QUESTION / ANSWER SHEET

ELECTENG 202

IRON MOUNTAIN FILE LABEL 5000025770560

THE UNIVERSITY OF AUCKLA

SEMESTER ONE 2017
Campus: City

ELECTRICAL & ELECTRONIC ENGINEERING

Circuits and Systems

(Time Allowed: THREE hours)

Instructions:

- Answer ALL **SIX** questions.
- All questions are of equal mark value.
- Each script contains **27 pages** including a final page for working. Do **NOT** remove the final page.
- Show **ALL** working.
- Use the back of the pages for continuation if needed.
- If you believe you need further information than that provided, make some appropriate engineering assumption(s), state them clearly, and continue with your answer.
- This exam is a "Restricted Calculator" exam and bound by the appropriate regulations.
- It is the responsibility of the student to ensure their **ID** are written on each page of the script.
- See the **Appendix** on Page 26 for a **table of formulae**

ID: 5 2 9 5 2 1 1 9 0

SURNAME: CHOY

FORENAMES: LINCOLN LAM-HANG

Question	Mark
Q1	26
Q2	20
Q3	18.5
Q4	19
Q5	15
Q6	17.5
Total	110

QUESTION / ANSWER SHEET

ELECTENG 202

ID: 529521190

QUESTION 1 (20 marks)

20

(a) For the circuit shown in Fig. 1.1, find i_1, i_2, i_3, and i_4. (5 marks)

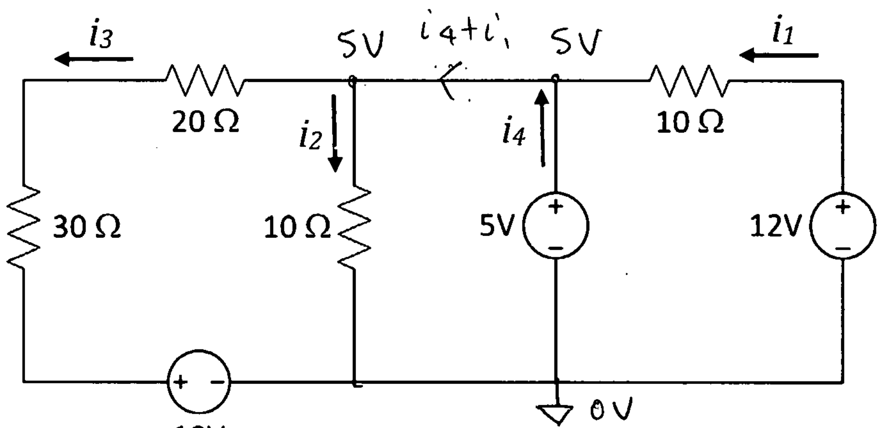

Fig. 1.1: Circuit 1-1.

5

From digram: Simply use Ohm's law

$i_1 = \frac{12-5}{10} = 0.7A$

$i_2 = \frac{5-0}{10} = 0.5A$

$i_3 = \frac{5-10}{50} = -0.1A$

Applying KCL:

$i_4 + i_1 = i_2 + i_3$

$i_4 + 0.7A = 0.5A + (-0.1A)$

$i_4 = -0.3A$

$i_1 =$ 0.7A $\quad i_2 =$ 0.5A $\quad i_3 =$ −0.1A $\quad i_4 =$ −0.3A

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(b) For the circuit shown in Fig. 1.2, find the equivalent resistance of the circuit, R_{eq}, as looked between terminals *a-b*. (5 marks)

Hint: You may find redrawing the circuit is useful to think about a short-cut solution.

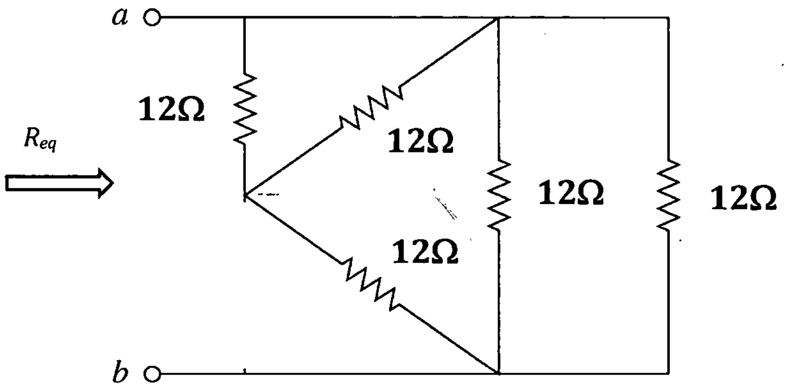

Fig. 1.2: Circuit 1-2.

