高等数学笔记

西安交通大学 智能制造工程 杨承轩

2025.6

目录

写在前面

啦啦啦啦啦，笔者工科数学分析-2 期中考试得分足足 81.5，所以这也要看嘛～
反正也没什么好东西 ()

杨承轩
乙巳，芒种，碑林

第 1 部分　一元微分与级数

1.1　函数的极限

I　两个重要极限

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to\infty}(1+\frac{1}{x})^x=e$$

II　无穷小量

$$\sin x\sim x\sim\tan x$$

$$1-\cos x\sim\frac{1}{2}x^2$$

$$\ln(1+x)\sim x$$

$$e^x-1\sim x$$

$$(1+x)^\alpha-1\sim\alpha x$$

1.2　求导法则

I　复合函数求导法则

$$\frac{\mathrm{d}y}{\mathrm{d}x}=f'(u)g'(x)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}u}\cdot\frac{\mathrm{d}u}{\mathrm{d}x}$$

II　反函数求导法则

$$(f^{-1})'(x)-\frac{1}{f'(y)}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}}$$

$(C)' = 0$
$(x^\alpha)' = \alpha x^{\alpha-1}$
$(\sin x)' = \cos x$
$(\cos x)' = -\sin x$
$(\tan x)' = \sec^2 x$
$(\cot x)' = -\csc^2 x$
$(\sec x)' = \sec x \tan x$
$(\csc x)' = -\csc x \cot x$
$(a^x)' = a^x \ln a (a > 0 且 a \neq 1)$
$(e^x)' = e^x$
$(\log_a x)' = \frac{1}{x \ln a} (a > 0 且 a \neq 1)$
$(\ln x)' = \frac{1}{x}$
$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
$(\arctan x)' = \frac{1}{1+x^2}$
$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$

表 1: 基本导数表

III 基本导数表

IV n 阶导数公式

$(e^x)^{(n)} = e^x$
$(\sin x)^{(n)} = \sin(x + n \cdot \frac{\pi}{2})$
$(\cos x)^{(n)} = \cos(x + n \cdot \frac{\pi}{2})$
$(x^\alpha)^{(n)} = \alpha(\alpha-1)\cdots(\alpha-n+1)x^{(\alpha-n)} (\alpha \in \mathbf{R}, x > 0)$
$[\ln(1+x)]^{(n)} = (-1)^{n-1} \frac{(n-1)!}{(1+x)^n} (x > -1)$

表 2: n 阶导数公式

V 参数方程求导法则

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\dot{y}}{\dot{x}}$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{\dot{x}^3}$$

1.3 微分

I 近似公式

$$e^x \approx 1 + x$$

$$\sin x \approx x$$

$$\tan x \approx x$$

$$(1+x)^\alpha \approx 1 + \alpha x$$

$$\ln(1+x) \approx x$$

II Lagrange 定理

$$f(b) - f(a) = f'(\xi)(b-a)$$

III Cauchy 定理

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(\xi)}{g'(\xi)}$$

IV L'Hospital 法则

$$\lim_{x \to x_0^+} \frac{f(x)}{g(x)} = \lim_{x \to x_0^+} \frac{f'(x)}{g'(x)} = a$$

1.4 Taylor 定理

I 带 Peano 余项的 Taylor 定理

$$f(x) = \Sigma_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k + o((x-x_0)^n)$$

II 带 Lagrange 余项的 Taylor 定理

$$f(x) = \Sigma_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k + \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}$$

III Maclaurin 公式

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \frac{x^{n+1}}{(n+1)!}e^{\theta x}, x \in (-\infty, +\infty), \theta \in (0,1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^{k-1}\frac{x^{2k-1}}{(2k-1)!} + (-1)^k \frac{\cos\theta x}{(2k+1)!}x^{2k+1}, x \in (-\infty, +\infty), \theta \in (0,1)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^k \frac{x^{2k}}{(2k)!} + (-1)^{k+1}\frac{\cos\theta x}{(2k+2)!}x^{2k+2}, x \in (-\infty, +\infty), \theta \in (0,1)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1}\frac{x^n}{n} + (-1)^n \frac{x^{n+1}}{(n+1)(1+\theta x)^{n+1}}, x \in (-1, +\infty), \theta \in (0,1)$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \cdots + \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n + \frac{\alpha(\alpha-1)\cdots(\alpha-n)}{(n+1)!}\frac{x^{n+1}}{(1+\theta x)^{n+1-\alpha}},$$

$$x \in (-1, +\infty), \theta \in (0,1)$$

1.5 幂级数

I 收敛半径

$$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|$$

$$R = \lim_{n\to\infty}\frac{1}{\sqrt[n]{|a_n|}}$$

II 函数展开成幂级数

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots, x \in (-\infty, +\infty)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^k \frac{x^{2k+1}}{(2k+1)!} + \cdots, x \in (-\infty, +\infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^k \frac{x^{2k}}{(2k)!} + \cdots, x \in (-\infty, +\infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1}\frac{x^n}{n} + \cdots, x \in (-1, 1]$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \cdots + \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n + \cdots, x \in (-1, 1)$$

