MSiA_420_HW1

2023-01-16

Question 1

See attached

Question 2

a

```
## New names:
## Rows: 18 Columns: 5
## -- Column specification
## -------------------------------------------------------- Delimiter: "," dbl
## (2): y, x lgl (3): ...3, ...4, ...5
## i Use `spec()` to retrieve the full column specification for this data. i
## Specify the column types or set `show_col_types = FALSE` to quiet this message.
## * `` -> `...3`
## * `` -> `...4`
## * `` -> `...5`
```

```
##
## Call:
## lm(formula = y ~ x, data = dat)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.8103 -1.3567  0.7347  1.5796  3.0106
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.42071    0.77963   6.953 3.25e-06 ***
## x            0.48964    0.04359  11.234 5.32e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.164 on 16 degrees of freedom
## Multiple R-squared:  0.8875, Adjusted R-squared:  0.8805
## F-statistic: 126.2 on 1 and 16 DF,  p-value: 5.317e-09
```

As seen above, the coefficients $\hat{\beta_0}$ and $\hat{\beta_1}$ are 5.4207 and 0.4896, respectively. Thus, the initial guesses of the form $\hat{\gamma_0} = \frac{1}{\hat{\beta_0}} = \frac{1}{5.4207} = 0.1845$, and $\hat{\gamma_1} = \frac{\hat{\beta_1}}{\hat{\beta_0}} = \frac{0.48964}{5.4207} = 0.0903$

b

```
## [1] 28.13688 12.57428
```

```
## Nonlinear regression model
##   model: y ~ ((p1 * x)/(p2 + x))
##    data: dat
##    p1    p2
## 28.14 12.57
##  residual sum-of-squares: 4.302
##
## Number of iterations to convergence: 9
## Achieved convergence tolerance: 9.921e-06
```

Question 3

a

```
##           [,1]      [,2]
## [1,]  15.37455 -13.73842
## [2,] -13.73842  13.92197
```

```
##           [,1]      [,2]
## [1,] 0.5502797 0.5430248
## [2,] 0.5430248 0.6076944
```

```
## [1] 0.7418084 0.7795475
```

b

```
##          p1        p2
## p1 0.529951 0.5202800
## p2 0.520280 0.5822471
```

Although not exactly the same, the covariance matrices are very similar.

c

```
## [1] 26.68294
```

```
## [1] 29.59083
```

```
## [1] 11.04637
```

```
## [1] 14.10219
```

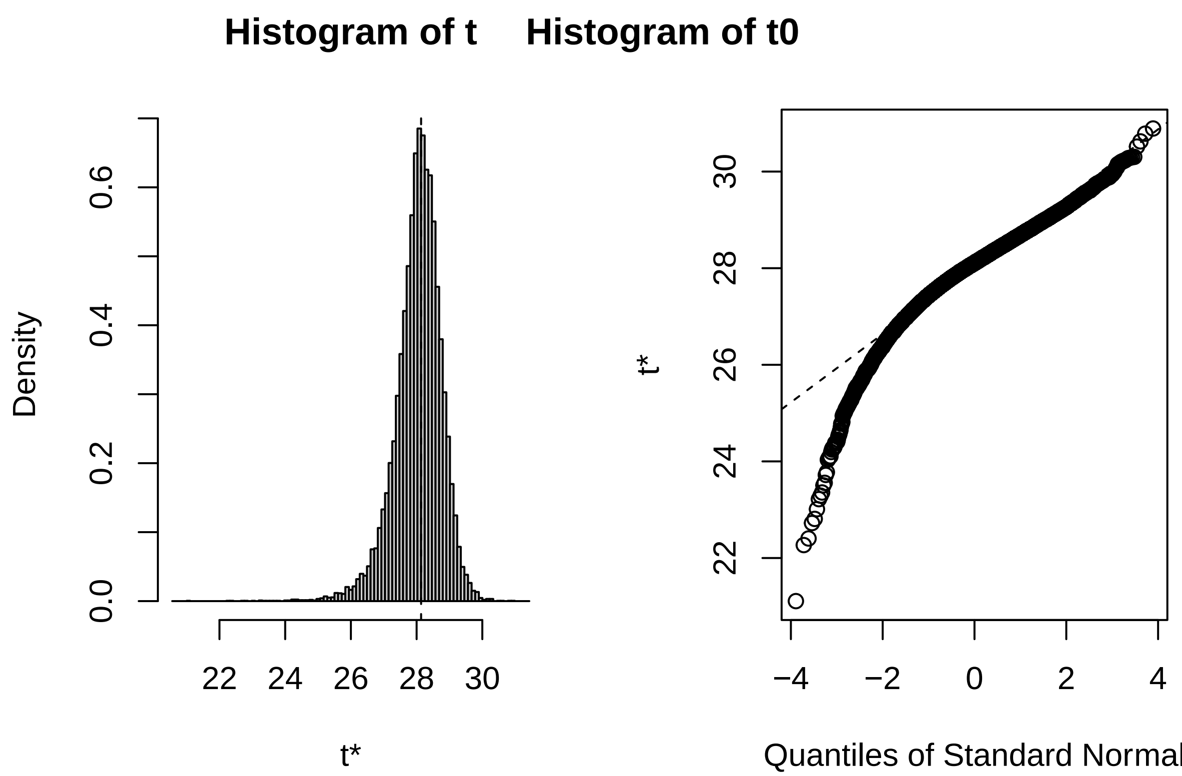
```
##       2.5 %   97.5 %
## p1 26.71021 29.56383
## p2 11.07887 14.06997
```

