# Toxicity-Aware Market-Making: Strategy Proposal

Henry Ramstad

February 3, 2026

## 1 Strategy Overview

This document specifies a market-making strategy that conditions quoting decisions on estimated order-flow toxicity. Let $\mathcal{F}_t$ denote the filtration of observed order flow up to time $t$. We define a toxicity indicator $Z_t \in \{0,1\}$ where $Z_t = 1$ indicates informed flow. The strategy computes $\hat{p}_{\text{toxic},t} = \mathbb{P}(Z_t = 1 \mid \mathcal{F}_t)$ and suppresses quotes when expected fill PnL is negative.

## 2 Stochastic Model Components

### 2.1 Latency

**Order send latency:**

$$L \sim \mathcal{N}^+(\mu_L, \sigma_L^2), \quad \mu_L = 10 \text{ ms}, \quad \sigma_L = 2.5 \text{ ms} \tag{1}$$

where $\mathcal{N}^+$ denotes a normal distribution truncated at zero.

**Cancel latency:**

$$L_{\text{cancel}} \sim \mathcal{N}^+(\mu_c, \sigma_c^2), \quad \mu_c = 8 \text{ ms}, \quad \sigma_c = 2 \text{ ms} \tag{2}$$

### 2.2 Queue Mechanics

**Initial queue position:**

$$Q_0 \sim \max\left(1, \mathcal{N}(\mu_Q, \sigma_Q^2)\right), \quad \mu_Q = 100 \text{ shares}, \quad \sigma_Q = 50 \text{ shares} \tag{3}$$

**Market order arrivals:**

$$N(t) \sim \text{Poisson}(\lambda_{\text{MO}} \cdot t) \tag{4}$$

**Fill probability over cancel window:**

$$P(\text{fill}) = 1 - e^{-\lambda_{\text{MO}} \cdot L_{\text{cancel}}} \tag{5}$$

### 2.3 Toxicity Classifier

Define rolling-window features over interval $\Delta t$:

$$r_{\text{cancel}} = \frac{\#\text{cancels}}{\Delta t} \tag{6}$$

$$\text{OBI} = \frac{Q_{\text{bid}} - Q_{\text{ask}}}{Q_{\text{bid}} + Q_{\text{ask}}} \tag{7}$$

**Toxicity estimate:**

$$\hat{p}_{\text{toxic}} = \sigma\left(\alpha_1 \cdot r_{\text{cancel}} + \alpha_2 \cdot |\text{OBI}| + \alpha_3 \cdot \hat{\sigma}_{\text{short}}\right) \tag{8}$$

where $\sigma(x) = (1 + e^{-x})^{-1}$.

### 2.4 Adverse Selection

**Post-fill price movement:**

$$\Delta S \mid Z = 1 \sim \mathcal{N}(-\mu_a, \sigma_a^2) \tag{9}$$

$$\Delta S \mid Z = 0 \sim \mathcal{N}(0, \sigma_b^2) \tag{10}$$

with $\mu_a \approx s/2$ where $s$ is the quoted spread.

### 2.5 Price Dynamics

**Midprice (short horizon):**

$$dS_t = \sigma\, dW_t \quad \Rightarrow \quad \Delta S \sim \mathcal{N}(0, \sigma^2 \Delta t) \tag{11}$$

### 2.6 Inventory

**Inventory process:**

$$I_{t+1} = I_t + F_t, \quad F_t \in \{-1, 0, +1\} \tag{12}$$

**Risk penalty:**

$$\phi(I) = \gamma I^2 \tag{13}$$

## 3 Quote Decision Rule

**Expected PnL per quote:**

$$\mathbb{E}[\text{PnL}] = P(\text{fill}) \cdot \left(\frac{s}{2} - \hat{p}_{\text{toxic}} \cdot \mu_a\right) - \gamma I_t^2 \tag{14}$$

**Decision:**

$$\mathbf{1}_{\text{quote}} = \begin{cases} 1 & \text{if } \mathbb{E}[\text{PnL}] > 0 \text{ and } |I_t| < I_{\max} \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

## 4 Hypotheses

Let $\pi^B$ denote the baseline policy (unconditional quoting) and $\pi^T$ the toxicity-aware policy. Let $\mathcal{P}$ denote a fixed realized market path.