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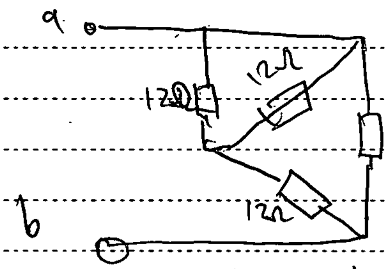
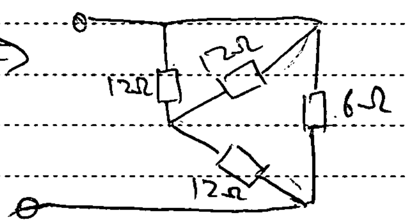
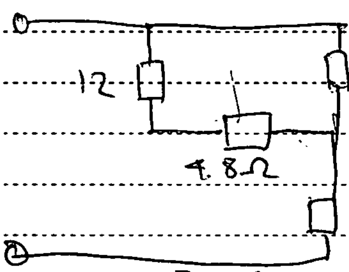
redraw

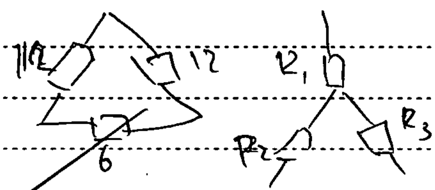


12Ω // 12 Ω ⇒



Convert to WYE





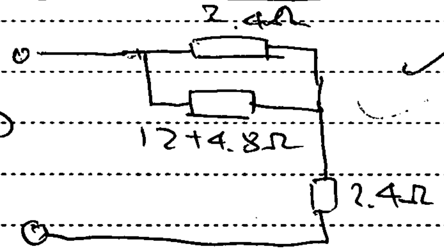
$R_1 = \frac{12 \times 12}{12+12+6} = \frac{144}{30}\,\Omega$

$= 4.8\Omega$

$R_2 = \frac{12 \times 6}{30} = 2.4\Omega$

$R_3 = \frac{12 \times 6}{30} = 2.4\Omega$

R_{TH} ⇒



$R_{TH} = 2.4 // (12 + 4.8) + 2.4$

$R_{EQ} = 4.5\Omega$

R_{eq} = ~~4.5~~ 4.5Ω

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(c) For the circuit shown in Fig. 1.3, find the equations relating the mesh currents i_1, i_2, and i_3. Simplify but do not solve. (4 marks)

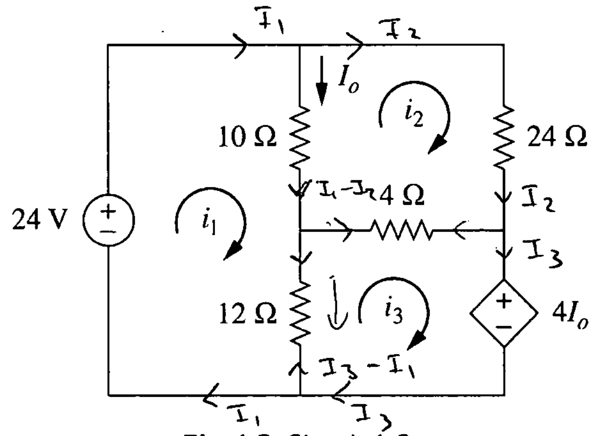

Fig. 1.3: Circuit 1-3.

