1. $\mathbb{P}(A \cap B \cap C) = \mathbb{P}(A \cap (B \cap C))$

$= \mathbb{P}(A \mid B \cap C)\,\mathbb{P}(B \cap C)$

$= \mathbb{P}(A \mid B \cap C)\,\mathbb{P}(B \mid C)\,\mathbb{P}(C)$

For $n \geq 2$

$$\mathbb{P}\left(\bigcap_{k=1}^{n} A_k\right) = \mathbb{P}(A_1) \prod_{k=2}^{n} \mathbb{P}\left(A_k \,\middle|\, \bigcap_{i=1}^{k-1} A_i\right)$$

Proof:

This is clearly true for base case $n=2$

Suppose true for fixed $n > 2$

$$\mathbb{P}\left(\bigcap_{k=1}^{n+1} A_k\right) = \mathbb{P}\left(A_{n+1} \,\middle|\, \bigcap_{k=1}^{n} A_k\right) \mathbb{P}\left(\bigcap_{k=1}^{n} A_k\right)$$

$$= \mathbb{P}\left(A_{n+1} \,\middle|\, \bigcap_{k=1}^{n} A_k\right) \mathbb{P}(A_1) \prod_{k=2}^{n} \mathbb{P}\left(A_k \,\middle|\, \bigcap_{i=1}^{k-1} A_i\right)$$

$$= \mathbb{P}(A_1) \prod_{k=2}^{n+1} \mathbb{P}\left(A_k \,\middle|\, \bigcap_{i=1}^{k-1} A_i\right)$$

Hence, true for $n+1$

Thus by induction true $\forall n \geq 2$