CV 大作业理论部分

卷积层：

前向传播：

$$O_{ij} = \sum_{b=1}^{2} \sum_{a=1}^{2} I_{i+a-1, j+b-1} \cdot W_{ab} + b$$

反向传播：

(1) $\dfrac{\partial L}{\partial b} = \sum_{i=1}^{2} \sum_{j=1}^{2} \dfrac{\partial L}{\partial O_{ij}} \cdot \dfrac{\partial O_{ij}}{\partial b} = \sum_{i=1}^{2} \sum_{j=1}^{2} \dfrac{\partial L}{\partial O_{ij}}$

(2) $\dfrac{\partial L}{\partial W_{ij}} = \sum_{a=1}^{2} \sum_{b=1}^{2} \dfrac{\partial L}{\partial O_{ab}} \cdot \dfrac{\partial O_{ab}}{\partial W_{ij}} = \sum_{a=1}^{2} \sum_{b=1}^{2} \dfrac{\partial L}{\partial O_{ab}} \cdot I_{i+a-1, j+b-1}$

(3) $\dfrac{\partial L}{\partial I_{ij}} = \dfrac{\partial L}{\partial O}$ 实 $\dfrac{\partial L}{\partial I}$ 的推导实在是不会了 ! !

计算如下：

(i) 对于角上元素：I_{11}、I_{13}、I_{31}、I_{33}

$\dfrac{\partial L}{\partial I_{11}} = \dfrac{\partial L}{\partial O_{11}} \cdot W_{11}$, $\dfrac{\partial L}{\partial I_{13}} = \dfrac{\partial L}{\partial O_{12}} \cdot W_{12}$

$\dfrac{\partial L}{\partial I_{31}} = \dfrac{\partial L}{\partial O_{21}} \cdot W_{21}$, $\dfrac{\partial L}{\partial I_{33}} = \dfrac{\partial L}{\partial O_{22}} \cdot W_{22}$

(ii) 对于中心元素：I_{22}

$\dfrac{\partial L}{\partial I_{22}} = \dfrac{\partial L}{\partial O_{11}} \cdot W_{22} + \dfrac{\partial L}{\partial O_{12}} \cdot W_{21} + \dfrac{\partial L}{\partial O_{21}} \cdot W_{12} + \dfrac{\partial L}{\partial O_{22}} \cdot W_{11}$

(iii) 对于其它元素：I_{12}, I_{21}, I_{23}, I_{32}

$\dfrac{\partial L}{\partial I_{12}} = \dfrac{\partial L}{\partial O_{11}} \cdot W_{12} + \dfrac{\partial L}{\partial O_{12}} \cdot W_{11}$

$\dfrac{\partial L}{\partial I_{21}} = \dfrac{\partial L}{\partial O_{11}} \cdot W_{21} + \dfrac{\partial L}{\partial O_{21}} \cdot W_{11}$

$\dfrac{\partial L}{\partial I_{23}} = \dfrac{\partial L}{\partial O_{12}} \cdot W_{22} + \dfrac{\partial L}{\partial O_{22}} \cdot W_{12}$

$\dfrac{\partial L}{\partial I_{32}} = \dfrac{\partial L}{\partial O_{21}} \cdot W_{22} + \dfrac{\partial L}{\partial O_{22}} \cdot W_{21}$

池化层（以最大池化为例）

模型结与卷积层一致

前向传播：

$$O_{ij} = \max_{\substack{1 \leq a \leq 2 \\ 1 \leq b \leq 2}} \{ I_{i+a-1, j+b-1} \}$$

反向传播：

$$\dfrac{\partial L}{\partial I} = \dfrac{\partial L}{\partial O} \cdot \dfrac{\partial O}{\partial I}$$

(i) 若是取最大值的位置，则 $\dfrac{\partial O}{\partial I} = 1$

则 $\dfrac{\partial L}{\partial I} = \dfrac{\partial L}{\partial O}$

(ii) 若不是取最大值的位置：则 $\dfrac{\partial O}{\partial I} = 0$

则 $\dfrac{\partial L}{\partial I} = 0$

综上，$\dfrac{\partial L}{\partial I} = \begin{cases} \dfrac{\partial L}{\partial O} & \text{，是最大值} \\ 0 & \text{，不是最大值} \end{cases}$