III　Euler 公式

$$e^{\mathrm{i}\pi} = \cos x + \mathrm{i} \sin x$$

1.6　Fourier 级数

I　Euler-Fourier 公式

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx \mathrm{d}x \quad (k = 0, 1, 2, \cdots)$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin kx \mathrm{d}x \quad (k = 1, 2, \cdots)$$

II　周期为 $2l$ 的函数展开

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} \mathrm{d}x \quad (n = 0, 1, 2, \cdots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} \mathrm{d}x \quad (n = 1, 2, \cdots)$$

第 2 部分　一元积分与微分方程

2.1　微积分基本公式

I　Newton-Leibniz 公式

$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a) = F(x)|_a^b$$

II　微积分第一基本定理

$$\Phi'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\int_a^x f(t)\mathrm{d}t = f(x)$$

III　基本积分表

$\int k\mathrm{d}x = kx + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1} + C(\alpha \neq 1)$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$
$\int \sin x\mathrm{d}x = \cos x + C$
$\int \sec^2 x\mathrm{d}x = \tan x + C$
$\int \csc^2 x\mathrm{d}x = -\cot x + C$
$\int \sec x \tan x\mathrm{d}x = \sec x + C$
$\int \csc x \cot x\mathrm{d}x + -\csc x + C$
$\int a^x\mathrm{d}x = \frac{a^x}{\ln a} + C(a > 0, a \neq 1)$
$\int e^x\mathrm{d}x = e^x + C$
$\int \cos x\mathrm{d}x = \sin x + C$
$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$
$\int \frac{\mathrm{d}x}{1+x^2} = \arctan x + C$
$\int \mathrm{sh}\, x\mathrm{d}x = \mathrm{ch}x + C$
$\int \mathrm{ch}\, x\mathrm{d}x = \mathrm{sh}x + C$

表 3: 基本积分表

2.2　基本积分法

I　换元积分法

$$\int f[\varphi(x)]\varphi'(x)\mathrm{d}x = (\int f(u)\mathrm{d}u)_{u=\varphi(x)}$$

$$\int f(x)\mathrm{d}x = (\int f[\varphi(t)]\varphi'(t)\mathrm{d}t)_{t=\varphi^{-1}(x)}$$

$$\int_a^b f(x)\mathrm{d}x = \int_\alpha^\beta f[\varphi(t)]\varphi'(t)t$$

II 分部积分法

$$\int u\mathrm{d}v = uv - \int v\mathrm{d}u$$

$$\int_a^b u\mathrm{d}v = uv|_a^b - \int_a^b v\mathrm{d}u$$

2.3 线性微分方程

I 一阶线性微分方程通解

$$y = e^{-\int P(x)\mathrm{d}x}(\int Q(x)e^{\int P(x)\mathrm{d}x}\mathrm{d}x + C)$$

II 特征根及其在通解中的对应项

特征根	通解中的对应项
单重根 λ	$Ce^{\lambda t}$
k 重实根 λ	$e^{\lambda t}(C_1 + C_2 t + \cdots + C_k t^{k-1})$
一对共轭单复根 $\alpha \pm \mathrm{i}\beta$	$e^{\alpha t}(C_1 \cos\beta t + C_2 \sin\beta t)$
一对 k 重共轭复根 $\alpha \pm \mathrm{i}\beta$	$e^{\alpha t}[(C_{11} + C_{12}t + \cdots + C_{1k}t^{k-1})\cos\beta t +$ $(C_{21} + C_{22}t + \cdots + C_{2k}t^{k-1})\sin\beta t]$

