LEO

***Abstract*—abstract**

I. Introduction

II. System Model

We consider a multi-LEO satellite communication system network. We assume that the ground projection points of all the LEOs form an independent HPPP of intensity λ_l. The set of all the LEO satellites is expressed as

$$\Phi = \left\{ S_i \in \mathbb{R}^3 : S_i = (X_i, Z_i), X_i \in \mathbb{R}^2, Z_i \in \mathbb{R}_+, i \in \mathbb{N}_+ \right\}. \tag{1}$$

where S_i is the location of the LEO satellite i, X_i is the ground projection point of S_i, and Z_i is the height of S_i. Assume that LEO satellites can have M maximum beams, and each beam is indexed by $m \in \mathcal{M} \triangleq \{1, ..., M\}$. Each LEO has a total number of J sub-channels, where each sub-channel has B_j bandwidth with a total bandwidth of $B = \sum_{j=1}^{J} B_j$.

A. Signal Model

In the LEO satellite systems, signal fading is a crucial factor due to the long distance of signal propagation. Unlike terrestrial networks, attenuation caused by tropospheric effects must be considered [1]. The total channel gain can be expressed as

$$H_{n,m,k} = H^{fs}_{n,m,k} \cdot H^{g}_{n,m,k} \cdot H^{sc}_{n,m,k} \cdot H^{sr}_{n,m,k}, \tag{2}$$

where free space path loss H_{fs} is given by

$$H^{fs}_{n,m,k} = \left(\frac{4\pi d_{n,m,k}}{c/f_{n,m,k}} \right)^2, \tag{3}$$

with $f_{n,m,k}$ as the central operating frequency and $d_{n,m,k}$ as the distance between the LEO and ground user. H^g is atmospheric absorption loss, with various types of gases and particles that absorb the electromagnetic wave of specific frequencies. H^{sc} denotes the tropospheric scintillation effect caused by the rapid variation in the refractive atmosphere. The PDF of shadowed Rician fading H^{sr} [2] can be expressed as

$$f_{H^{sr}}(x) = \alpha_1 e^{-\beta_1 x} {}_1F_1(m_1; 1; \delta x), \quad x > 0 \tag{4}$$

where $\alpha_1 = \frac{1}{2b}(2bm_1/(2bm_1 + \Omega_1))^{m_1}$, $\beta_1 = \frac{1}{2b}$, $\delta = (2b^2 m_1 + b\Omega)\Omega/2$, the parameter Ω_1 is the average power of the LOS component, $2b$ is the average power of the multipath component, m_1 is the Nakagami parameter, for $m_1 = 0$ and $m_1 = \infty$, the channel follows Rayleigh and Rician distribution, and ${}_1F_1(m_1; 1; \delta x)$ is the confluent hypergeometric [3], which is given by

$${}_1F_1(x; y; z) = \sum_{n=0}^{\infty} \frac{x^{(n)} z^n}{y^{(n)} n!} \tag{5}$$

where $x^{(n)} = \Pi_{k=0}^{n-1}(x + k)$ is the rising factorial.

We adopted the antenna radiation pattern in [4] for the LEO satellites denoted as

$$G_{n,m,k} = G_0 \left[\frac{J_1\left(\mu\left(\theta_{n,m,k}\right)\right)}{2\mu(\theta_{n,m,k})} + 36 \frac{J_3\left(\mu\left(\theta_{n,m,k}\right)\right)}{\mu(\theta_{n,m,k})^3} \right]^2, \tag{6}$$

where

$$\mu(\theta_{n,m,k}) = \frac{2.07123 \cdot \sin(\theta_{n,m,k})}{\sin(\theta^{bw}_{n,m})}. \tag{7}$$

$\theta_{n,m,k}$ is the boresight angle, and $\theta^{bw}_{n,m}$ is the 3 dB half-power beamwidth angle of the antenna. G_0 is the maximum antenna gain defined as $G_0 = \rho \frac{4\pi A}{(c/f_c)^2}$, where ρ is the antenna efficiency, A is the aperture size of the antenna, c is the velocity of light. $J_1(\cdot)$ and $J_3(\cdot)$ respectively represent the Bessel functions of the first kind of orders 1 and 3.

