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专业： 自动化控制
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课程名称： 优化实用算法 指导老师： 王何宇 成绩：

Problem 3.7: Prove the result (3.28)

Proof:

First, $\because x_{k+1} = x_k - \alpha_k \nabla f_k$

$$\begin{aligned}||x_{k+1} - x^*||_Q^2 =&(x_{k+1} - x^*)^T Q (x_{k+1} - x^*)\\ =&(x_k - x^* - \alpha_k \nabla f_k)^T Q (x_k - x^* - \alpha_k \nabla f_k)\\ =&(x_k - x^*)^T Q (x_k - x^*) - (x_k - x^*)^T Q \alpha_k \nabla f_k - \alpha_k \nabla f_k^T Q (x_k - x^*) + \alpha_k \nabla f_k^T Q \alpha_k \nabla f_k\\ =&||x_k - x^*||_Q^2 - 2\alpha_k \nabla f_k^T Q (x_k - x^*) + \alpha_k^2 \nabla f_k^T Q \nabla f_k\end{aligned}$$

$\therefore$ we could derive

$$||x_k - x^*||_Q^2 - ||x_{k+1} - x^*||_Q^2 = 2\alpha_k \nabla f_k^T Q (x_k - x^*) - \alpha_k^2 \nabla f_k^T Q \nabla f_k \tag{1}$$

Second, $\because \nabla f = Q(x_k - x^*)$, we select α to minimize $f = \frac{1}{2} x^T Q x + bx$. In class we have use differential criteria to get the result, where

$$\alpha_k = \frac{\nabla f_k^T \nabla f_k}{\nabla f_k^T Q \nabla f_k}$$

Then we substitude them in formula (1):

$$||x_k - x^*||_Q^2 - ||x_{k+1} - x^*||_Q^2 = \frac{2(\nabla f_k^T \nabla f_k)^2}{\nabla f_k^T Q \nabla f_k} - \frac{(\nabla f_k^T \nabla f_k)^2}{\nabla f_k^T Q \nabla f_k} = \frac{(\nabla f_k^T \nabla f_k)^2}{\nabla f_k^T Q \nabla f_k}$$

$$||x_k - x^*||_Q^2 = (x_k - x^*)^T Q (x_k - x^*) = [(x_k - x^*)^T Q] Q^{-1} [Q(x_k - x^*)] = \nabla f_k^T Q^{-1} \nabla f_k$$

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