

BUSN 33946 & ECON 35101  
International Macroeconomics and Trade  
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Autumn 2019, Week 2



# Outline of today

- ▶ Preview of assignments
- ▶ Overview of neoclassical models
- ▶ Canonical Ricardian model: DFS 1977
- ▶ A Ricardian assignment model: Costinot 2009
- ▶ DFS with non-homothetic preferences: Matsuyama 2000
- ▶ Trade and growth: Acemoglu & Ventura 2002, briefly

# Preview of assignments

Assignments are available at

<https://github.com/jdingel/econ35101>

- ▶ Due week 4: DFS exercise
- ▶ Due week 6: gravity estimation exercise
- ▶ Due week 10: theory exercise
- ▶ Due week 11: referee report and final exam

Submit assignments via Canvas.

Comprehension checks are due in weeks 4, 6, and 10.

Due dates are backloaded: don't procrastinate.

# Taxonomy of neoclassical trade models

- ▶ In a neoclassical model, comparative advantage (lower relative autarkic marginal cost) is the basis for trade
- ▶ Autarky costs might reflect demand or supply differences
- ▶ Demand differences typically neglected by assumption
- ▶ Supply-side explanations for autarkic cost differences:
  - ▶ Technological differences (Ricardian theory)
  - ▶ Factor-endowment differences (Ricardo-Viner and Heckscher-Ohlin, week 5)
  - ▶ Increasing returns to scale (beyond neoclassical scope)
- ▶ In theoretical models, the roles of factor proportions and technological differences are typically kept separate:
  - ▶ Ricardian model assumes one factor of production
  - ▶ Factor-proportions theory typically assumes common production function

# Technology vs factors

Different models for different questions?<sup>1</sup>

- ▶ What is the effect of rising Chinese productivity on US real wages? (DFS 1977, [its interpretation](#), [Hsieh and Ossa 2016](#))
- ▶ What are distributional consequences of trade? Need multiple factors

Interaction of technology and factors might matter

- ▶ Does fact of intra-industry trade necessitate increasing returns in theory? No, says [Davis \(1995\)](#).
- ▶ [Chor \(JIE 2010\)](#) and [Morrow \(JIE 2010\)](#)
- ▶ Factor-biased technical change

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<sup>1</sup>Jones & Neary (1980): “positive trade theory uses a variety of models, each one suited to a limited but still important range of questions”

# Canonical Ricardian model of DFS 1977

- ▶ Two countries, Home and Foreign; asterisk denotes latter
- ▶ One factor of production, call it labor, endowed in amounts  $L$  and  $L^*$  and paid wages  $w$  and  $w^*$   
[efficiency units, and see “Hicksian composite”]
- ▶ Unit labor costs for good  $z$  are  $a(z)$  and  $a^*(z)$
- ▶ WLOG, order goods such that  $A(z) \equiv \frac{a^*(z)}{a(z)}$  is decreasing
- ▶ Home has comparative advantage in low- $z$  goods

## Recall the concepts and insights of two-good case

Consider two goods,  $z$  and  $z'$

- ▶ Home has *absolute advantage* in  $z$  when  $a(z) < a^*(z)$
- ▶ Home has *comparative advantage* in  $z$  when its relative autarkic marginal cost is lower:  $\frac{a(z)}{a(z')} < \frac{a^*(z)}{a^*(z')}$

What is equilibrium pattern of specialization?

- ▶ For factor markets to clear, Home cannot be least-cost provider of both goods. It is not possible that

$$wa(z) < w^*a^*(z) \text{ and } wa(z') < w^*a^*(z')$$

- ▶ If Home has comparative advantage in  $z$ , it must be that

$$\frac{a(z)}{a^*(z)} \leq \frac{w^*}{w} \leq \frac{a(z')}{a^*(z')}$$

- ▶ Absolute advantage determines wages; comparative advantage determines specialization