The SINR of the ground user k served by the beam m from LEO n at the sub-channel j can be formulated as

$$\gamma_{n,m,k,j} = \frac{P_{n,m} H_{n,m,k} G_{n,m,k} \alpha_{n,m,k,j}}{I^a_{n,m,k,j} + I^b_{n,m,k,j} + \sigma^2}, \tag{8}$$

where $P^{LEO}_{n,m}$ is the transmit power of beam m at satellite n, G_{n,m,k_1} is the antenna gain, and σ^2 is the power of the thermal noise. The LEO association indicator $\alpha_{n,m,k_1} = 1$ when user k_1 is served by the beam m of satellite n operating in subband j. We further define the exclusive set of $\mathcal{N}' \triangleq \mathcal{N} \backslash n$, $\mathcal{M}' \triangleq \mathcal{M} \backslash m$, and $\mathcal{K}' \triangleq \mathcal{K} \backslash k_1$. The intra-LEO interference I^a_{n,m,k_1} in (8) is given by

$$I^a_{n,m,k,j} = \sum_{m' \in \mathcal{M}'} \sum_{k' \in \mathcal{K}'} P_{n,m'} H_{n,m',k} \cdot G_{n,m',k} \alpha_{n,m',k',j}. \tag{9}$$

Similarly, inter-LEO interference $I^b_{n,m,k}$ can be expressed as

$$I^b_{n,m,k,j} = \sum_{n' \in \mathcal{N}'} \sum_{m' \in \mathcal{M}} \sum_{k' \in \mathcal{K}'} P_{n',m'} H_{n',m',k} \cdot G_{n',m',k} \alpha_{n',m',k',j}. \tag{10}$$

B. Beam Training

The training overhead of an LEO in a single transmission frame can be expressed as

$$T^{bt}_n = |\phi_n| \cdot T_{beam} + \sum_{m \in \mathcal{M}} |\mathcal{K}_{n,m}| \cdot (T_{fb} + T_{ack}), \tag{11}$$

where T_{beam} is the time duration of one beam beaconing. $\mathcal{K}_{n,m}$ is the set of users served by the m-th beam of the n-th satellite. T_{fb} and T_{ack} denote the feedback and acknowledgment intervals, respectively. ϕ^{LEO}_n is the beam training set containing the beams that need to be scanned by the n-th satellite, which can be expressed as

$$\phi_n = \{(n, m) \in \mathbf{\Phi} \mid, \forall m \in \mathcal{M}\}, \tag{12}$$

where $\mathbf{\Phi}$ is the beam training set of the LEO constellation. Furthermore, we define a successful beam training if the optimal beam SINR is greater than a threshold γ_{thr}, namely $\gamma_{n,m,k} \geq \gamma_{thr}$. A weak beam with a low SINR will result in failure of beam training. We define beam alignment accuracy as

$$Q_{n,m,k,t} = \frac{1}{W} \sum_{\tau=t-W+1}^{t} \max_j \left(\mathbb{1}(\gamma_{n,m,k,j}(\tau) \geq \gamma_{thr}) \right), \tag{13}$$

where W is the historical observable window size and the value of the indicator function $\mathbb{1}(\cdot)$ is one if the statement is true.

The total overhead can be expressed as

$$T^o_n = T^{bt}_n + T^{tran}_n + T^{prop}_n \tag{14}$$

Therefore, the latency-aware throughput of the k_1-th user which is served by the m-th beam of the n-th satellite is given by

$$R_{n,m,k} = \left(1 - \frac{T_n^o}{T_f}\right) \sum_{j=1}^{J} \alpha_{n,m,k,j} B_j \log_2 \left(1 + \gamma_{n,m,k,j}\right), \quad (15)$$

where T_f is the time interval of the entire transmission frame and $W_{n,m}$ is the bandwidth of the m-th beam in n-th LEO. We allocate bandwidth equally among users served by the same beam to ensure fairness. The achievable sum-rate of all users under the LEO satellite constellation is denoted as

$$R_{tot} = \sum_{n\in\mathcal{N}} \sum_{m\in\mathcal{M}} \sum_{k\in\mathcal{K}} R_{n,m,k}, \quad (16)$$

and the total beam alignment accuracy is expressed as

$$Q_{tot,t} = \sum_{n\in\mathcal{N}} \sum_{m\in\mathcal{M}} \sum_{k\in\mathcal{K}} Q_{n,m,k,t}. \quad (17)$$

