Suppose that the utility function is

$$U = \left[\int_{\omega \in \Omega} \varphi(\omega)^{\epsilon} q(\omega)^{\rho} \mathrm{d}\omega \right]^{1/\rho}$$

where $\varphi(\omega)^{\epsilon}$ is the "quality" of variety ω. Show that the price index associated with this utility function is

$$P = \left[\int_{\omega \in \Omega} \varphi(\omega)^{\epsilon\sigma} p(\omega)^{1-\sigma} \mathrm{d}\omega \right]^{\frac{1}{1-\sigma}}$$

Derive the optimal consumption and expenditure decisions for individual varieties analogous to equation (2) of Melitz (2003).

Now, let the firm's production function be $l = f + q$. Note the absence of firm heterogeneity in costs. Derive expressions analogous to equations (4) and (5) of Melitz (2003).