## Pattern of specialization in DFS 1977

- ▶ Let  $p(z)$  denote the price of good  $z$  under free trade
- ▶ Profit maximization and factor-market clearing require

$$p(z) \leq wa(z) \text{ and } p(z) \leq w^*a^*(z)$$

with equality if produced in Home or Foreign, respectively

- ▶ There exists  $\tilde{z}$  such that Home produces all of  $z < \tilde{z}$  and Foreign produces all of  $z > \tilde{z}$  (proof by contradiction)
- ▶ Countries specialize according to comparative advantage
- ▶ Define relative wage  $\omega \equiv \frac{w}{w^*}$
- ▶ Given relative wages, cost-minimizing specialization is  $[0, \tilde{z}]$  at Home and  $[\tilde{z}, 1]$  in Foreign such that  $A(\tilde{z}) = \omega$
- ▶ Continuum of  $z$  and  $A'(\tilde{z}) < 0$  makes  $\tilde{z} = A^{-1}(\omega)$
- ▶ Second curve in  $z$ - $\omega$  space requires demand



# Cobb-Douglas preferences

Identical Cobb-Douglas preferences with expenditure shares  $b(z)$

$$b(z) = \frac{p(z) c(z)}{wL} = b^*(z) = \frac{p^*(z) c^*(z)}{w^* L^*}$$
$$\int_0^1 b(z) dz = \int_0^1 b^*(z) dz = 1$$

Denote the share of expenditure on Home goods by  $\theta(\tilde{z})$

$$\theta(\tilde{z}) = \int_0^{\tilde{z}} b(z) dz \quad \text{and} \quad 1 - \theta(\tilde{z}) = \int_{\tilde{z}}^1 b(z) dz$$

Trade balance then requires  $\theta(\tilde{z}) w^* L^* = [1 - \theta(\tilde{z})] wL$ , which implies

$$\omega = \frac{\theta(\tilde{z})}{1 - \theta(\tilde{z})} \frac{L^*}{L} \equiv B(\tilde{z})$$

# Gains from trade in DFS 1977

- ▶ It's a neoclassical model with a free-trade equilibrium, so last week's results apply: there are gains from trade
- ▶ Given functional forms, we can speak to magnitudes

$$\ln(U/L) = \ln w - \int_0^1 b(z) \ln p(z) dz$$

- ▶ Choose  $w = 1$  in both autarky and trade equilibria

$$\int_0^1 b(z) \ln a(z) dz \text{ vs } \int_0^{\tilde{z}} b(z) \ln a(z) dz + \int_{\tilde{z}}^1 b(z) \ln [w^* a^*(z)] dz$$

- ▶ Connect to **double-factoral terms of trade** and dissimilarity as source of GFT

# Comparative statics for population growth

An increase in  $L^*/L$ , DFS Figure 2:

- ▶ Equilibrium is a decrease in  $\bar{z}$  and increase in  $\bar{\omega}$
- ▶ At initial  $\bar{\omega}$ , larger  $L^*/L$  means trade surplus for Home, so its terms of trade must improve
- ▶ Goods produced at Home before and after shock have no change in price
- ▶ Goods produced in Foreign before and after shock become cheaper for Home consumers
- ▶ What about the goods that switch?
- ▶ Each good is produced using CRS, but akin to country-level DRS

# Comparative statics for technical change

What happens with each of the following shocks?

- ▶ Uniform global technical progress:  $d \ln a(z) = d \ln a^*(z) = x < 0$
- ▶ Uniform Foreign technical progress:  $d \ln a^*(z) = x < d \ln a(z) = 0$
- ▶ Technical transfer: Convergence to  $a(z) = a^*(z)$

# Trade costs

“Iceberg” trade costs  $g(z) = g$  (in fact, [shipping ice is IRS](#))

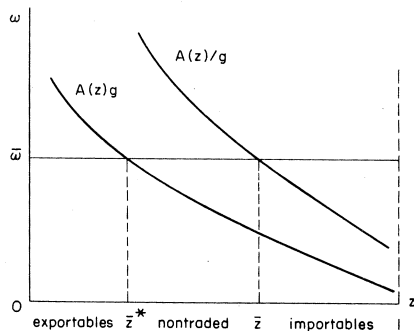


FIGURE 3

- ▶ Home produces if  $wa(z) \leq (1/g)w^*a^*(z)$
- ▶ Foreign produces if  $w^*a^*(z) \leq (1/g)wa(z)$
- ▶ Trade balance is  $(1 - \lambda)wL = (1 - \lambda^*)w^*L^*$

# What about three or more countries?

Wilson (Ecma, 1980):

