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MODELING AND PREDICTION OF SOFT TISSUE DIRECTIONAL STIFFNESS USING *IN-VITRO* FORCE-DISPLACEMENT DATA

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ABSTRACT

Due to the nonlinear behavior of soft tissue properties, static and dynamic analysis of these materials has been always a challenge for scientists, especially in large deformation where viscoelastic nature of these materials plays an important role in the analysis. Human body organ tissues are mostly different in mechanical properties and a unique behavior cannot be generalized for all of them. Numerous efforts have been made to predict the nonlinear force-displacement and stress-strain behavior of soft tissues. A fine modeling of the nonlinear stiffness coefficient for soft tissues is an important parameter in the understanding and treatment of these materials. This is important especially in fine robotic surgery and haptic processes. In this paper, based on experimental results in the literature, two empirical equations of uniaxial force *versus* displacement for two categories of soft tissues are developed. Plots of force *versus* displacement, based on experimental data and empirical equations for some tissues, are presented and compared. The results obtained using these equations when compared with those of experimental data indicate not more than 2 to 5 percent error. By applying the derivative of these equations, the directional stiffness of the two categories of soft tissues is found as a function of displacement.

INTRODUCTION

The biomechanical model of soft tissue is a key requirement for developing a reality-based model for minimally invasive surgical simulation. In addition, the soft tissue model can also be used in real-time for local tool-tissue interaction tasks through a haptic display. Nonlinear viscoelastic properties of soft tissues have complicated construction of a unique constitutive equation to represent the biomechanical behavior of these materials. Classical viscoelastic models of soft tissues generally use mechanical elements including the massless linear spring and viscous dashpot with spring and viscosity constants respectively.

However, experimental results of displacement *versus* external force on a variety of soft tissues indicate a nonlinear stiffness coefficient behavior for these materials, [1-11]. To achieve a more accurate model considering the nonlinear nature of soft tissues, results from *in-vitro* experimental data can be used for soft tissue viscoelastic modeling. In addition, with consideration to the force-displacement behavior in the *Toe Region* (Figure 1), soft tissues may be categorized into two main groups.

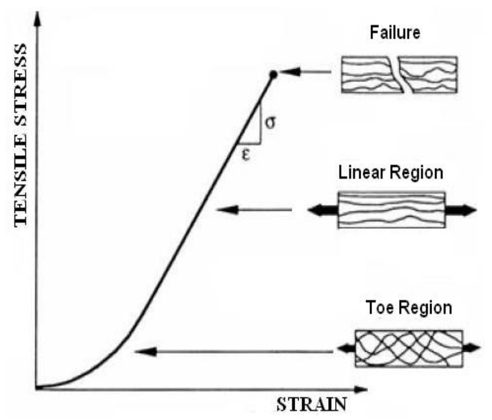

Fig. 1. Typical stress-strain curve for soft tissue.

In this paper we attempt to find appropriate empirical equations for one-dimensional stiffness coefficient, using experimental data from the literature. These empirical equations can be used to serve the nonlinear viscoelastic behavior in soft tissue analysis.

CATEGORIZATION OF SOFT TISSUES

Study and review of more than 40 experimental force-displacement and stress-strain curves for a variety of soft tissues from the literature indicates that these materials in loading may be categorized into two different groups based on the toe region length.

In the first group (with short toe-region), nonlinearity may be followed by a simple exponential function. In this group, displacement smoothly follows the force monotonically.

The second group (with a long toe-region) follows a rather complicated exponential curvature. In this group force *versus* displacement changes rapidly and stiffness increases sharply.

It is clear that almost none of the organs in general and tissues in particular are exactly the same in mechanical behavior. Considering the complication of mechanical properties for measurement of living tissue, it is too difficult to achieve a uniform formulation based on the soft tissue structure or position in the human body. Although the test conditions for experimental data may be different, but the general behavior of the soft tissues remains unchanged with unique physical properties. In these experiments, force-displacements are uniaxial but sample sizes, strain rates and maximum deformation may vary. It should be noted that this categorization eases the prediction of the one-dimensional nonlinear stiffness coefficient for soft tissue with higher accuracy. It is clear that this categorization is not unique and some specific tissues may not match in this model with acceptable minimum error. However, this is a step forward to ease the rocky rode of the nature of non-linearity.