4

Loop i_1:

~~$-24 + 10(I_1 - I_2) + 12(I_1 - I_2 - I_3) = 0$~~

~~$10I_1 - 10I_2 + 12I_1 - 12I_2 - 12I_3 = 24$~~

~~$22I_1 - 22I_2 - 12I_3 = 24$~~

~~$11I_1 - 11I_2 - 6I_3 = 12$~~

Note $I_1 - I_2 = I_0$

Loop i_2:

$10(I_2 - I_1) + 24I_2 + 4(I_2 - I_3) = 0$

$10I_2 - 10I_1 + 24I_2 + 4I_2 - 4I_3 = 0$

$38I_2 - 10I_1 - 4I_3 = 0$

$19I_2 - 5I_1 - 2I_3 = 0$

Loop i_3:

Recall: $I_0 = I_1 - I_2$

$4I_0 + 12(I_3 - I_1) + 4(I_3 - I_2) = 0$

$4I_0 + 12I_3 - 12I_1 + 4I_3 - 4I_2 = 0$

$4I_1 - 4I_2 + 12I_3 - 12I_1 + 4I_3 - 4I_2 = 0$

$-8I_1 - 8I_2 + 16I_3 = 0$

$2I_3 - I_1 - I_2 = 0$

Loop i_1:

$-24 + 10(I_1 - I_2) + 12(I_1 - I_3) = 0$

$-24 + 10I_1 - 10I_2 + 12I_1 - 12I_3 = 0$

$22I_1 - 10I_2 - 12I_3 = 24$

$11I_1 - 5I_2 - 6I_3 = 24$

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(d) For the circuit shown in Fig. 1.4, find v_{ab} between terminals a and b. (6 marks)

Hint: You may think of source transformation approach.

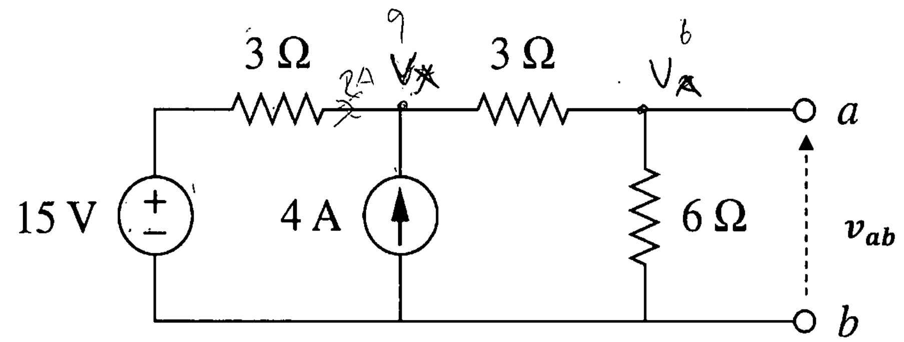

Fig. 1.4: Circuit 1-4.

6

Note $V_A = V_{ab}$

~~Vx~~ Solve using nodal analysis. Node V_x

$\frac{V_x - 15}{3} - 4 + \frac{V_x - V_A}{3} = 0$

$V_x - 15 - 12 + V_x - V_A = 0$

~~$2V_x - 12 = V_A$~~ $2V_x - 27 = V_A$

Node V_A

$\frac{V_A}{6} + \frac{V_a - V_x}{3} = 0$

$V_A + 2V_a - 2V_x = 0$

$3V_A = 2V_x$

From earlier ~~$V_A = 2V_x - 12$~~ $V_A = 2V_x - 27$

~~$6V_x - 36 = 2V_x$~~ ~~$\Rightarrow V_x = 9V$~~

~~$4V_x = 36$~~

So $\Rightarrow$ ~~$V_A = 2(9) - 12$~~ ~~$= 6V$~~ ~~$V_A = V_{ab}$~~

$3(2x - 27) = 2V_x$

~~$4V_x = 5$~~ $6V_x - 81 = 2V_x$

$4V_x = 81$

$V_x = 20.25V$

$V_A = 2(20.25) - 27$

$= 13.5V$

$v_{ab} =$ ~~6V~~ 13.5V

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20

Question 2 (20 marks)