*The DFS paper represents a significant contribution in demonstrating how one might modify the standard Ricardian model in order to make it more tractable for comparative statics analysis. Their assumptions are so restrictive, however, that the extent to which their approach can be generalized is not readily apparent. Besides the possibility of relaxing their assumptions on demand, it is not at all clear from their examples how the analysis would proceed if we wished to allow for more than two countries.*

Costinot (Ecma, 2009): Log-supermodularity, a strong assumption borrowed from the *monotone comparative statics* literature, delivers the two-country logic in a many-country setting

# Ricardo-Roy assignment models

Costinot and Vogel (2015) survey Ricardo-Roy models

- ▶ Ricardo: Linear production functions
- ▶ Roy: Multiple factors of production ( $\omega$ )

Output in sector  $\sigma$  in country  $c$  is

$$Q(\sigma, c) = \int_{\Omega} A(\omega, \sigma, c) L(\omega, \sigma, c) d\omega$$

Ricardo 1817: England =  $c > c' =$  Portugal and  
cloth =  $\sigma > \sigma' =$  wine

$$A(\sigma, c)/A(\sigma', c) \geq A(\sigma, c')/A(\sigma', c')$$

Dornbusch, Fischer & Samuelson 1977:

$$A(\sigma) \equiv \frac{A(\sigma, c)}{A(\sigma, c')} \quad A'(\sigma) < 0$$

# Log-supermodularity (1/2)

## Definition (Log-supermodularity)

A function  $g : \mathbb{R}^n \rightarrow \mathbb{R}^+$  is *log-supermodular* if  $\forall x, x' \in \mathbb{R}^n$

$$g(\max(x, x')) \cdot g(\min(x, x')) \geq g(x) \cdot g(x')$$

where  $\max$  and  $\min$  are component-wise operators.

- Example:  $A : \Sigma \times \mathbb{C} \rightarrow \mathbb{R}^+$ , where  $\Sigma \subseteq \mathbb{R}$  and  $\mathbb{C} \subseteq \mathbb{R}$ , with  $\sigma > \sigma'$  and  $c > c'$

$$A(\sigma, c)A(\sigma', c') \geq A(\sigma', c)A(\sigma, c')$$

- $g(x)$  is LSM in  $(x_i, x_j)$  if  $g(x_i, x_j; x_{-i, -j})$  is LSM
- $g(x)$  is LSM  $\iff g(x)$  is LSM in  $(x_i, x_j) \forall i, j$
- $g > 0$  and  $g$  is  $C^2 \implies \frac{\partial^2 \ln g}{\partial x_i \partial x_j} \geq 0 \iff g(x)$  is LSM in  $(x_i, x_j)$



## Log-supermodularity (2/2)

Three handy properties:

1. If  $g, h : \mathbb{R}^n \rightarrow \mathbb{R}^+$  are log-supermodular, then  $gh$  is log-supermodular.
2. If  $g : \mathbb{R}^n \rightarrow \mathbb{R}^+$  is log-supermodular, then  $G(x_{-i}) \equiv \int g(x_i, x_{-i}) dx_i$  is log-supermodular.
3. If  $g : \mathbb{R}^n \rightarrow \mathbb{R}^+$  is log-supermodular, then  $x_i^*(x_{-i}) \equiv \arg \max_{x_i \in \mathbb{R}} g(x_i, x_{-i})$  is increasing in  $x_{-i}$ .

# The Ricardo-Roy setup in one slide

Primitives:

- ▶ Technologies  $A(\omega, \sigma, \gamma_{A,c})$
- ▶ Endowments  $L(\omega, \gamma_{L,c})$
- ▶ Demands  $D(p, I_c | \sigma, \gamma_{D,c})$

Equilibrium:

- ▶ Profit maximization by firms

$$Q(\sigma, c) = \int_{\Omega} A(\omega, \sigma, \gamma_{A,c}) L(\omega, \sigma, c) d\omega$$

$$\Rightarrow p(\sigma) \leq \min_{\omega \in \Omega} \{w(\omega, c) / A(\omega, \sigma, \gamma_{A,c})\}$$

$$\Omega(\sigma, c) \equiv \{\omega \in \Omega : L(\omega, \sigma, c) > 0\} \subseteq \arg \min_{\omega \in \Omega} \{w(\omega, c) / A(\omega, \sigma, \gamma_{A,c})\}$$

