a unit of labor is W. These prices may vary across farms (e.g., due to transportation costs, labor market segmentation etc.). We will treat D as a fixed factor; A captures sources of farm-level differences in farm productivity due to unobserved differences in, for example, soil quality and managerial capacity. Farm owners choose the level of employed labor to maximize profits. The observed values of L are therefore solutions to the optimization problem:

$$L = \arg\max_{l} P \cdot A l^{\alpha_0} D^{1-\alpha_0} - W \cdot l.$$

[a] Show that the amount of employed labor is given by

$$L = \left\{ \alpha_0 \frac{P}{W} A \right\}^{\frac{1}{1-\alpha_0}} D. \tag{2}$$

[b] Let $a_0 = \frac{1}{1-\alpha_0} \ln \alpha_0 + \frac{1}{1-\alpha_0} \mathbb{E}[\ln A]$, $b_0 = \frac{1}{1-\alpha_0}$, and $V = \frac{1}{1-\alpha_0} \{\ln A - \mathbb{E}[\ln A]\}$. Show that the log of the labor-land ratio is given by

$$\ln\left(\frac{L}{D}\right) = a_0 + b_0 \ln\left(\frac{P}{W}\right) + V \tag{3}$$

and that, letting $c_0 = \mathbb{E}[\ln A]$ and $U = \ln A - \mathbb{E}[\ln A]$, the log of planation yield (output per unit of land) is given by

$$\ln\left(\frac{Y}{D}\right) = c_0 + \alpha_0 \ln\left(\frac{L}{D}\right) + U. \tag{4}$$

[c] Briefly discuss the content and plausibility of the restriction

$$\mathbb{E}\left[\ln A \middle| \ln\left(P/W\right)\right] = \mathbb{E}\left[\ln A\right]. \tag{5}$$

[d] Using (3), (4) and (5) show that the coefficient on $\ln\left(L/D\right)$ in $\mathbb{E}^*\left[\ln\left(Y/D\right)\middle| \ln\left(L/D\right)\right]$ equals

$$\alpha_0 + \left(1-\alpha_0\right)\frac{\mathbb{V}\left(\ln A\right)}{\mathbb{V}\left(\ln A\right)+\mathbb{V}\left(\ln\left(P/W\right)\right)}.$$

Provide some economic intuition for this result.

[e] Using (3), (4) and (5) show that the coefficient on $\ln\left(L/D\right)$ in $\mathbb{E}^*\left[\ln\left(Y/D\right)\middle| \ln\left(L/D\right), V\right]$ equals α_0. Provide some economic intuition for this result.

[f] Assume that all farms face the same output price (P) and labor cost (W). What value does the coefficient on $\ln\left(L/D\right)$ in $\mathbb{E}^*\left[\ln\left(Y/D\right)\middle| \ln\left(L/D\right)\right]$ equal now? Why?

[g] Using the restriction in part [c], construct an instrumental variables estimate of α_0.

[3] Consider the following model of supply and demand:

$$\begin{aligned}\ln Q_i^D\left(p\right) &= \alpha_1 + \alpha_2\ln\left(p\right) + U_i^D \\ \ln Q_i^S\left(p\right) &= \beta_1 + \beta_2\ln\left(p\right) + U_i^S,\end{aligned}$$

with i indexing a generic random draw from a population of 'markets' ; U_i^D and U_i^S are market-specific demand and supply shocks. We assume that $\left(U_i^S, U_i^D\right) \overset{i.i.d}{\sim} F$ for $i = 1, 2, \ldots, N$. In each market the observed price and quantity pair $\left(P_i, Q_i\right)$ coincides with the solution to market clearing condition

$$Q_i^D\left(P_i\right) = Q_i^S\left(P_i\right) = Q_i.$$

[a] Provide an economic interpretation of the parameters α_2 and β_2. What signs do you expect them to take? Why?

[b] Depict the market equilibrium graphically. Solve for the equilibrium values of $\ln Q_i$ and $\ln P_i$ algebraically. How is the market price and quantity related to the demand and supply shocks, U_i^D and U_i^S? Provide some economic content for your answer. Can you use a figure to illustrate it?

[c] Calculate $\mathbb{E}^*\left[\ln Q\middle| \ln P\right]$. You may assume that $\mathbb{C}\left(U^D, U^S\right) = 0$. Evaluate the coefficient on $\ln\left(P\right)$, does it coincide with an economically interpretable parameter? Assume that $\mathbb{V}\left(U_i^S\right)/\left(\mathbb{V}\left(U_i^S\right)+\mathbb{V}\left(U_i^D\right)\right) \approx 1$, does your answer change? Why?

