FFM:

损失函数:

$$FM: \hat{z}(x) = w_0 + \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \langle v_i, v_j \rangle x_i x_j$$

$$FFM: \hat{z}(x) = w_0 + \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \langle v_{i,fj}, v_{j,fi} \rangle x_i x_j$$

$$\hat{y} = sigmoid(z) = \frac{1}{1+e^{-z}}$$

$$J(w_0 \dots w_n, v) = -\frac{1}{m} \sum_{i=1}^{m} y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)}) \log(1-\hat{y}^{(i)})$$

梯度计算:

(1) $\frac{\partial J}{\partial w_0}$ 同 LR、POLY2,FM

(2) $\frac{\partial J}{\partial w_j}$ 同 LR,POLY2,FM

(3) $\frac{\partial J}{\partial v_{i,fj}}$ (也可写作 $\frac{\partial J}{\partial v_{i,field_j}}$)

$$\frac{\partial J}{\partial v_{i,fj}} = -\frac{1}{m} \sum_{l=1}^{m} (y^{(l)} - \hat{y}^{(l)}) \times v_{\theta,fi} x_i^{(l)} x_\theta^{(l)}$$

其中 f_j, $f_\theta \in field\_f_j$

$field\_f_j$ 内只有 x_θ 不为0.

推导过程

以 $\frac{\partial J}{\partial V_{1,f_3}} \Rightarrow -\frac{1}{m}\sum_{i=1}^{m}(y^{(i)}-\hat{y}^{(i)}) \times \boxed{\frac{\partial z^{(i)}}{\partial V_{1,f_3}}}$

$$\frac{\partial z}{\partial V_{1,f_3}} = \frac{\partial z}{\partial V_{1,field3}} = \frac{\partial\left(w_0+\sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n}\sum_{j=i+1}^{n} \langle V_{i,f_j}, V_{j,f_i}\rangle x_i x_j\right)}{\partial V_{1,field3}}$$

field3 = $\{f_3, f_4\}$ $\quad\therefore j \in \{3,4\}$

$$= \frac{\partial\left(\sum_{j=\{3,4\}} \langle V_{1,f_j}, V_{j,f_i}\rangle x_i x_j\right)}{\partial V_{1,field3}}$$

（i确定为1，j为{3,4}）

$$= \frac{\partial\left(\langle V_{1,f_3}, V_{3,f_1}\rangle x_1 x_3 + \langle V_{1,f_4}, V_{4,f_1}\rangle x_1 x_4\right)}{\partial V_{1,field3}}$$

$$= \frac{\partial\left(\langle V_{1,field3}, V_{3,f_1}\rangle x_1 x_3 + \langle V_{1,field3}, V_{4,f_1}\rangle x_1 x_4\right)}{\partial V_{1,field3}}$$

$$= x_1\left(V_{3,f_1}\cdot x_3 + V_{4,f_1}\cdot x_4\right)$$

$$= x_1 V_{?,f_1} x_?$$

（$\underbrace{x_3, x_4}_{\text{同一个field}}$ 不能同时为1）

$$= V_{?,f_1} x_1 x_?$$

$$\frac{\partial J}{\partial V_{1,f_3}} = -\frac{1}{m}\sum_{i=1}^{m}(y^{(i)}-\hat{y}^{(i)})\cdot x_1\left(V_{3,f_1}\cdot x_3 + V_{4,f_1}\cdot x_4\right)$$

更一般的情况：

$$\frac{\partial z}{\partial v_{i,fj}} = \frac{\partial z}{\partial v_{i,field\text{-}fj}} = v_{\theta,fi}\, x_i \cdot x_\theta$$

$fj,\ f\theta \in field\text{-}fj$

$field\text{-}fj$ 内为1的特征下标为 θ.

（对应前面 $f3$、$f4$ 中不为0的特征）.

$$\therefore \frac{\partial J}{\partial v_{i,fj}} = -\frac{1}{m}\sum_{l=1}^{m}(y^{(l)}-\hat{y}^{(l)}) \times v_{\theta,fi}\, x_i^{(l)} x_\theta^{(l)}$$

一定不为0

全局的样本

总能找到不为0的样本