- ▶ Market clearing

$$\int_{\Sigma} L(\omega, \sigma, c) d\sigma = L(\omega, \gamma_{L,c}) \quad \forall \omega, c$$

$$\int_{\mathbb{C}} D(p, I_c | \sigma, \gamma_{D,c}) dc = \int_{\mathbb{C}} Q(\sigma, c) dc \quad \forall \sigma$$

# The Ricardian case of Ricardo-Roy

Costinot (2009) introduces a particular neoclassical environment:

- ▶ Linear production functions
- ▶ Assumption 0 is that the solution to the revenue-maximization problem is unique. This will help pin down assignments.
- ▶ Assumption 0 always holds if there is a continuum of distinct factors.

Ricardian case (ignore Roy until week 5):

- ▶  $A(\omega, \sigma, \gamma_{A,c}) = h(\omega)A(\sigma, \gamma)$
- ▶ Given this technology and free trade, endowment and demand shifters don't matter
- ▶ Assumption 0 implies that each country produces one good

# The Ricardian case of Ricardo-Roy

Assume that  $A(\sigma, \gamma)$  is log-supermodular

- ▶ Then, for  $\sigma > \sigma'$  and  $\gamma > \gamma'$  and non-zero productivities,

$$\frac{A(\sigma, \gamma)}{A(\sigma, \gamma')} \geq \frac{A(\sigma', \gamma)}{A(\sigma', \gamma')}$$

- ▶ Return to the two-good logic: there must be an ordering in which higher- $\gamma$  countries produce higher- $\sigma$  goods
- ▶ This doesn't require Assumption 0: the set  $\Sigma(\gamma)$  is increasing in the strong set order (intervals of specialization ordered by  $\gamma$ )

Two payoffs to this section of Costinot (2009):

- ▶ You can deliver a many-country Ricardian model with sufficiently strong assumptions
- ▶ A number of 2005-2007 papers on trade and institutions amount to microfoundations for LSM of “institutional dependence”  $\sigma$  and “institutional quality”  $\gamma$

How would you introduce trade costs?

# Matsuyama (2000): The DFS model without homotheticity

Why might income elasticities of goods be interesting?

- ▶ Terms of trade might be shaped by global growth
- ▶ Product cycles in which rich buy innovations first
- ▶ “Neutral” productivity shifts aren’t neutral
- ▶ Scope for normative implications

Some themes in this paper are echoed in [Matsuyama \(2019\)](#) [week 6]

- ▶ Product cycles via income elasticities
- ▶ Demand: Gross substitutes and “quality” or not

## Matsuyama (2000) set up

Mostly as in DFS 1977, however

- ▶  $z \in [0, \infty)$  and cutoff good  $m$  given by  $w = A(m)$  with  $w^* = 1$
- ▶  $A(z)$  schedule in Figure 1 same as DFS Figure 1
- ▶ Hierarchical demand of Murphy, Shleifer, Vishny (1989):

$$V = \int_0^\infty b(z)x(z)dz \text{ where } x(z) \in \{0, 1\}$$

- ▶ Assume  $b(z)/a(z)$  and  $b(z)/a^*(z)$  both decreasing so that households “prioritize” lower- $z$  goods
- ▶ The expenditure needed to consume goods  $[0, z]$  is  $E(z) \equiv \int_0^z p(s)ds$ . This is monotone in the utility level.
- ▶ The CDFs of household effective labor are  $F(h)$  and  $F(h^*)$

# Matsuyama (2000) trade-balance equation

Equation (8) is

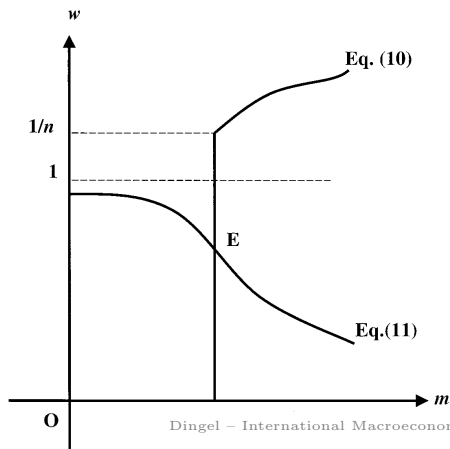
$$\begin{aligned} N \int_0^\infty \max \left\{ h - \int_0^m a(s) ds, 0 \right\} dF(h) \\ = N^* \int_0^\infty \min \left\{ \frac{h^*}{w}, \int_0^m a(s) ds \right\} dF^*(h^*) \end{aligned}$$

