Systems of Supply and Demand

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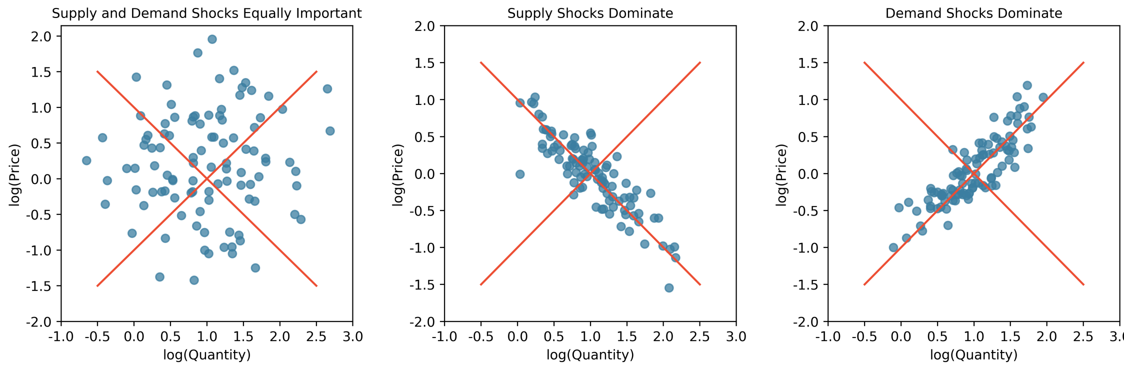
The graphical supply and demand model of price and quantity determination, introduced by Fleeming Jenkins in 1870, is familiar to all students of economics (Brownlie & Prichard, 1963). This model begins with a *demand schedule*, $Q^D(p; W_1)$, which gives the amount of a commodity consumers would wish to purchase at the given price, p. In general, the demand schedule slopes downward: consumers will wish to purchase less when prices are higher. Here W_1 equals a $J_1 \times 1$ vector of observed *demand shifters*: variables which capture observable heterogeneity in the demand schedule across markets. In the market for soda, for example, a component of W_1 might be a heat wave indicator, since demand for soda is higher when it is hot outside. During heat waves the demand schedule shifts rightward, since at any given price, p, consumers will demand great quantities of soda when it is hot out.

The model similarly posits the existence of a *supply schedule* $Q^S(p; W_2)$. This function gives how much of the commodity producers are willing to supply at sales price, p. In a competitive market, the supply curve can be derived from firms' cost functions (specifically by an aggregation of their marginal cost curves). In general, since marginal costs are increasing in production, the supply schedule will slope upwards. Here W_2 is a vector of *supply shifters*: variables which capture observable heterogeneity in the supply schedule across markets. In the market for fish, for example, a component of W_2 might be weather conditions at sea. It is more costly to bring fresh fish to the market when fishing is dangerous due to stormy conditions (Graddy, 1995). Sometimes, reflecting the close connection between the supply schedule and the costs of production, we will call the elements of W_2 *cost shifters*. Note that, depending on the market and data available, W_1 and W_2 may have elements in common.

In this lecture we will study whether the form of the demand and supply schedules can be consistently recovered from a dataset of market-clearing prices and quantities. Let $i = 1, \ldots, N$ index a random sample of markets. For example markets for fresh oranges in January across different cities in the United States. In each market we observe P_i, the average price of an orange in city i during the month of January, and Q_i, the total number

Figure 1: Supply, demand and market equilibrium



SOURCE: Author's calculations.
NOTES: Market clearing prices and quantities under various demand and supply shock configurations. In all three panels the observed demand and supply shocks, U_i^D and U_i^S, vary independently of one another. In Panel A $\mathbb{V}\left(U_i^D\right) = \mathbb{V}\left(U_i^S\right)$, in Panel B $\mathbb{V}\left(U_i^D\right) \ll \mathbb{V}\left(U_i^S\right)$, while in Panel C $\mathbb{V}\left(U_i^S\right) \ll \mathbb{V}\left(U_i^D\right)$. The red lines depict the average supply and demand schedules.

of oranges sold during the month. To clarify the issues involved, imagine we have an infinite amount of market-clearing price and quantity pairs. In such a setting is it possible to recover the forms of the demand and supply schedules? Our answer will be, generally, no, it is not possible.

