SUBCONVEXITY FOR SECOND MOMENTS OF L-FUNCTIONS TWISTED BY CHARACTERS

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Abstract.

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1. Introduction

Our aim is to prove the following subconvexity result:

Theorem 1.1. *Let f be a cusp form on $\mathrm{PSL}_2(\mathbb{Z})\backslash\mathbb{H}$ of trivial character and let χ be a Dirichlet character modulo Q. Then*

$$L\left(\frac{1}{2}, f \otimes \chi\right) \ll_\epsilon Q^{\frac{1}{3}+\frac{\theta}{3}+\epsilon},$$

where θ is the best bound towards the Ramanujan-Petersson conjecture for Maass forms.

2. Preliminaries

Sums. Let $q \geq 1$ and let χ be a Dirichlet character modulo q. For $n \in \mathbb{Z}$, the Ramanujan sum associated to χ is

$$c_\chi(n) = \sum_{a \mod q} \chi(a) e^{2\pi i n \frac{a}{q}}.$$

Then $\tau(\chi) = c_\chi(1)$ is the usual Gauss sum associated to χ. When χ is primitive, we have

$$c_\chi(m) = \overline{\chi}(m)\tau(\chi),$$

and $|\tau(\chi)| = \sqrt{q}$. When χ is the trivial character, we obtain the usual Ramanujan sum

$$c(n) = \sum_{a \mod q} e^{2\pi i n \frac{a}{q}}.$$

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The Shifted Dirichlet Series $D_{f,v}(w; n, \ell_1, \ell_2)$.

The Multiple Dirichlet Series $Z_{f,g}(s, v, u, \ell_1, \ell_2)$.

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