From the circuit shown in Fig. 2.1,

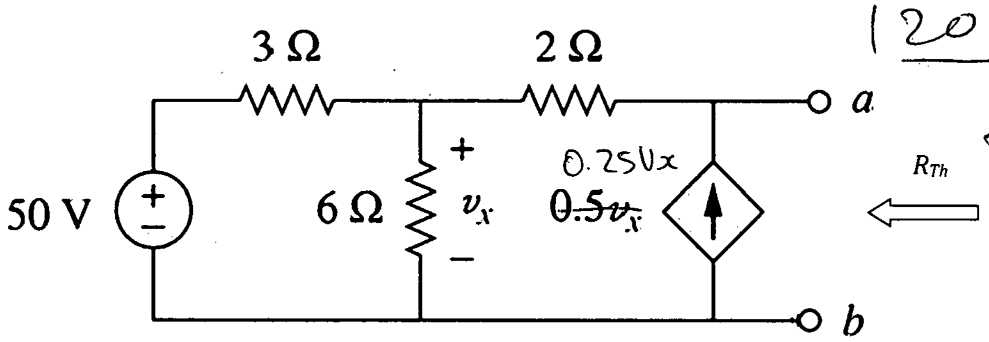
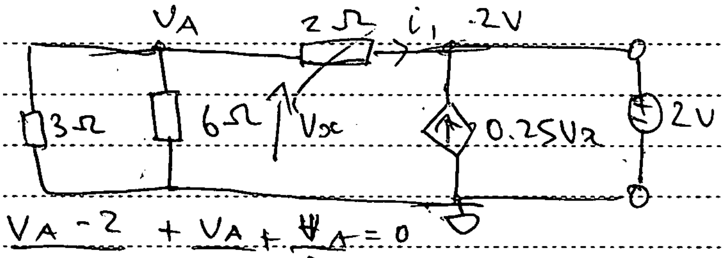


Fig. 2.1: Circuit 2-1.

(a) Obtain the Thevenin equivalent resistance R_{Th} between terminals a and b. (5 marks)

Set all independent sources to 0, and add source



$V_x = V_A$

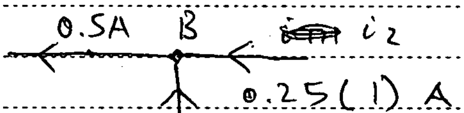
$\frac{V_A - 2}{2} + \frac{V_A}{6} + \frac{V_A}{3} = 0$

$3V_A - 6 + V_A + 2V_A = 0$

$6V_A = 6$, $V_A = 1V$

$i_1 = \cancel{1A}\ 0.5A$

Apply KCL



$i_2 = 0.25A$ into node B

$R_{TH} = \frac{2}{0.25} = 8\Omega$

$R_{Th} =$ 8Ω

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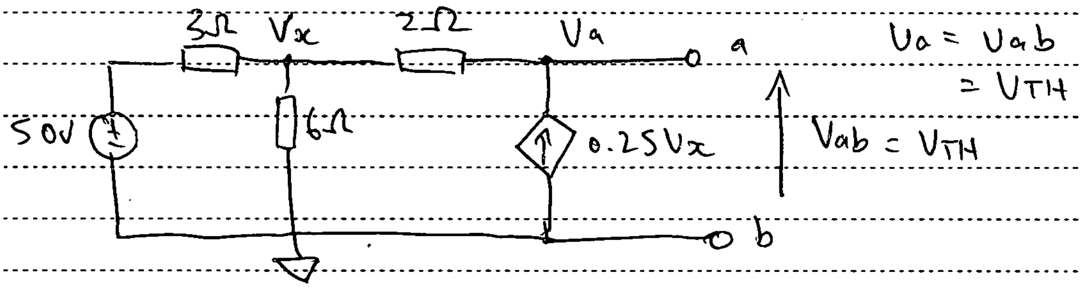
(b) Find the Thevenin voltage V_{Th} between terminals a and b. (5 marks)



Va = Vab
= VTH

Use Nodal : @ node Vx

$$\frac{V_x - V_a}{2} + \frac{V_x - 50}{3} + \frac{V_x}{6} = 0$$

$$3V_x - 3V_a + 2V_x - 100 + V_x = 0$$

$$6V_x - 100 = 3V_A \quad — ①$$

@ node VA

$$-0.25V_x + \frac{V_A - V_x}{2} = 0$$

5

$$-0.5V_x + V_A - V_x = 0$$

$$V_A = 1.5V_x$$

Sub into equation ①

$$6V_x - 100 = 3(1.5V_x)$$

$$1.5V_x = 100, \quad V_x = 66.66V$$

$$V_A = \frac{6V_x - 100}{3} \Rightarrow 100V$$

$$V_{TH} = 100V$$

V_{Th} = 100V

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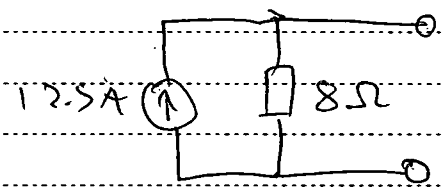
(c) Find the equivalent Norton circuit from the Thevenin equivalent circuit you found in Q2(a) and Q2(b). (2 marks)