Equilibrium in a simple log-linear model of supply and demand

Panel A of Figure 1 depicts a simple supply and demand system. The market-clearing price and quantity pair, denoted by $(P_i, Q_i)'$, is defined by the intersection of the two schedules. This intersection defines a price at which the amount of the commodity desired by consumers exactly coincides with the amount suppliers are willing to sell. Mathematically this pair is defined as the solution to the pair of equations

$$Q_i = Q^D\left(P_i, W_{1i}\right) = Q^S\left(P_i, W_{2i}\right), \tag{1}$$

for $i = 1, \ldots, N$.

For the majority of this lecture we will specialize to the log-linear case where

$$\begin{aligned}\ln Q^D\left(p, w_2\right) &= \alpha^D + \epsilon^D \ln p + w_2'\gamma^D + U^D \\ \ln Q^S\left(p, w_1\right) &= \alpha^S + \epsilon^S \ln p + w_1'\gamma^S + U^S.\end{aligned}$$

This specification highlights at additional feature of our setup that was left implicit above.

The demand and supply schedules are *random functions*. This means that we allow for the quantity demanded at price p to vary across markets homogenous in $W_1 = w_1$. Similarly we allow for heterogeneity in supply schedules across markets homogenous in $W_2 = w_2$. In the above system this variation is generated by the demand and supply shocks, U_i^D and U_i^S. These shocks induce parallel shifts in the (log-) demand and supply schedules across otherwise identical markets. This feature of our setup is important for real world application; it is common that two sampled units – in this case markets – might vary in their "outcomes" – in this case $(P_i, Q_i)'$ – even when identical otherwise (here having the same values of W_{1i} and W_{2i}).

Observe that the coefficients, ϵ^D and ϵ^S, equal the price elasticities of demand and supply respectively. Below we will show that having knowledge of these two elasticities facilitates tax policy analysis.

With log-linear supply and demand, the market-clearing condition (1) generates the system of linear equations

$$\begin{aligned} \ln Q_i &= \alpha^S + \epsilon^S \ln P_i + U_i^S \\ \ln Q_i &= \alpha^D + \epsilon^D \ln P_i + U_i^D. \end{aligned}$$

Here, for expositional purposes, I further specialize to the case with no observed demand and supply shifters (i.e., $\dim(W_{1i}) = \dim(W_{2i}) = 0$).

In matrix form we have

$$\begin{bmatrix} 1 & -\epsilon^S \\ 1 & -\epsilon^D \end{bmatrix} \begin{bmatrix} \ln Q_i \\ \ln P_i \end{bmatrix} = \begin{bmatrix} \alpha^S + U_i^S \\ \alpha^D + U_i^D \end{bmatrix}.$$

Solving for $(\ln P_i, \ln Q_i)$ yields

$$\begin{aligned} \begin{bmatrix} \ln Q_i \\ \ln P_i \end{bmatrix} &= \begin{bmatrix} 1 & -\epsilon^S \\ 1 & -\epsilon^D \end{bmatrix}^{-1} \begin{bmatrix} \alpha^S + U_i^S \\ \alpha^D + U_i^D \end{bmatrix} \\ &= \frac{1}{\epsilon^S - \epsilon^D} \begin{bmatrix} -\epsilon^D & \epsilon^S \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \alpha^S + U_i^S \\ \alpha^D + U_i^D \end{bmatrix} \\ &= \frac{1}{\epsilon^S - \epsilon^D} \begin{bmatrix} \epsilon^S \left(\alpha^D + U_i^D\right) - \epsilon^D \left(\alpha^S + U_i^S\right) \\ \alpha^D + U_i^D - \left(\alpha^S + U_i^S\right) \end{bmatrix}. \end{aligned}$$

