3. i Let $X_1 \sim N(\mu_1, \sigma_1^2)$, $X_2 \sim N(\mu_2, \sigma_2^2)$ be independent

Take $X = X_1 + X_2$

Then $\varphi_X(t) = \mathbb{E}(e^{itX}) = \mathbb{E}(e^{it(X_1+X_2)})$

$= \mathbb{E}(e^{itX_1})\,\mathbb{E}(e^{itX_2})$ by independence

$= e^{i\mu_1 t - \frac{1}{2}\sigma_1^2 t}\, e^{i\mu_2 t - \frac{1}{2}\sigma_2^2 t}$

$= e^{i(\mu_1+\mu_2) - \frac{1}{2}(\sigma_1^2+\sigma_2^2)t}$

Hence $X \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$

ii Let $Y_1 \sim \Gamma(\alpha_1, \beta)$, $Y_2 \sim \Gamma(\alpha_2, \beta)$ be independent

Take $Y = Y_1 + Y_2$

Similarly $\varphi_Y(t) = \mathbb{E}(e^{itY_1})\,\mathbb{E}(e^{itY_2})$

$= (1 - \frac{it}{\beta})^{-\alpha_1} (1 - \frac{it}{\beta})^{-\alpha_2}$

$= (1 - \frac{it}{\beta})^{-(\alpha_1+\alpha_2)}$

Hence $Y \sim \Gamma(\alpha_1+\alpha_2, \beta)$