表 4: 特征根及其在通解中的对应项

III 特解的设置形式

$F(t)$ 的类型	应设置特解 $x^*(t)$ 的形式	
m 次多项式 $\varphi(t)$	0 不是特征值	$x^* = Z(t)$
	0 是 k 重特征值	$x^* = t^k Z(t)$
$\varphi(t)e^{\mu t}$(t)	μ 不是特征值	$x^* = Z(t)e^{\mu t}$
	μ 是 k 重特征值	$x^* = t^k Z(t)e^{\mu t}$
$\mu(t)e^{\mu t}\cos\nu t$ 或 $\varphi(t)e^{\mu t}\sin\nu t$	$\mu + \mathrm{i}\nu$ 不是特征值	$x^* = e^{\mu t}[Z_1(t)\cos\nu t + Z_2(t)\sin\nu t]$
	$\mu + \mathrm{i}\nu$ 是 k 重特征值	$x^* = t^k e^{\mu t}[Z_1(t)\cos\nu t + Z_2(t)\sin\nu t]$

表 5: 特解的设置形式

2.4 线性微分方程组

I 通解表达式

$$\boldsymbol{x}(t) = \boldsymbol{X}(t)\boldsymbol{C} + \boldsymbol{X}(t)\int_{t_0}^{t} \boldsymbol{X}^{-1}(\tau)\boldsymbol{f}(\tau)\mathrm{d}\tau$$

II 特解表达式

$$\boldsymbol{x}(t) = \boldsymbol{X}(t)\boldsymbol{X}^{-1}(t_0)\boldsymbol{x}_0 + \boldsymbol{X}(t)\int_{t_0}^{t} \boldsymbol{X}^{-1}(\tau)\boldsymbol{f}(\tau)\mathrm{d}\tau$$

III 矩阵有 n 个线性无关的特征向量

$$\boldsymbol{X}(t) = (\boldsymbol{r}_1 e^{\lambda_1 t}, \boldsymbol{r}_2 e^{\lambda_2 t}, \cdots, \boldsymbol{r}_n e^{\lambda_n t})$$

IV 矩阵没有 n 个线性无关的特征向量

$$\boldsymbol{x}(t) = e^{\lambda_i t}(\boldsymbol{r}_0 + \frac{t}{1!}\boldsymbol{r}_1 + \frac{t^2}{2!}\boldsymbol{r}_2 + \cdots + \frac{t^{n_i-1}}{(n_i-1)!}\boldsymbol{r}_{n_i-1})$$

第 3 部分　多元微积分

3.1　导数与微分

I　方向导数

$$\left.\frac{\partial f}{\partial \boldsymbol{l}}\right|_{(x_0,y_0)} = f_x(x_0,y_0)\cos\alpha + f_y(x_0,y_0)\cos\beta$$

II　梯度

$$\mathbf{grad} f(x_0,y_0) = f_x(x_0,y_0)\boldsymbol{i} + f_y(x_0,y_0)\boldsymbol{j}$$

III　全微分

$$\mathrm{d}y = \frac{\partial y}{\partial x_1}\mathrm{d}x_1 + \frac{\partial y}{\partial x_2}\mathrm{d}x_2 + \cdot + \frac{\partial y}{\partial x_n}\mathrm{d}x_n$$

IV　偏导数

$$\frac{\partial y}{\partial x_j} = \frac{\partial y}{\partial u_1}\frac{\partial u_1}{\partial x_j} + \frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial x_j} + \cdots + \frac{\partial y}{\partial u_m}\frac{\partial u_m}{\partial x_j}, j = 1,2,\cdots,n$$

3.2　几何应用

I　曲线的切线方程

$$\boldsymbol{\rho} = \boldsymbol{r}(t_0) + t\dot{\boldsymbol{r}}(t_0)$$

$$\frac{x - x(t_0)}{\dot{x}(t_0)} = \frac{y - y(t_0)}{\dot{y}(t_0)} = \frac{z - z(t_0)}{\dot{z}(t_0)}$$

$$\frac{x - x_0}{1} = \frac{y - y(x_0)}{\dot{y}(x_0)} = \frac{z - z(x_0)}{\dot{z}(x_0)}$$

II　弧长

$$s = \int_\alpha^\beta \|\dot{\boldsymbol{r}}(t)\| = \int_\alpha^\beta \sqrt{[\dot{x}(t)]^2 + [\dot{y}(t)]^2 + [\dot{z}(t)]^2}\mathrm{d}t$$

$$s = \int_\alpha^\beta \sqrt{[\dot{x}(t)]^2 + [\dot{y}(t)]^2}\mathrm{d}t$$

$$s = \int_a^b \sqrt{1 + [y'(x)]^2}\mathrm{d}x$$

III 曲面的切平面方程

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

$$F_x(P_0)(x - x_0) + F_y(P_0)(y - y_0) + F_z(P_0)(z - z_0) = 0$$

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

IV 曲率

$$\kappa(s) = \|\boldsymbol{r}''(s)\| = \frac{\|\dot{\boldsymbol{r}}(t) \times \ddot{\boldsymbol{r}}(t)\|}{\|\dot{\boldsymbol{r}}(t)\|^3}$$