The equilibrium log-quantity sold is

$$\ln Q_i = \frac{\epsilon^S}{\epsilon^S - \epsilon^D}\left(\alpha^D + U_i^D\right) - \frac{\epsilon^D}{\epsilon^S - \epsilon^D}\left(\alpha^S + U_i^S\right), \tag{2}$$

while the equilibrium log-price is

$$\ln P_i = \frac{1}{\epsilon^S - \epsilon^D}\left(\alpha^D + U_i^D\right) - \frac{1}{\epsilon^S - \epsilon^D}\left(\alpha^S + U_i^S\right). \tag{3}$$

The distribution of the supply and demand shocks $\left(U_i^S, U_i^D\right)$, as well as the form of the supply and demand schedules, determines the cross-market distribution of $(\ln P_i, \ln Q_i)$. The market clearing price and quantity, P_i and Q_i, are *simultaneously* determined.

Panel A of Figure 1 plots a random sample of market clearing prices when U_i^S and U_i^D vary independently. The deterministic components of the underlying demand and supply schedules, specifically the coefficient vectors $\theta^D = \left(\alpha^D, \epsilon^D\right)'$ and $\theta^S = \left(\alpha^S, \epsilon^S\right)'$, are the same across markets.

In looking at Panel A of Figure 1 we would be hard pressed to deduce the form of the underlying demand and supply schedules.

The ordinary least squares fit

Given a dataset of market-clearing prices and quantities $\{(\ln P_i, \ln Q_i)\}_{i=1}^N$ we might be tempted to try to equate the price elasticity of demand with the coefficient on $\ln P_i$ in the ordinary least squares (OLS) fit of $\ln Q_i$ onto a constant and $\ln P_i$. The OLS fit provides a consistent estimate of the linear regression of $\ln Q_i$ onto a constant and $\ln P_i$:

$$\mathbb{E}^*\left[\ln Q_i \middle| \ln P_i\right] = a_0 + b_0 \ln P_i$$

with

$$b_0 = \frac{\mathbb{C}\left(\ln Q_i, \ln P_i\right)}{\mathbb{V}\left(\ln P_i\right)},\ a_0 = \mathbb{E}\left[\ln Q_i\right] - b_0 \mathbb{E}\left[\ln P_i\right].$$

Assuming that $\mathbb{C}\left(U_i^D, U_i^S\right) = 0$ (is this realistics?), equations (2) and (3) give

$$\mathbb{C}\left(\ln Q_i, \ln P_i\right) = \frac{1}{\left(\epsilon^S - \epsilon^D\right)^2}\left[\epsilon^S \mathbb{V}\left(U_i^D\right) + \epsilon^D \mathbb{V}\left(U_i^S\right)\right]$$

$$\mathbb{V}\left(\ln P_i\right) = \frac{1}{\left(\epsilon^S - \epsilon^D\right)^2}\left[\mathbb{V}\left(U_i^S\right) + \mathbb{V}\left(U_i^D\right)\right],$$

such that the coefficient on $\ln P_i$ in the linear regression of $\ln Q_i$ onto a constant and $\ln P_i$ is therefore

$$b_0 = \epsilon^S \frac{\mathbb{V}\left(U_i^D\right)}{\mathbb{V}\left(U_i^S\right) + \mathbb{V}\left(U_i^D\right)} + \epsilon^D \frac{\mathbb{V}\left(U_i^S\right)}{\mathbb{V}\left(U_i^S\right) + \mathbb{V}\left(U_i^D\right)}, \tag{4}$$

or a weighted average of the price elasticities of supply and demand (see Working (1927) for an early analysis along these lines).

Panel B of Figure 1 plots a sample of market-clearing price and quantity pairs $(\ln P_i, \ln Q_i)$ when $\mathbb{V}\left(U_i^D\right) \ll \mathbb{V}\left(U_i^S\right)$. Panel C depicts the case where $\mathbb{V}\left(U_i^S\right) \ll \mathbb{V}\left(U_i^D\right)$. Problem Set 2 asks you to think about when these different cases might arise in real world settings. The main take-away so far is that an OLS fit of $\ln Q_i$ onto a constant and $\ln P_i$ does not, in general, recover an economic quantity of interest.