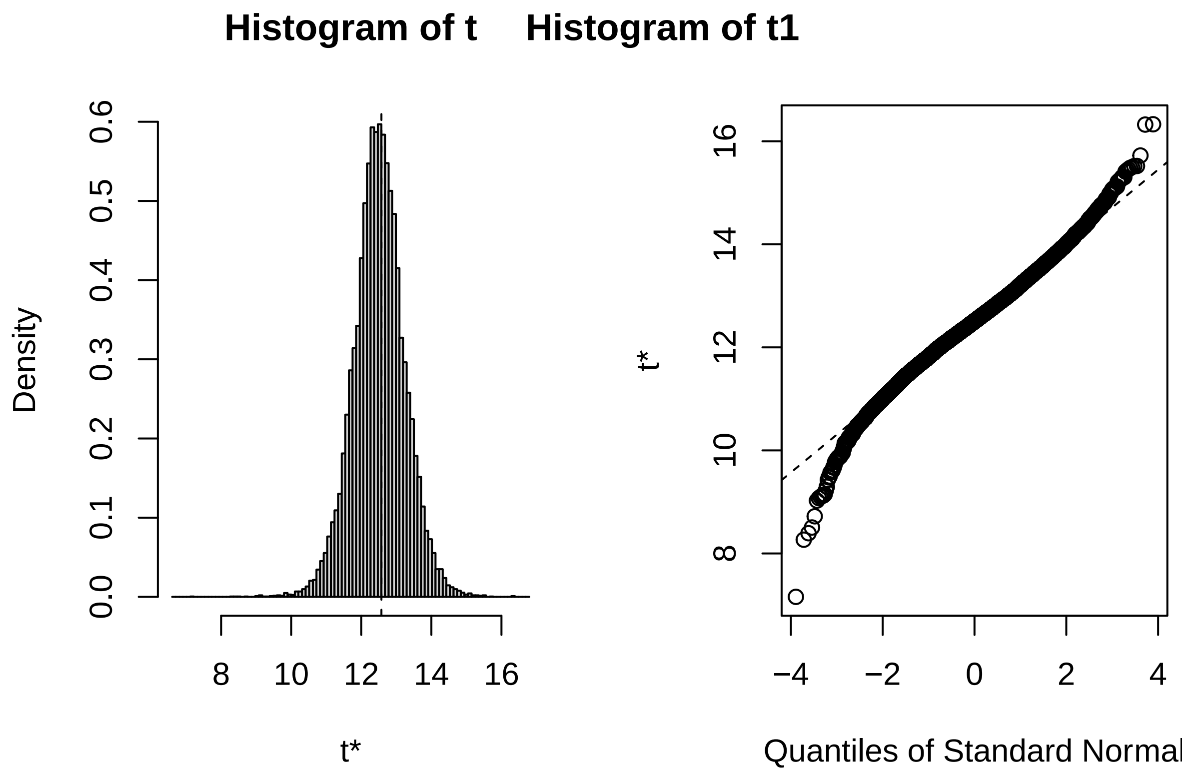
As seen from the above comparison, the “crude” confidence interval is similar to the output by R, but it is wider.