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{[(\dot{x})^2 + (\dot{y})^2]^{\frac{3}{2}}} = \frac{|y''|}{[1 + (y')^2]^{\frac{3}{2}}}$$

V 曲率半径与曲率圆

$$\boldsymbol{r}_Q = \boldsymbol{r}_P(s) + R(s)\boldsymbol{N}(s)$$

$$R = \frac{1}{\kappa}$$

VI 渐伸线与渐屈线

$$\boldsymbol{\rho}(s) = \boldsymbol{r}(s) + (a - s)\boldsymbol{T}(s)$$

VII 挠率

$$\tau(s) = \frac{[\boldsymbol{r}'(s) \quad \boldsymbol{r}''(s) \quad \boldsymbol{r}'''(s)]}{\|\boldsymbol{r}''(s)\|^2}$$

$$\tau(t) = \frac{[\dot{\boldsymbol{r}}(t) \quad \ddot{\boldsymbol{r}}(t) \quad \dddot{\boldsymbol{r}}(t)]}{\|\dot{\boldsymbol{r}}(t) \times \ddot{\boldsymbol{r}}(t)\|^2}$$

3.3 二重积分

I 直角坐标系

$$V = \iint_{(\sigma)} f(x, y)\mathrm{d}\sigma = \int_a^b [\int_{y_1(x)}^{y_2(x)} f(x, y)\mathrm{d}y]\mathrm{d}x$$

$$V = \iint_{(\sigma)} f(x, y)\mathrm{d}\sigma = \int_c^d [\int_{x_1(y)}^{x_2(y)} f(x, y)\mathrm{d}x]\mathrm{d}y$$

II 极坐标系

$$\iint_{(\sigma)} f(x,y)\mathrm{d}\sigma = \iint_{(\sigma)} f(\rho\cos\theta, \rho\sin\theta)\rho\mathrm{d}\rho\mathrm{d}\theta$$

III 曲线坐标

$$\iint_{(\sigma)} f(x,y)\mathrm{d}\sigma = \iint_{(\sigma')} f[x(u,v), y(u,v)]\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\mathrm{d}\sigma'$$

3.4 三重积分

I 直角坐标系

$$\iiint_{(V)} f(x,y,z)\mathrm{d}V = \iint_{(\sigma)} [\int_{z_1(x,y)}^{z_2(x,y)} f(x,y,z)\mathrm{d}z]\mathrm{d}\sigma$$

$$\iiint_{(V)} f(x,y,z)\mathrm{d}V = \int_a^b [\iint_{(\sigma_z)} f(x,y,z)\mathrm{d}\sigma]\mathrm{d}z$$

II 曲线坐标

$$\iiint_{(V)} f(x,y,z)\mathrm{d}V = \iiint_{(V)} f[x(u,v,w), y(u,v,w), z(u,v,w)]\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right|\mathrm{d}u\mathrm{d}v\mathrm{d}w$$

III 柱面坐标

$$\iiint_{(V)} f(x,y,z)\mathrm{d}V = \iiint_{(V)} f(\rho\cos\theta, \rho\sin\theta, z)\rho\mathrm{d}\rho\mathrm{d}\theta\mathrm{d}z$$

IV 球面坐标

$$\iiint_{(V)} f(x,y,z)\mathrm{d}V = \iiint_{(V)} f(r\sin\varphi\cos\theta, r\sin\varphi\sin\theta, r\cos\varphi)r^2\sin\varphi\mathrm{d}r\mathrm{d}\varphi\mathrm{d}\theta$$

3.5 第一型线面积分

I 第一型线积分

$$\int_{(C)} f(x,y,z)\mathrm{d}s = \int_\alpha^\beta f[x(t), y(t), z(t)]\sqrt{\dot{x}^2(t) + \dot{y}^2(t) + \dot{z}^2(t)}\mathrm{d}t$$