The market for flash memory

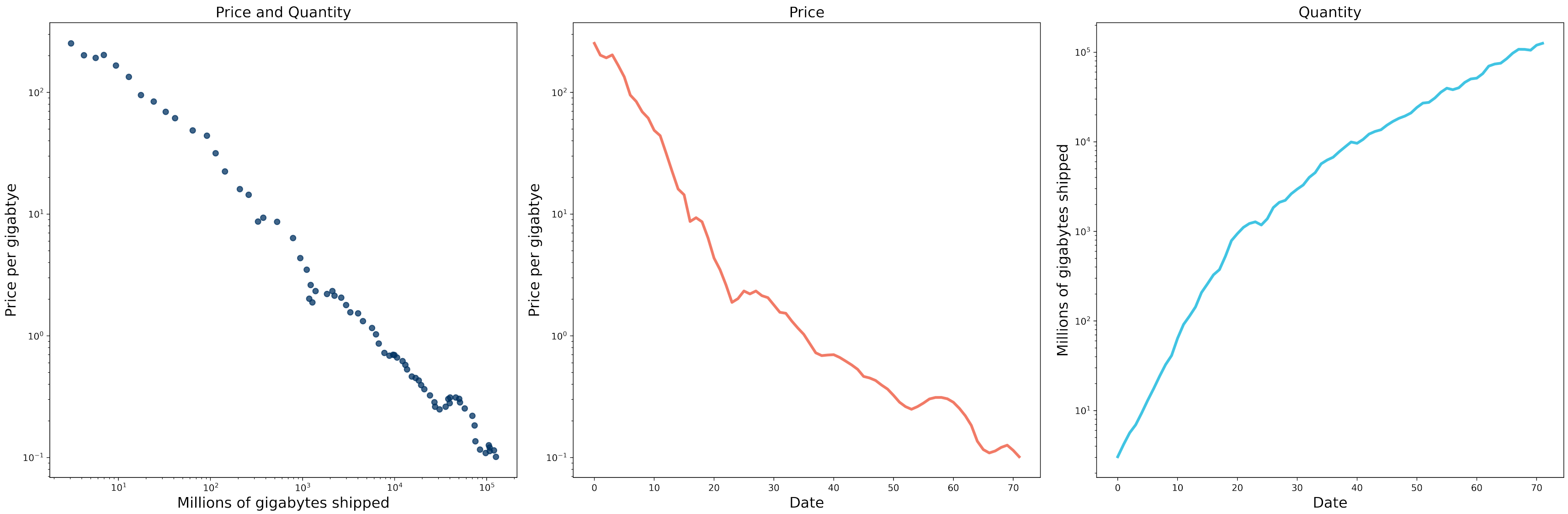
Figure 2 plots monthly price and sales data for gigabytes of flash memory from 2003 to 2020. Flash memory is used in a variety of consumer products (see Yinug (2008) for basic information about the flash memory market). Panel A of the figure plots monthly log price and quantity pairs. A clear downward relationship is visible in the figure. It is tempting to conclude that the plotted data traces out a demand schedule (as in Panel B of Figure 1 above). This is certainly possible, but it need not be.

Semiconductor production is characterized by rapid technical change, allowing for production at lower unit costs over time. If we think about each month as a separate flash memory market, then – over time – the intercept of the supply schedule, α^S, has been shifting upwards so that at any given price semiconductor manufacturers have been willing – due to declining unit costs – to supply more units of flash memory. This corresponds to a rightward shift in the supply schedule over time.

At the same time the demand for flash memory has increased as well. More and more consumer products incorporate flash memory each year. We can think of this process as inducing an upward shift in the demand schedule intercept, α^D. Over time the demand schedule for flash memory has also been shifting rightwards.

The sequence of market clearing price and quantity pairs shown in Figure 2 reflects the realization of these secular supply and demand shifts. Panel A 3 plots a hypothetical sequence of supply and demand schedules consistent with declining unit costs and growing demand over time. Panel B of the figure shows that this can easily induce a scatter of price and quantity pairs that "slope" downwards. However the slope of the corresponding regression fit does not recover the price elasticity of demand. We should be cautious about interpreting even very strong patterns in price and quantity relationships.