Since LEO satellites are dependent on solar power, optimizing EE becomes a crucial aspect of the resource allocation problem in LEO networks. The beam training energy in a single transmission frame can be expressed as

$$E_{n,m}^{bt} = T_n^{bt} P_{n,m}^{bt}. \quad (18)$$

The transmission energy in a single transmission frame can be expressed as

$$E_{n,m}^{tran} = \left(T_f - T_n^{bt}\right) P_{n,m}. \quad (19)$$

The EE performance of the whole system is expressed as

$$E_{eff} = \frac{R_{tot}}{\sum_{n\in\mathcal{N}} \sum_{m\in\mathcal{M}} \frac{E_{n,m}^{tran} + E_{n,m}^{bt}}{T_f}}. \quad (20)$$

III. Problem Formulation

We define the solutions of power $\boldsymbol{P} = \{P_{n,m} \mid \forall n \in \mathcal{N}, \forall m \in \mathcal{M}\}$, association indicator $\boldsymbol{\alpha} = \{\alpha_{n,m,k,j} \in \{0,1\} \mid \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}, \forall j \in \mathcal{J}\}$, beam training set $\boldsymbol{\Phi} = \{(n,m) \mid \forall n \in \mathcal{N}, m \in \phi_n\}$ containing the pairs of beam index m of LEO and BS n, and beamwidth $\boldsymbol{\theta} = \{\theta_{n,m,k} \mid \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}\}$. To maximize energy efficiency, our objective is for the satellites to allocate power to each beam in a way that balances high data rates with low interference. Accordingly, the optimization problem can be formulated as

$$\max_{\boldsymbol{P},\boldsymbol{\theta}.\boldsymbol{\alpha},\boldsymbol{\Phi}} \quad E_{eff} \quad (21a)$$

$$\text{s.t.} \quad \alpha_{n,m,k,j} \in \{0,1\}, \quad \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}, \forall j \in \mathcal{J} \quad (21b)$$

$$\sum_{k\in\mathcal{K}} \alpha_{n,m,k,j} \leq 1, \quad \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \forall j \in \mathcal{J}, \quad (21c)$$

$$R_{n,m,k} \geq R_{min}, \quad \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}, \quad (21d)$$

$$Q_{n,m,k,t} \geq Q_{min}, \quad \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}, \quad (21e)$$

$$T_n^{bt} \leq C_{max}, \quad \forall n \in \mathcal{N}, \quad (21f)$$

$$0 \leq P_{n,m} \leq P_{beam}, \quad \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \quad (21g)$$

$$0 \leq P_n^{bt} \leq P_{beam}, \quad \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \quad (21h)$$

$$0 \leq \sum_{m\in\mathcal{M}} P_{n,m} \leq P_{max}, \quad \forall n \in \mathcal{N}. \quad (21i)$$

$$\theta_{n,m}^{bw} \geq \frac{70c}{f_c A}, \quad \forall n \in \mathcal{N}. \quad (21j)$$

Constraint (21c) satisfy the unicast constraint. Constraint (21j) limit the minimum 3dB beamwidth under the digital beamforming method [5].

The CCDF of SINR can be formulated as (**??**)

IV. Digital Twin

The real-world wireless communication channel is determined by the distance, dynamics of the medium (e.g. temperature and water vapor density), and other blockages or scatterers in the environment. The real-world channel can be expressed as

$$H = g(\mathcal{E}) \quad (25)$$

where $g(\cdot)$ is the wireless signal propagation law and $\mathcal{E}$ is the communication environment. Since the physic law of signal propagation is difficult to characterize in real-world scenario, we use the deep neural network to model the wireless signal propagation law. The digital twin will be trained by the collected data which contain the information of the environment and CSI. The collectable environment data can be expressed as

$$\tilde{\mathcal{E}} = [S, X_u, \boldsymbol{\kappa}, \mathbf{V}] \quad (26)$$

where $\boldsymbol{\kappa}$ is the temperature data and $\mathbf{V}$ is the water vapor density data. The estimated CSI generated by the digital twin can be expressed as

$$\tilde{H} = \tilde{g}(\tilde{\mathcal{E}}) \quad (27)$$

The transmission delay between the real world satellites and digital twin can be formulated as

$$T_n^{tran} = \frac{\mathcal{D}_n}{R_n}, \quad (28)$$

where $\mathcal{D}_n$ is the size of the data transmitted. The propagation delay can be formulated as

$$T_n^{prop} = \frac{d_n}{c}, \quad (29)$$

V. Federated Learning

All LEO satellites train their model w_n based on the local observations denoted as

$$w_n = w_n(t-1) - \eta \nabla F_n(w_n(t-1)) \quad (30)$$

and they share their models with the period of T_s, the global aggregation model can be expressed as

$$w = \frac{1}{\sum_{i=1}^{N} a_i} \sum_{i=1}^{N} a_i w_n \quad (31)$$

where the sharing weight of each agent can be expressed as

$$a_i = \frac{R}{R_h}, \quad (32)$$

and historical average reward is denoted as

$$R_h = \beta \cdot R + (1-\beta) \cdot R_h(t-1), \quad (33)$$

where β is the soft update factor of the reward and R is the reward of current time slot.