[4] Consider two large populations of households and firms, both of which may costlessly

migrate across $i = 1, \ldots, N$ different localities. Conditional on choosing to reside in location i, households $t = 1, \ldots, M_i$, choose the amount of unimproved land, l, and composite commodity, x, to consume in order to maximize utility:

$$\max_{l,x} Q_i l^\gamma x^{1-\gamma} \ s.t. \ H_{it} W_i \leq R_i l + x, \tag{6}$$

where Q_i, W_i and R_i respectively denote the "quality of life", wage rate per efficiency unit of labor, and rent per unit of unimproved land, in locality i. The price of the composite commodity is fixed on world markets and normalized to one. A household's endowment of efficiency units of labor is given by H_{it} so that their total budget conditional on residence in locality i is $H_{it} W_i$. Let L_{it} and X_{it} denote the household's utility-maximizing land and composite commodity consumption.

Equilibrium requires that households are indifferent between locations, or that the indirect utility associated with one efficiency unit of labor is constant across communities. Under (6) this condition takes the form

$$V(W, R; Q) = \psi \frac{QW}{R^\gamma} = \bar{u}, \ \psi = \gamma^\gamma (1-\gamma)^{1-\gamma}, \tag{7}$$

where $\bar{u}$ it the common, nationwide, utility level (available to an owner of an efficiency unit of labor).

Firm $f = 1, \ldots, L_i$ in locality i produces X_{if} units of the composite commodity using a constant returns to scale production technology requiring three inputs: (i) efficiency units of labor, n ; (ii) units of improved land, l , and (iii) capital, k. Labor and land prices in a locality are determined in equilibrium, while the cost of capital is constant and equal to σ. The firm's problem is to minimize costs:

$$\min_{n,l,k} W_i n + R_i l + \sigma k, \ s.t. \ A_i n^\alpha l^\beta k^{1-\alpha-\beta} = X_{if}, \tag{8}$$

where A_i is locality-specific total factor productivity. Free entry ensures that profits are zero in equilibrium. The zero profit condition requires that unit costs equal the normalized price of the composite good:

$$C(W, R, A) = \xi \frac{W^\alpha R^\beta}{A} = 1, \ \xi = \left(\frac{1}{\alpha}\right)^\alpha \left(\frac{1}{\beta}\right)^\beta \left(\frac{1}{1-\alpha-\beta}\right)^{1-\alpha-\beta} \sigma^{1-\alpha-\beta} \tag{9}$$

[a] Use (7) and (9) to show that the equilibrium wage and rent in locality i is given by

$$
\begin{aligned}
\ln W_i &= \kappa_W + \frac{\gamma}{\alpha\gamma+\beta}\ln A_i - \frac{\beta}{\alpha\gamma+\beta}\ln Q_i \\
\ln R_i &= \kappa_R + \frac{1}{\alpha\gamma+\beta}\ln A_i + \frac{\alpha}{\alpha\gamma+\beta}\ln Q_i,
\end{aligned} \tag{10}
$$

with

$$
\begin{aligned}
\kappa_W &= \frac{1}{\alpha\gamma+\beta}\left(\beta\ln\bar{u} - \beta\ln\psi - \gamma\ln\xi\right) \\
\kappa_R &= \frac{1}{\alpha\gamma+\beta}\left(\alpha\ln\bar{u} - \alpha\ln\psi - \ln\xi\right).
\end{aligned}
$$

HINT: You do not need to verify the form of the intercepts in (10).

[b] Assume that A_i and Q_i vary independently. Let $\phi = \frac{\gamma^2\mathbb{V}(\ln A_i)}{\gamma^2\mathbb{V}(\ln A_i)+\beta^2\mathbb{V}(\ln Q_i)}$. Show that the coefficient on $\ln W_i$ in the (mean squared error minimizing) linear predictor of $\ln R_i$ onto a constant and $\ln W_i$ equals

$$
b = \frac{1}{\gamma}\phi - \frac{\alpha}{\beta}\left(1-\phi\right).
$$

Interpret this expression. Under what conditions is b positive? Negative? Why?

[c] Assume that the $\gamma = 1/3$, $\alpha = 3/5$, and $\beta = 1/5$. A macro economist, with considerable central banking experience, asserts that the "quality-of-life" differences across cities are "overblown". An econometrician, with almost no "real world" experience, asserts that "firms are equally unproductive in all places". An urban economist claims that "actually both are equally important".

Using a random sample of US cities you compute an estimate of b equal to -3. Who is correct? The macro-economist, econometrician or urban economist?

[d] It turns out that you made a Python coding error and that the correct estimate of b is zero. Who do you believe to be correct now?

[e] Use (10) to solve for $\ln A_i$ and $\ln Q_i$ (again you may ignore constants). Use the parameter values given in (c) to simplify your expressions.

[f] You observe mean January temperature for each city in your sample. Can you compute consistent estimates of the semi-elasticity of total factor productivity and quality of life with respect to temperature? Describe your procedure.