Figure 2: The Market for NAND Flash Memory, 2003 to 2020



Source: Gartner Market Statistics

Excise taxes

There are many reasons why a policy-maker might require knowledge of the form of the supply and demand schedules in a given market. A canonical example involves predicting the market effects of taxation. Excise taxes are taxes imposed on each unit of a good sold. Cilluffo (2021) provides an overview of Federal excise taxes. Examples include the gasoline excise tax, per package taxes on cigarettes and alcohol, or Berkeley's one cent per one ounce "soda tax". The motivations for such taxes vary. A primary goal may be raising revenue to fund government expenditures. In other cases the goal may be to correct a perceived negative externality. For example, advocates of higher excise taxes on gasoline emphasize their effects of carbon emissions.

Consider the case where an excise tax of τ_0 is paid by the producer for each unit sold. In this case market equilibrium is defined by the pair of conditions

$$Q_0 = Q^D\left(P_0, W_1\right) = Q^S\left(P_0 - \tau_0, W_2\right).$$

Now consider the effects on an increase in the excise tax. When the excise tax increases from τ_0 to $\tau_1 \overset{def}{\equiv} \tau_0 + \mathrm{d}\tau$, the market-clearing price and quantity, respectively $P_1 \overset{def}{\equiv} P_0 + \mathrm{d}P$ and $Q_1 \overset{def}{\equiv} Q_0 + \mathrm{d}Q$, will instead satisfy

$$Q_1 = Q^D\left(P_0 + \mathrm{d}P, W_1\right) = Q^S\left(P_0 + \mathrm{d}P - \tau_0 - \mathrm{d}\tau, W_2\right).$$

For $\mathrm{d}\tau \to 0$ and $\mathrm{d}P \to 0$ we have, by Taylor's Theorem

$$Q_1 = Q^D\left(P_0, W_1\right) + \frac{\partial Q^D\left(P_0, W_1\right)}{\partial P}\mathrm{d}P \tag{5}$$

and

$$Q_1 = Q^S\left(P - \tau, W_2\right) + \frac{\partial Q^S\left(P_0, W_1\right)}{\partial P}\mathrm{d}P - \frac{\partial Q^S\left(P_0, W_1\right)}{\partial P}\mathrm{d}\tau. \tag{6}$$