- ▶ Thus the “B” schedule in our figure will have a vertical portion for low values of  $w$
- ▶ Upward-sloping portion is where an increase in  $w$  causes poor Foreign households to cut imports. Greater  $m$  balances.
- ▶ Compared to DFS,  $F(h)$  and  $F^*(h^*)$  matter
- ▶ Compared to DFS, “B” schedule depends on  $a(z)$

# Matsuyama (2000): Homogeneous households

- ▶ “North-South trade” is  $a(z) > a^*(z) \forall z$ . Implies  $w < 1$ .
- ▶ With degenerate  $F$  and  $F^*$ , equation (8) simplifies to

$$\int_0^m a(s)ds = \begin{cases} \frac{N}{N+N^*} & \text{if } w \leq 1 + N^*/N \\ 1 - \frac{N^*}{wN} & \text{if } w > 1 + N^*/N \end{cases}$$



- ▶ In equilibrium,  $u^* > u > m$
- ▶  $\int_0^m a(s)ds = n \equiv \frac{N}{N+N^*}$   
[equation (10)]
- ▶  $wa(m) = a^*(m)$  [equation (2)  
or (11)]



# Matsuyama (2000): Comparative statics

Increase in  $N \rightarrow$

- ▶ increase in  $m$ : shift “B” to right, since  $n$  increases
- ▶ fall in  $w$ : walk down  $A$  schedule  $\rightarrow$  worse Home terms of trade
- ▶  $du < 0$  and  $du^* > 0$ : via factor terms of trade [differentiate (12) and (13)]

Northern productivity growth (which affects only  $A$ ):

- ▶  $dw < 0$  and  $du^* > 0$ : factor terms of trade improve for North and new goods are produced
- ▶  $du$  is more complicated, see uniform export productivity growth for  $du = 0$

Southern productivity growth (which affects both  $A$  and “B”):

- ▶  $dm > 0$  due to  $A$  rising and “B” shifting right
- ▶ ambiguous effects on  $w$ ,  $u$ , and  $u^*$
- ▶ immiserizing growth is possible

Uniform growth (for countries, not for  $z$ ):  $du$  depends on  $A$  slope

# Matsuyama (2000): A Multicountry World

- ▶ Claim (p.1116): “One advantage of the present model over the Dornbusch et al. model is that it is relatively straightforward to extend the model to incorporate more than two countries.”
- ▶ Assumption 4: For all  $j = 1, \dots, J - 1$ ,  $a_{j+1}(z)/a_j(z)$  is continuous and decreasing in  $z$ .
- ▶ Assumption 5:  $a_3(z) < a_2(z) < a_1(z) \forall z$ .

Evaluate the claim.

# Acemoglu and Ventura (2002): World Income Distribution

- ▶ Armington (1969) assumption is a Ricardian assumption
- ▶ Demands is CES with  $\epsilon > 1$  ruling out immiserizing growth
- ▶ Terms of trade are a source of DRS returns at a country level despite CRS technologies of AK model
- ▶ Empirics on terms of trade hampered by OVB of  $\Delta \ln \mu$  (technical progress in number of varieties)
- ▶ Conditional convergence is basis for initial level of income (relative to steady state) as IV
- ▶ Perhaps higher output growth due to accumulation worsens terms of trade

# Wrapping up

## Theory

- ▶ DFS 1977 is an extremely elegant version of the Ricardian model
- ▶ Strong assumptions allow extension to many countries
- ▶ More interesting patterns of demand make comparative statics more interesting
- ▶ In theory, terms of trade key to many of these questions

## Whither empirics?

- ▶ How to avoid two-country or MCS assumptions?
- ▶ How to take highly stylized model to the data?
- ▶ How to measure relevant objects?

## Food for thought:

- ▶ Matsuyama, “[Ricardian Trade Theory](#)”, 2008
- ▶ Eaton and Kortum, “[Putting Ricardo to Work](#)”, 2012