$I_N = \frac{V_{TH}}{R_{TH}}$ $V_{TH} = 100V$

$R_{TH} = 8\Omega$

$I_N = \frac{100}{8}$

$= 12.5A$ $R_N = R_{TH}$



2

$R_N =$ 8Ω $I_N =$ 12.5A

(d) Find the value of the load resistor R_{max}, which when connected between terminals a and b, results in maximum power transfer. Calculate the maximum power transfer P_{max} to that load resistor R_{max}. (8 marks)

$P_{max\ transfer} = \frac{(V_{TH})^2}{4R_L}$

For ~~100~~ max power transfer

$R_L = R_{TH} = R_{max}$

$P_{max} = \frac{(V_{TH})^2}{4R_{TH}}$

$P_{max} = \frac{100^2}{4 \times 8}$

$= 312.5W$

8

$R_{max} =$ 8Ω $P_{max} =$ 312.5W

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QUESTION 3 (20 marks)

An RC circuit is shown in Fig. 3.1. Assume that the switch has been closed for a very long time before it is opened at time $t = 0$.

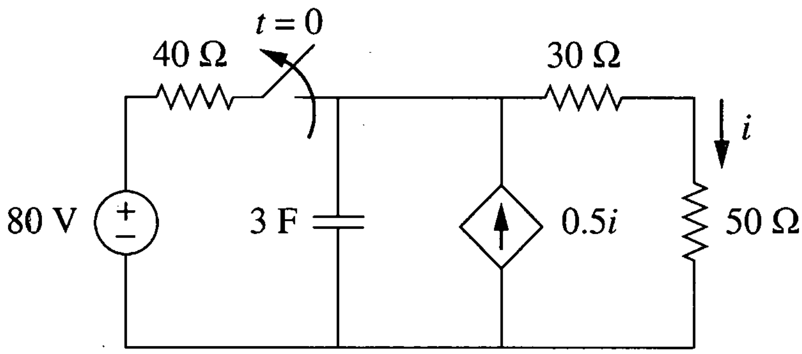
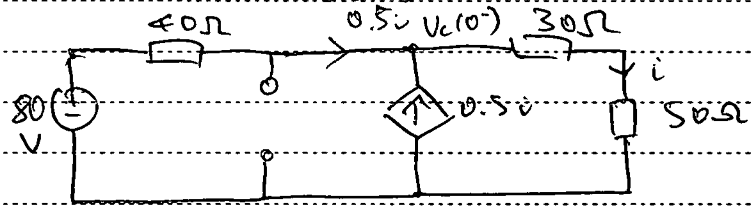


Fig. 3.1: Circuit 3-1.

(a) At time $t = 0^+$ (i.e., just after the switch is opened), determine the voltage $v_C(0^+)$ across the capacitor. Then, find the voltage across the capacitor after the switch is opened for a very long time $v_C(\infty)$. (4 marks)

3.5

$V_c(0^+) = V_c(0^-)$



$0.5i = \frac{80 - V_c(0^-)}{40}$

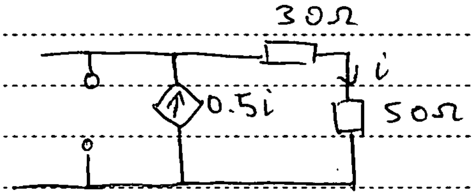
$\frac{V_c(0^-) - 80}{40} + \frac{V_c(0^-)}{80} - 0.5i = 0$

$\frac{2V_c(0^-) - 160}{40} + \frac{V_c(0^-)}{80} = 0$

$4V_c(0^-) - 320 + V_c(0^-) = 0$

$V_c(0^-) = 64V \rightarrow V_c(0^+) = 64V$

$v(\infty)$



current (i) → 0 as $t \rightarrow \infty$

therefore $v(\infty) = 0V$

$v_C(0^+) =$ 64V

$v_C(\infty) =$ 0V Explanation ?