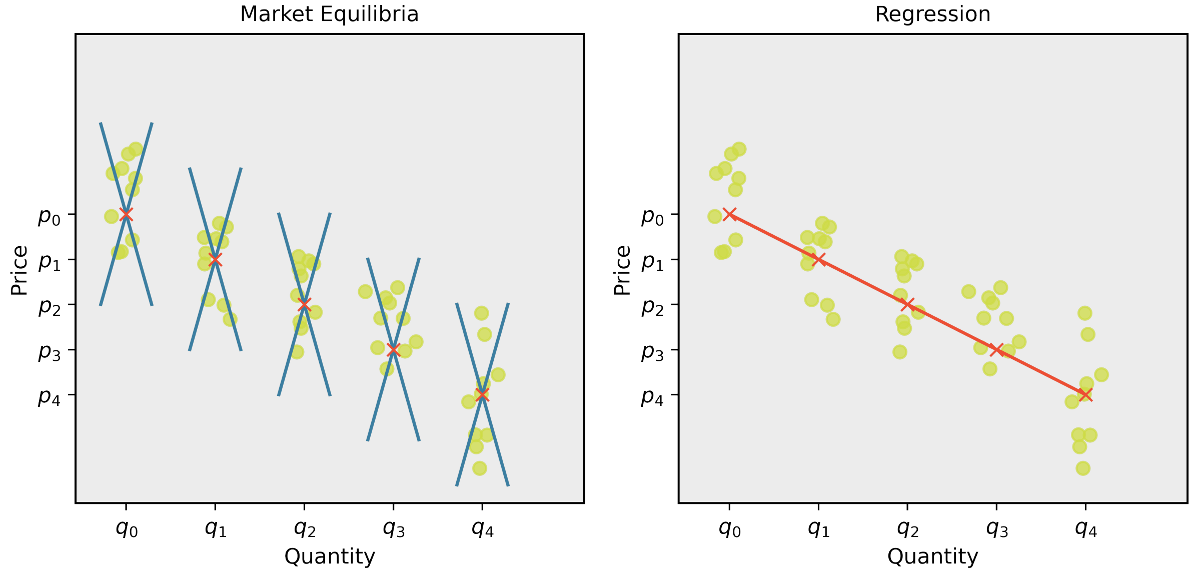
Tax incidence

Solving (5) and (6) for $\frac{\mathrm{d}P}{\mathrm{d}\tau}$ yields

$$\frac{\mathrm{d}P}{\mathrm{d}\tau} = \frac{\frac{\partial Q^S(P_0,W_1)}{\partial P}}{\frac{\partial Q^S(P_0,W_1)}{\partial P} - \frac{\partial Q^D(P_0,W_1)}{\partial P}}.$$

Recalling that the price elasticity of demand is $\epsilon^D = \frac{\partial Q^D(P_0,W_1)}{\partial P}\frac{P_0}{Q_0}$ and the price elasticity of

Figure 3: Supply and demand schedules vs. the $\mathbb{E}^*[\ln P|\ln Q]$ linear regression function



SOURCE: Author's calculations.
NOTES: Panel A depicts a sequence of average supply and demand schedules. Panel B shows the induced pattern of market clearing prices and quantities.

supply is $\epsilon^S = \frac{\partial Q^S(P_0,W_1)}{\partial P}\frac{P_0}{Q_0}$, we get the so called *incidence formula*

$$\frac{\mathrm{d}P}{\mathrm{d}\tau} = \frac{\epsilon^S}{\epsilon^S - \epsilon^D}. \tag{7}$$

Since $\epsilon^D \leq 0$ ("demand curves slope downwards") and $\epsilon^S \geq 0$ ("supply curves slope upwards") we have that $0 \leq \frac{\epsilon^S}{\epsilon^S-\epsilon^D} \leq 1$.[1]

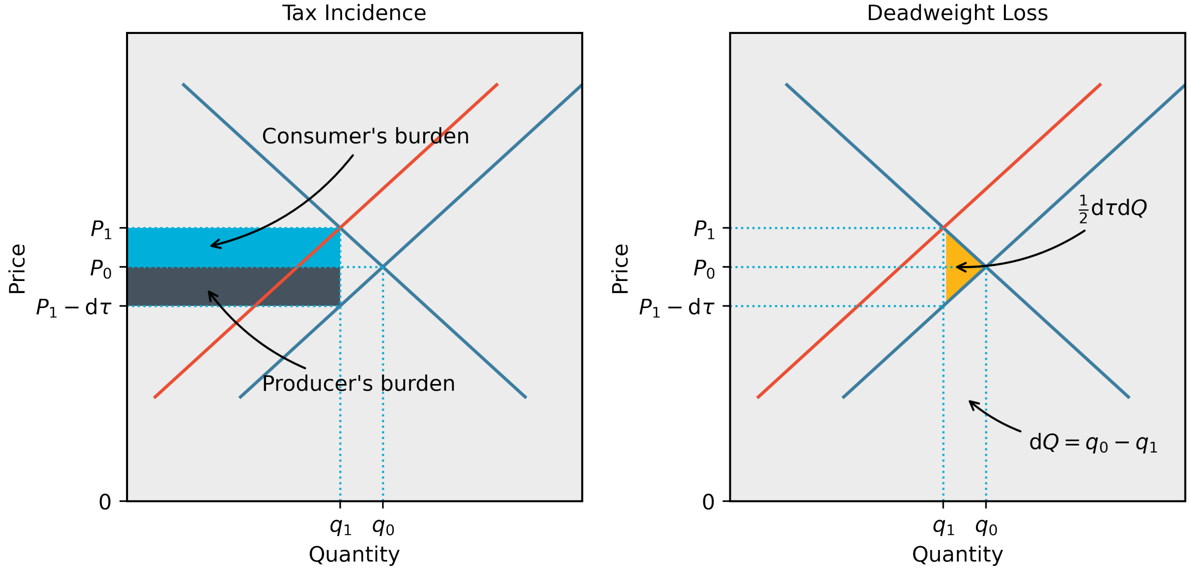
If $\frac{\epsilon^S}{\epsilon^S-\epsilon^D}$ is close to one, then the increase in the tax is almost fully passed onto consumers in the form of high prices. In this case consumers bear the burden of the tax. In contrast if $\frac{\epsilon^S}{\epsilon^S-\epsilon^D}$ is close to zero, then the tax increase is not meaningfully reflected in prices and producers largely bear the burden of the tax. With knowledge of the price elasticities of supply and demand a policy-maker can form predictions about tax incidence.