FORCE-DISPLACEMENT FORMULATION

Different mathematical modeling using exponential, polynomial and a combination of these models, were implemented to predict the two distinct categories of soft tissue behavior. It is clear, the best fitted model with minimization of the error while applicable and simple was the purpose of the modeling. The following equations were obtained based on error minimization between the experimental data and the empirical results:

Fitted equation for category *I* may be written as:

$$F_I(\delta) = \sum_{i=1}^{2} a_i e^{b_i \delta} \qquad (1)$$

Where F is the exerted force in $mN(10^{-3}N)$, δ is the displacement in $mm(10^{-3}m)$, a_i and b_i are tissue constants that are presented in Table 1. for some tissues, [1]-[2] in category *I*. Results of applying equation (1) of the first category along with experimental data for some tissues in Table 1 are presented in Figure 2 and Figure 3.

Table (1). Coefficients for some tissues in Category *I*.

Ref.	Tissue Name
[1]	Breast (sample 1): $a_1 = 1926.000, b_1 = 1.399$ $a_2 = -1926.000, b_2 = 1.393$
[1]	Breast (sample 2): $a_1 = 1.670, b_1 = 3.843$ $a_2 = -1.685, b_2 = -1.844$
[1]	Breast (sample 3): $a_1 = 7.355, b_1 = 4.829$ $a_2 = -7.183, b_2 = -8.368$
[2]	Kidney (sample 1): $a_1 = 102.700, b_1 = 2.666$ $a_2 = -105.800, b_2 = -1.483$
[2]	Kidney (sample 2): $a_1 = 81.960, b_1 = 2.949$ $a_2 = -84.930, b_2 = -2.971$
[2]	Kidney (sample 3): $a_1 = 77.980, b_1 = 3.150$ $a_2 = -79.320, b_2 = -2.447$

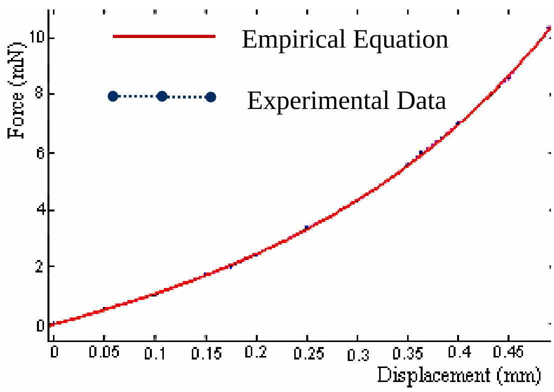

Fig. 2. Force-displacement curves for human breast (fibrograndular tissue).

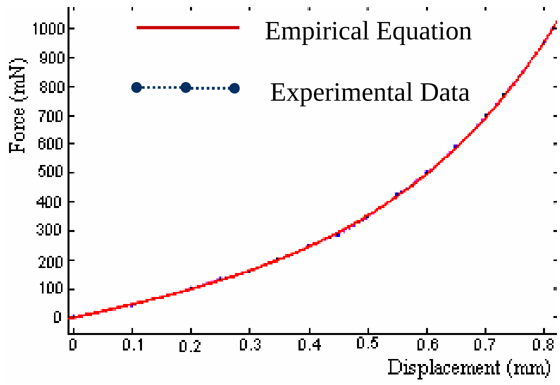

Fig. 3. Force-displacement curves for canine kidney (sample 3).

Comparison between the empirical findings and experimental results indicates high resolution of the empirical equation and error not more than 5 percent.

The empirical equation for category *II* was found to be as follows:

$$F_{II}(\delta) = \sum_{i=1}^{2} a_i e^{-\left(\frac{b_i-\delta}{c_i}\right)^2} \qquad (2)$$

Where F is exerted force in mN(*or* $10^{-3}N$), δ is displacement in mm(*or* $10^{-3}m$), a_i, b_i and c_i are tissue constants that are presented in Table 2 for some tissues, [3-5], in category *II*. Application of equation (2) for the second category along with experimental data is presented for some tissues in Figure 4 and Figure 5. Applying this equation to the soft tissues in category *II* and comparing the findings with experimental results indicates a fine match with error between 2 to 5 percent.

Table (2). Coefficients for some tissues in Category *II*.