Question 4

a

```
##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
##
## Call:
## boot(data = dat, statistic = dat_fit, R = 20000, theta0 = c(0.1845,
##     0.0903))
##
##
## Bootstrap Statistics :
##     original       bias    std. error
## t1* 28.13688 -0.08847224   0.7065388
## t2* 12.57428 -0.06407480   0.7347188
```





b

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 20000 bootstrap replicates
##
## CALL :
## boot.ci(boot.out = dat_boot, conf = 0.95, type = "norm", index = 1)
##
## Intervals :
## Level      Normal
## 95%   (26.84, 29.61 )
## Calculations and Intervals on Original Scale
```

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 20000 bootstrap replicates
##
## CALL :
## boot.ci(boot.out = dat_boot, conf = 0.95, type = "norm", index = 2)
##
## Intervals :
## Level      Normal
## 95%   (11.20, 14.08 )
## Calculations and Intervals on Original Scale
```

c

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 20000 bootstrap replicates
##
## CALL :
## boot.ci(boot.out = dat_boot, conf = 0.95, type = "basic", index = 1)
##
## Intervals :
## Level      Basic
## 95%   (27.03, 29.83 )
## Calculations and Intervals on Original Scale
```

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 20000 bootstrap replicates
##
## CALL :
## boot.ci(boot.out = dat_boot, conf = 0.95, type = "basic", index = 2)
##
## Intervals :
## Level      Basic
## 95%   (11.18, 14.11 )
## Calculations and Intervals on Original Scale
```

d

For the most part, the confidence intervals in part b and part c agree. The confidence interval for γ_1 is much closer since the histogram is pretty much completely normal. This means in part b where we use normal approximation, the confidence intervals will be much closer. For γ_0, we can see from part a that the histogram is slightly left skewed. Thus, the reflected confidence interval is slightly shifted to the right compared to the "crude" confidence interval. Note that the histogram for γ_0 is still very close to normal despite skewedness, and thus the confidence intervals do not differ by much.

Appendix

This section is to be used for including your R code. The following lines of code will take care of it. Please make sure to comment your code appropriately - in particular, demarcating codes belonging to different questions. Among other things, it will be easier for you to debug your own code.

```
labs = knitr::all_labels()
labs = labs[!labs %in% c("setup","getlabels", "allcode")]
```

```
########### QUESTION 2 begins here ###################
dat <- read_csv("HW1_data.csv")
dat <- dat %>%
  select(y,x)
m1 <- lm(y~x, data = dat)
summary(m1)
est <- c(0.1845, 0.0903)
x_i <- dat$x
y_i <- dat$y
```

```r
fn <- function(p) {
    yhat <- (p[1] * x_i) / (p[2] + x_i)
    sum((y_i-yhat)^2)
}
out_nlm <- nlm(fn, p = est,hessian=TRUE)
theta_nlm <- out_nlm$estimate  #parameter estimates
theta_nlm
out_nls <- nls(y~((p1 * x) / (p2 + x)), data = dat, start = list(p1 = 0.1845, p2 = 0.0903))
out_nls
########### QUESTION 3 begins here ###################
mse <- out_nlm$minimum / (length(y_i) - length(theta_nlm))
info_mat <- out_nlm$hessian / 2 / mse
cov_theta <- solve(info_mat)
se <- sqrt(diag(cov_theta))
info_mat
cov_theta
se
vcov(out_nls)
p1_left <- theta_nlm[1] - 1.96 * se[1]
p1_right <- theta_nlm[1] + 1.96 * se[1]
p2_left <- theta_nlm[2] - 1.96 * se[2]
p2_right <- theta_nlm[2] + 1.96 * se[2]
p1_left
p1_right
p2_left
p2_right
confint.default(out_nls)
dat_fit <- function(Z, i, theta0){
  Zboot <- Z[i, ]
  x <- Zboot[[2]]
  y <- Zboot[[1]]
  fn <- function(p){
    yhat <- (p[1] * x) / (p[2] + x)
    sum((y - yhat)^2)
  }
  out <- nlm(fn, p = theta0)
  theta <- out$estimate
}
dat_boot <- boot(dat, dat_fit, R = 20000, theta0 = c(0.1845, 0.0903))
dat_boot
plot(dat_boot, index = 1)
title(main = "Histogram of t0")
plot(dat_boot, index = 2)
title(main = "Histogram of t1")
# gamma 0
boot.ci(dat_boot, conf = 0.95, type = "norm", index = 1)

#gamma 1
boot.ci(dat_boot, conf = 0.95, type = "norm", index = 2)
# gamma 0
boot.ci(dat_boot, conf = 0.95, type = "basic", index = 1)

#gamma 1
```

```r
boot.ci(dat_boot, conf = 0.95, type = "basic", index = 2)
```