Figure 4 depicts the effect of introducing an excise tax of $\mathrm{d}\tau$ on the market clearing price and quantity (assume that initially there is no tax such that $\tau_0 = 0$). For any given market price p,

[1] It is also possible to derive (7) by applying the implicit function theorem to $Q^D(P,W_1) - Q^S(P-\tau,W_2) = 0$:

$$\frac{\mathrm{d}P}{\mathrm{d}\tau} = -\frac{\frac{\partial}{\partial \tau}\left\{Q^D(P,W_1) - Q^S(P-\tau,W_2)\right\}}{\frac{\partial}{\partial P}\left\{Q^D(P,W_1) - Q^S(P-\tau,W_2)\right\}} = -\frac{-\frac{\partial Q^S(P,W_1)}{\partial P}\times -1}{\frac{\partial Q^D(P,W_1)}{\partial P} - \frac{\partial Q^S(P,W_1)}{\partial P}} = \frac{\epsilon^S}{\epsilon^S-\epsilon^D}.$$

Figure 4: Excise tax incidence and deadweight loss



SOURCE: Author's calculations.
NOTES: The left panel shows the effect of introducing a producer-paid excise tax of $\mathrm{d}\tau$ per unit sold on the market clearing price and quantity. Prior to the introduction of the tax the market clears at (p_0, q_0), whereas after the tax it clears at (p_1, q_1). The light-blue shaded region shows the portion of the tax revenue paid for by consumers due to higher after-tax prices, whereas the dark-blue shaded region shows the portion paid for by producers in the form of lower post-tax revenue per unit sold. Total revenue is equal to the sum of the two shaded regions (i.e, $\mathrm{d}\tau \times q_1$). The right panel depicts the deadweight loss (i.e., the reduction in consumer and producer surplus) associated with the tax.

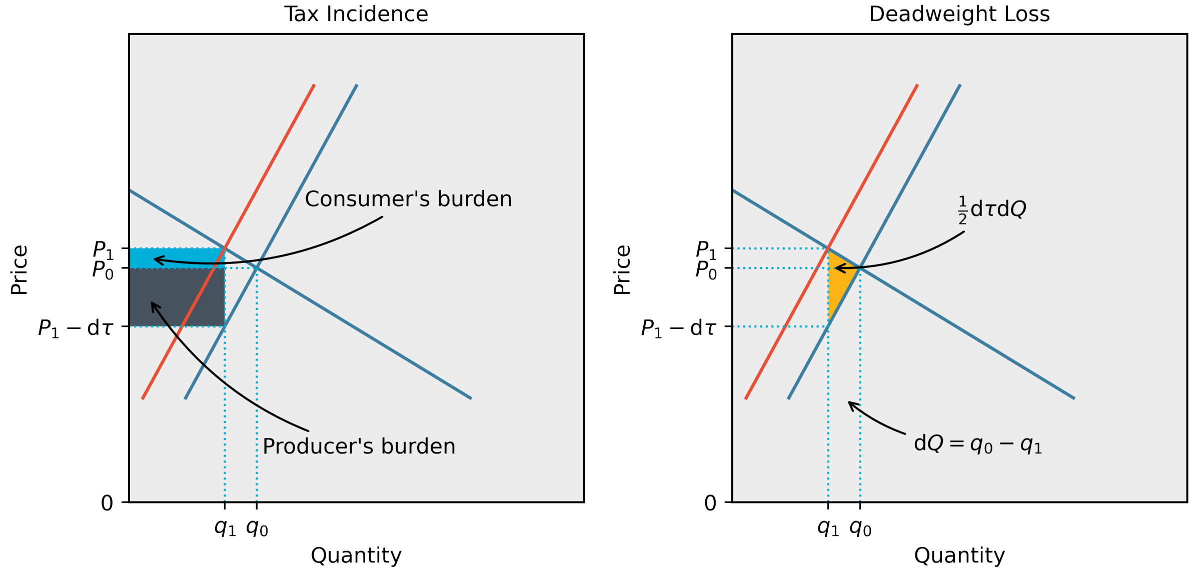
the introduction of the tax causes producers' to supply less to the market; effectively shifting the supply schedule inward. In this example of linear demand and supply schedules, the vertical shift in the supply schedule is exactly $\mathrm{d}\tau$. In the general case the linear approximation is only accurate for small changes in the excise tax.