II　第一型面积分

$$\iint_{(S)} f(x,y,z)\mathrm{d}S = \iint_{(\sigma)} f[x(u,v),y(u,v),z(u,v)]\|\boldsymbol{r}_u \times \boldsymbol{r}_v\|\mathrm{d}u\mathrm{d}v$$

$$\iint_{(S)} f(x,y,z)\mathrm{d}S = \iint_{(\sigma)} f[x,y,z(x,y)]\sqrt{1+z_x^2+z_y^2}\mathrm{d}x\mathrm{d}y$$

3.6　第二型线面积分

I　第二型线积分

$$\int_{(C)} \boldsymbol{A}(M)\cdot\mathbf{d}\boldsymbol{s} = \int_{(C)} P(x,y,z)\mathrm{d}x + Q(x,y,z)\mathrm{d}y + R(x,y,z)\mathrm{d}z$$

$$\int_{(C)} \boldsymbol{A}(M)\cdot\mathbf{d}\boldsymbol{s} = \int_{(C)} (P\cos\alpha + Q\cos\beta + R\cos\gamma)\mathrm{d}s$$

II　第二型面积分

$$\iint_{(S)} \boldsymbol{A}(M)\cdot\mathbf{d}\boldsymbol{S} = \iint_{(S)} P(x,y,z)\mathrm{d}y\wedge\mathrm{d}z + Q(x,y,z)\mathrm{d}z\wedge\mathrm{d}x + R(x,y,z)\mathrm{d}x\wedge\mathrm{d}y$$

$$\iint_{(S)} R(x,y,z)\mathrm{d}x\wedge\mathrm{d}y = \pm\iint_{(\sigma_{xy})} R[x,y,z(x,y)]\mathrm{d}x\mathrm{d}y$$

$$\iint_{(S)} \boldsymbol{A}(M)\cdot\mathbf{d}\boldsymbol{S} = \iint_{(S)} P(x,y,z)\cos\alpha + Q(x,y,z)\cos\beta + R(x,y,z)\cos\gamma$$

3.7　三大积分公式

I　Green 公式

$$\iint_{(\sigma)} (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})\mathrm{d}\sigma = \oint_{(+C)} P(x,y)\mathrm{d}x + Q(x,y)\mathrm{d}y$$

II　Gauss 公式

$$\iiint_{(V)} (\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z})\mathrm{d}V = \oiint_{(S)} P\mathrm{d}y\wedge\mathrm{d}z + Q\mathrm{d}z\wedge\mathrm{d}x + R\mathrm{d}x\wedge\mathrm{d}y$$

$$\iiint_{(V)} \nabla\cdot\boldsymbol{A}\mathrm{d}V = \oiint_{(S)} \boldsymbol{A}\cdot\mathbf{d}\boldsymbol{S}$$

III 散度

$$\mathrm{div}\boldsymbol{A} = \nabla \cdot \boldsymbol{A} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\iiint_{(V)} \mathrm{div}\boldsymbol{A}\mathrm{d}V = \oiint_{(S)} \boldsymbol{A} \cdot \mathbf{d}\boldsymbol{S}$$

IV Stokes 公式

$$\oint_{(C)} P\mathrm{d}x + Q\mathrm{d}y + R\mathrm{d}z = \iint_{(S)} (\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z})\mathrm{d}y \wedge \mathrm{d}z + (\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x})\mathrm{d}z \wedge \mathrm{d}x + (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})\mathrm{d}x \wedge \mathrm{d}y$$

$$\oint_{(C)} \boldsymbol{A} \cdot \mathbf{d}\boldsymbol{s} = \iint_{(S)} (\nabla \times \boldsymbol{A}) \cdot \mathbf{d}\boldsymbol{S} = \iint_{(S)} (\nabla \times \boldsymbol{A}) \cdot \boldsymbol{e}_n \mathrm{d}S$$

V 旋度

$$\mathbf{rot}\boldsymbol{A} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = (\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z})\boldsymbol{i} + (\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x})\boldsymbol{j} + (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})\boldsymbol{k}$$

$$\oint_{C} \boldsymbol{A} \cdot \mathbf{d}\boldsymbol{s} = \iint_{(S)} \mathbf{rot}\boldsymbol{A} \cdot \mathbf{d}\boldsymbol{S}$$

写在后面

确实什么都没有哈哈哈哈哈哈哈哈哈哈哈，618 快乐（bushi）。

本资料的封面为 word 排版制作，目录和正文部分则使用 latex 自创模板排版制作。在此感谢这两款软件的支持。

如有任何问题，请加 qq：1743495223 与笔者联系，互相交流学习或者扩列也行。

祝各位学业有成！

杨承轩
乙巳，五月廿三，碑林