**Hypothesis 1** (Sharpe Improvement)**.**

$$\mathbb{E}\left[\text{Sharpe}(\pi^T) \mid \mathcal{P}\right] > \mathbb{E}\left[\text{Sharpe}(\pi^B) \mid \mathcal{P}\right] \tag{16}$$

**Hypothesis 2** (Adverse Selection Reduction)**.**

$$\mathbb{E}\left[\sum_t \mathbf{1}_{Z_t=1} \cdot |\Delta S_t| \;\middle|\; \pi^T, \mathcal{P}\right] < \mathbb{E}\left[\sum_t \mathbf{1}_{Z_t=1} \cdot |\Delta S_t| \;\middle|\; \pi^B, \mathcal{P}\right] \tag{17}$$

**Hypothesis 3** (Inventory Variance Reduction)**.**

$$Var(I_t \mid \pi^T, \mathcal{P}) < Var(I_t \mid \pi^B, \mathcal{P}) \tag{18}$$

**Hypothesis 4** (Cross-Sectional Robustness)**.** *For securities $i = 1, \ldots, N$:*

$$\frac{1}{N} \sum_{i=1}^{N} \mathbf{1} \left\{ \mathrm{PnL}(\pi_i^T) > \mathrm{PnL}(\pi_i^B) \right\} > 0.8 \tag{19}$$

**Hypothesis 5** (Monte Carlo Dominance)**.** *Across $M = 100$ replications:*

$$\frac{1}{M} \sum_{m=1}^{M} \mathbf{1} \left\{ \mathrm{PnL}^{(m)}(\pi^T) > \mathrm{PnL}^{(m)}(\pi^B) \right\} \geq 0.9 \tag{20}$$

# 5 Testing Procedure

## 5.1 Data

One trading day of NYSE Integrated Feed ($\approx 10^8$ order events). The realized path $\mathcal{P} = \{S_t, \mathcal{O}_t\}_{t \in [0,T]}$ is held fixed.

## 5.2 Monte Carlo Resampling

For each security $i$ and replication $m = 1, \ldots, M$:

1. Sample $L^{(m)} \sim \mathcal{N}^+(10, 2.5^2)$
2. Sample $Q_0^{(m)} \sim \max(1, \mathcal{N}(100, 50^2))$
3. Generate $N^{(m)}(t) \sim \mathrm{Poisson}(\hat{\lambda}_{\mathrm{MO}} t)$
4. Simulate execution for $\pi^B$ and $\pi^T$
5. Record $\mathrm{PnL}_i^{(m)}$, $\mathrm{Sharpe}_i^{(m)}$, $\mathrm{Var}(I)_i^{(m)}$

## 5.3 Stratification

**Cross-sectional:** Securities grouped by ADV, mean spread, and mean queue depth. Median $\Delta$PnL and win-rate reported per bucket.

# 6 Constraints

- Latency: $L \sim \mathcal{N}^+(10, 2.5^2)$ ms, $L_c \sim \mathcal{N}^+(8, 2^2)$ ms
- Queue: FIFO priority, stochastic initial position
- Inventory: $|I_t| \leq I_{\max}$
- Fills: Partial fills via Poisson depletion

## 7 Benchmark

Baseline policy $\pi^B$: continuous quoting on all securities without conditioning on $\hat{p}_{\text{toxic}}$.

## 8 Objectives

1. $\text{Sharpe}(\pi^T) > \text{Sharpe}(\pi^B)$
2. $\text{Var}(I \mid \pi^T) < \text{Var}(I \mid \pi^B)$
3. Win-rate $\geq 0.9$ across Monte Carlo replications

## 9 Scope

Evaluation is conducted on a single realized market path. Results characterize policy performance conditional on observed order flow and do not extend to out-of-sample market conditions. Cross-sectional variation across securities reflects heterogeneity in microstructure characteristics rather than independent regime sampling.