The introduction of the excise tax induces a new market clearing equilibrium. This equilibrium coincides with a movement along the demand schedule from (p_0, q_0) to (p_1, q_1).

The two shaded regions in Panel A of Figure 4 collectively show the total revenue raised by the tax, namely $\mathrm{d}\tau \times q_1$. The portion borne by consumers, in the form of higher market prices per unit sold, is given by the light-blue shaded region, $[p_1 - p_0] \times q_1$. The portion borne by producers, in the form of lower post-tax revenue per unit sold, is given by the dark-blue shaded region, $[p_0 - (p_1 - \mathrm{d}\tau)] \times q_1$.

Figure 6 illustrates how the effect of the excise tax introduction depends on the price elasticity of demand. When consumer demand is price sensitive, producers are less able to pass on the costs of the excise tax to buyers. In this case the incidence of the tax fall heavily onto

Figure 5: The price elasticity of demand and excise taxes



SOURCE: Author's calculations.
NOTES: See the notes to Figure 4. Relative to Figure 4, the price elasticity of demand is larger (in absolute value) here. This increases, as per expression (7), the incidence of the tax on producers.

producers (see equation (6)). It is a useful exercise to consider different price elasticity of demand and supply scenarios and work out their implications for tax incidence (and deadweight loss, discussed below).

Deadweight loss

Associated with the introduction of an excise tax will be a reduction in consumer and producer surplus; called deadweight loss (note there may be other benefits of the tax which more than offset these losses). The introduction of a producer-paid excise taxes induces a shift along the demand schedule of

$$\mathrm{d}Q = Q^D\left(P_0 + \mathrm{d}P, W_1\right) - Q^D\left(P_0, W_1\right),$$

which for a small tax change gives

$$\mathrm{d}Q = \frac{\mathrm{d}Q^D\left(P, W_1\right)}{dP}\mathrm{d}P.$$

The total loss of consumer and producer surplus is given by the yellow triangle in Panel B of Figures 4 and 5. The area of this triangle equals

$$\begin{aligned}
\frac{1}{2}\mathrm{d}Q\mathrm{d}\tau &= \frac{1}{2}\frac{\mathrm{d}Q^D}{\mathrm{d}P}\frac{\mathrm{d}P}{\mathrm{d}\tau}\left(\mathrm{d}\tau\right)^2 \\
&= \frac{1}{2}\frac{Q}{P}\frac{\mathrm{d}Q^D}{\mathrm{d}P}\frac{P}{Q}\frac{\mathrm{d}P}{\mathrm{d}\tau}\left(\mathrm{d}\tau\right)^2 \\
&= \frac{1}{2}\frac{\epsilon^S\epsilon^D}{\epsilon^S-\epsilon^D}\frac{Q}{P}\left(\mathrm{d}\tau\right)^2.
\end{aligned}$$

Again, the price elasticities of demand and supply, respectively, ϵ^D and ϵ^S, are key inputs into this calculation.

Note also that the equilibrium effects on quantity sold equals

$$\frac{\mathrm{d}Q}{\mathrm{d}\tau} = \frac{\epsilon^S\epsilon^D}{\epsilon^S-\epsilon^D}\frac{Q}{P}.$$

This object can be important for predicting the overall benefits of a tax when a reduction in consumption is desired (as proponents of increasing gasoline excise taxes emphasize).

Further reading

In our next lecture we will discuss how the method of instrumental variables can, under certain assumptions, be used to recover the price elasticities of demand and supply.

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