VI. Rocket propulsion

Based on the Hohmann transfer equation, the required impulses at the perigee and apogee are

$$\Delta v_p = \sqrt{\frac{\mu}{r_1}} \left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right), \quad (34)$$

$$\Delta v_a = \sqrt{\frac{\mu}{r_2}} \left(1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right). \quad (35)$$

where $\mu = GM$ is the standard gravitational parameter, r_1 is the distance between the perigee and the Earth center and r_2 is the distance between the apogee and the Earth center. The final mass of the satellite in target orbit is calculated from the rocket equation [6]:

$$m_1 = m_0 \exp\left(\frac{-(\Delta v_p + \Delta v_a)}{g I_{sp}}\right). \tag{36}$$

where m_0 is the initial mass of the satellite, m_1 is the mass of the satellite when reaching the target position, g is the gravity of Earth, and I_{sp} is the specific impulse. By Kepler's third law, the transfer time can be expressed as

$$t_f = \pi\sqrt{\frac{(r_1 + r_2)^3}{8\mu}} \tag{37}$$

The transfer time from rocket equation can be expressed as

$$t_f = \frac{m_0 - m_1}{\dot{m}}. \tag{38}$$

where the mass flow rate $\dot{m}$ is

$$\dot{m} = \frac{2\rho P}{(g I_{sp})^2}. \tag{39}$$

VII. Temp

$$\boldsymbol{P} = \{P_{n,m} \mid \forall n \in \mathcal{N}, \forall m \in \mathcal{M}\} \tag{40}$$

$$\boldsymbol{\alpha} = \{\alpha_{n,m,k,j} \in \{0,1\} \mid \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}, \forall j \in \mathcal{J}\} \tag{41}$$

$$\boldsymbol{\Phi} = \{(n,m) \mid \forall n \in \mathcal{N}, m \in \phi_n\} \tag{42}$$

$$\boldsymbol{\theta} = \{\theta_{n,m,k} \mid \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}\} \tag{43}$$

References

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$$\begin{aligned}
\mathbb{P}(\gamma_{n,m,k,j} > \tau) &= \mathbb{E}_{h^{rc},\theta_{n,m,k},d_{n,k}}\left\{\mathbb{P}\left[h^{rc} > \tau\left(\frac{I^{a,s}_{n,m,k,j} + I^{b,s}_{n,m,k,j} + I^{c,s}_{n,m,k,j}}{S_{n,m,k,j}} + \frac{\sigma^2 + I^{ICI}_{k_s}}{S_{n,m,k,j}}\right)\right]\right\} \\
&= \mathbb{E}_{h^{rc},\theta_{n,m,k},d_{n,k}}\left\{Q_1\left(\sqrt{2K}, \sqrt{2(1+K)}\lambda_{n,m,k,j}\right)\right\} \\
&\geq \mathbb{E}_{h^{rc},\theta_{n,m,k},d_{n,k}}\left\{\exp\left(-\frac{1}{2}\left(\sqrt{2K} + \sqrt{2(1+K)}\lambda_{n,m,k,j}\right)^2\right)\right\}
\end{aligned} \tag{22}$$

$$S_{n,m,k,j} = P_{n,m,k,j}\left(\frac{c}{4\pi d_{n_{LEO},k_{LEO}} f_c}\right)^2 A^{-1}(d_{n_{LEO},k_{LEO}}) G_{n,m,k,j} \alpha_{n,m,k,j} \tag{23}$$

$$\lambda_{n,m,k,j} = \tau \left(\frac{I_{n,m,k,j}^{a,s} + I_{n,m,k,j}^{b,s} + I_{n,m,k,j}^{c,s}}{S_{n,m,k,j}} + \frac{\sigma^2 + I_{k_s}^{ICI}}{S_{n,m,k,j}} \right) \tag{24}$$