Ref.	Sample's Name & Coefficients
[3]	Heel Pad (sample 1): $a_1 = 2.637\times10^6, b_1 = 6.508, c_1 = 1.820$ $a_2 = 8.753\times10^5, b_2 = 5.566, c_2 = 2.280$
[3]	Heel Pad (sample 2): $a_1 = 4.328\times10^5, b_1 = 4.233, c_1 = 0.590$ $a_2 = 6.808\times10^5, b_2 = 7.238, c_2 = 2.978$
[3]	Heel Pad (sample 3): $a_1 = 2.294\times10^5, b_1 = 3.499, c_1 = 0.234$ $a_2 = 1.792\times10^6, b_2 = 4.468, c_2 = 1.431$
[4-5]	Liver (sample 1): $a_1 = 937.300, b_1 = 23.730, c_1 = 1.955$ $a_2 = 998.000, b_2 = 28.160, c_2 = 10.350$
[4-5]	Liver (sample 2): $a_1 = 6693.300, b_1 = 29.510, c_1 = 6.251$ $a_2 = 905.900, b_2 = 29.010, c_2 = 14.580$
[4-5]	Liver (sample 3): $a_1 = 1473.000, b_1 = 24.670, c_1 = 4.081$ $a_2 = 1822.000, b_2 = 36.630, c_2 = 17.270$

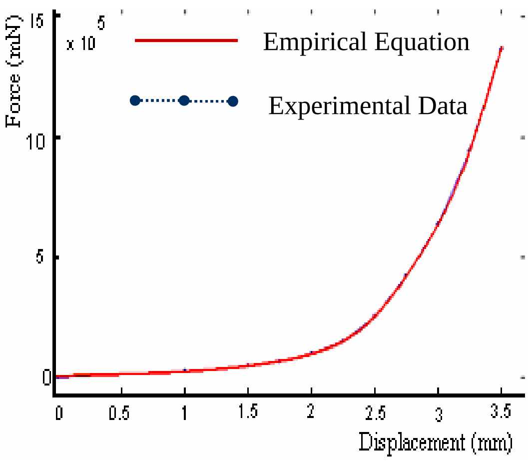

Fig. 4. Force-displacement curves for human heel pad (sample 3).

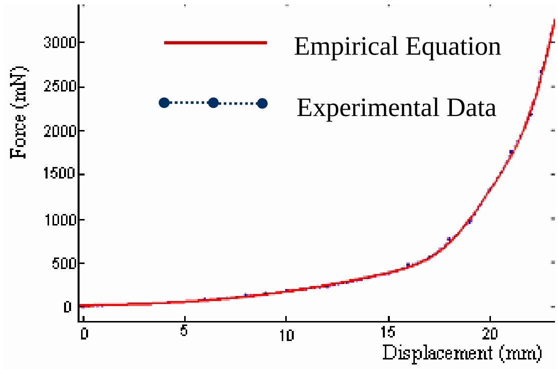

Fig. 5. Force-displacement curves for pig liver (sample 2).

RESULTS

Considering the nonlinear relationship between force and displacement, it is clear that the stiffness coefficient of the soft tissues has nonlinear characteristics. Applying the differential equation between force and displacement, $K = (\partial F/\partial \delta)$ with the use of equations (1) and (2), the one-dimensional stiffness coefficient for soft tissues of categories *I* and *II* may be written as:

For category *I*:

$$K_I(\delta) = \sum_{i=1}^{2} a_i b_i e^{b_i \delta} \qquad (3)$$

For category *II*:

$$K_{II}(\delta) = \sum_{i=1}^{2} \left[2\, a_i \left(\frac{b_i - \delta}{c_i^2} \right) e^{-\left(\frac{b_i - \delta}{c_i} \right)^2} \right] \qquad (4)$$

Where K is the one-dimensional stiffness coefficient in (*N/m*) and δ is displacement in (*mm*).

It is clear that these equations are highly nonlinear with respect to displacement and can be used in nonlinear viscoelastic modeling of soft tissues.

DISCUSSION AND CONCLUSIONS

Soft tissues may be divided in different categories; these categories may be based on physical properties and force-displacement or stress-strain configurations. In this study an extensive effort has been made to categorize a series of soft tissues, presented by different scientists in the literature, in two different categories. Formulating and testing the force-displacement equations has led us to model the soft tissues behavior under two different categories namely *I* and *II*. Categorization and formulation was based on error minimization between the experimental findings in the literature and the results of empirical equations. The allowable error between experimental data and the results of empirical equations was defined to be not more than 5 percent. Finally, the one-dimensional stiffness coefficient *versus* displacement was developed for the two categories of soft tissues. Based on these findings, it is much easier to model the soft tissue nonlinear behavior.

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