

Problem 1c.

$$L(\theta | x_1=18) = \binom{100}{18} \theta_1^{18} (1-\theta_1)^{82}$$

$\vdots$

$$\begin{aligned} L(\theta) = \prod_{i=1}^n f(x_i | \theta) &= \binom{100}{18} \cdot \binom{100}{11} \binom{100}{9} \binom{100}{25} \binom{100}{18} \binom{100}{19} \\ &\quad \cdot \theta_1^{18} (1-\theta_1)^{82} \\ &\quad \cdot \theta_2^{11} (1-\theta_2)^{89} \\ &\quad \cdot \theta_3^9 (1-\theta_3)^{91} \\ &\quad \cdot \theta_4^{25} (1-\theta_4)^{75} \\ &\quad \cdot \theta_5^{18} (1-\theta_5)^{82} \\ &\quad \cdot \theta_6^{19} (1-\theta_6)^{81} \end{aligned}$$

$$\begin{aligned} \ell(\theta) = \log L(\theta) &= C + 18 \log(\theta_1) + 82 \log(1-\theta_1) \\ &\quad + 11 \log(\theta_2) + 89 \log(1-\theta_2) \\ &\quad + 9 \log(\theta_3) + 91 \log(1-\theta_3) \\ &\quad + 25 \log(\theta_4) + 75 \log(1-\theta_4) \\ &\quad + 18 \log(\theta_5) + 82 \log(1-\theta_5) \\ &\quad + 19 \log(\theta_6) + 81 \log(1-\theta_6) \end{aligned}$$

$$\frac{\partial \ell}{\partial \theta_1} = 18 \cdot \frac{1}{\theta_1} + 82 \cdot \frac{-1}{1-\theta_1}$$

$$\text{To find MLE} \Rightarrow \frac{\partial \ell}{\partial \theta_1} = 0 \Rightarrow 0 = \frac{18}{\theta_1} - \frac{82}{1-\theta_1}$$

$$\Rightarrow \frac{82}{1-\theta_1} = \frac{18}{\theta_1}$$

$$82\theta_1 = 18 - 18\theta_1$$

$$\theta_1 = \frac{18}{100}$$

using same logic:

$$MLE(\theta_2) = \frac{11}{100}$$

$$MLE(\theta_3) = \frac{9}{100}$$

$$MLE(\theta_4) = \frac{25}{100}$$

$$MLE(\theta_5) = \frac{18}{100}$$

$$MLE(\theta_6) = \frac{19}{100}$$

Problem 16.

$$\text{we know that: } P(\text{model} | \text{data}) = \frac{P(\text{data} | \text{model}) \cdot P(\text{model})}{P(\text{data})}$$

$$\Rightarrow \theta | X \propto X | \theta \cdot \theta$$

$$= \prod_{i=1}^6 \binom{100}{x_i} \cdot \theta_1^{18} (1-\theta_1)^{82} \cdot \theta_2^{11} (1-\theta_2)^{89} \cdot \theta_3^9 (1-\theta_3)^{91} \\ \cdot \theta_4^{25} (1-\theta_4)^{75} \cdot \theta_5^{18} (1-\theta_5)^{82} \cdot \theta_6^{19} (1-\theta_6)^{81} \cdot \frac{1}{B(\alpha)}$$

$$L(\theta | X) = \sum_{i=1}^6 \log \binom{100}{x_i} + \frac{1}{B(\alpha)} + \\ 18 \log(\theta_1) + 82 \log(1-\theta_1) \\ + 11 \log(\theta_2) + 89 \log(1-\theta_2)$$

$$+ 9 \log(\theta_3) + 9 \log(1 - \theta_3)$$

$$+ 25 \log(\theta_4) + 75 \log(1 - \theta_4)$$

$$+ 18 \log(\theta_5) + 82 \log(1 - \theta_5)$$

$$+ 17 \log(\theta_6) + 81 \log(1 - \theta_6)$$