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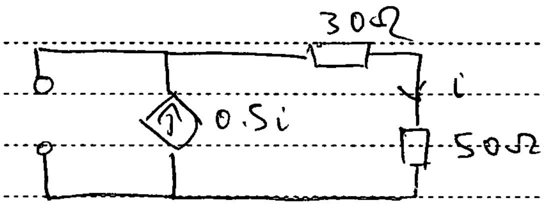
(b) Calculate the time constant of the circuit in Fig. 3.1 after the switch is opened (i.e., for $t > 0$).

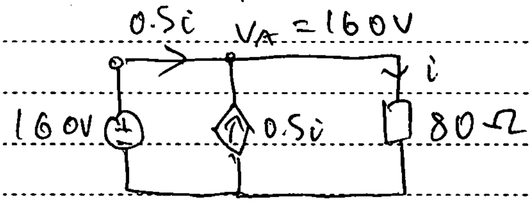
(4 marks)

4

$\tau = RC$ Find R



Add ~~so~~ independent source



$V_A = 160V$

$i = \frac{V_A}{80} = 2A$

$0.5i = 1A$

$R_{TH} = \frac{160}{1} = 160\Omega$

$\tau = 3 \times 160 = 480$ seconds

$\tau =$ 480s

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(c) For time $t > 0$ seconds (i.e., after the switch is opened), determine expressions (i.e., equations) for the voltage $v_C(t)$ across the capacitor, and the current $i_C(t)$ through the capacitor. (4 marks)

3

For $t > 0$ $\tau = 480s$

$v_c(\infty) = 0$

$v_c(0^+) = 12$

$v_c(t) = v_c(\infty) + [v_c(0^+) - v_c(\infty)]\, e^{-\frac{t}{\tau}}$

$v_c(t) = 0 + [12 - 0]\, e^{-\frac{t}{480}}$

$v_c(t) = 12e^{-\frac{t}{480}}$ V

$i_c = -C \frac{dv_c}{dt}$

$i_c = C \times \left(12e^{-\frac{t}{480}} \cdot dt\right)$

$i_c(t) = 3 \times -\frac{1}{40} e^{-\frac{t}{480}}$

$i_c(t) = -\frac{3}{40} e^{-\frac{t}{480}}$ A

$v_C(t) =$ $12e^{-\frac{t}{480}}$ V $i_C(t) =$ $-\frac{3}{40} e^{-\frac{t}{480}}$ A

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(d) For the RLC circuit shown in Fig. 3.2,

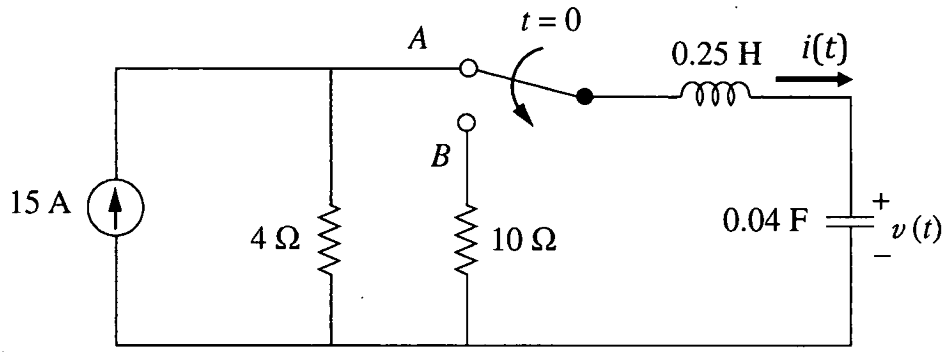
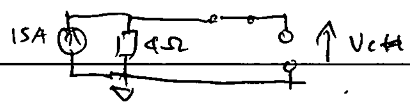


Fig. 3.2: Circuit 3-2.

answer the following questions:

(i) Find the voltage across the capacitor just after the switch be in contact with position B; $v(0^+)$. (2 marks)



$V(0^+) = v(0^-)$

$v(0^-) = 15 \times 4 = 60V$

$v(0^+) = 60V$

Assuming that the circuit was in steady state for $t < 0$

2

$v(0^+) =$ 60V

(ii) Find the current through the inductor just after the switch be in contact with position B; $i(0^+)$. (2 marks)

$i(0^+) = i(0^-)$ Assuming steady state for $t<0$

At $t<0$, $i(0^-)$, capacitor open circuited, so

current through inductor is 0A

$i(0^-) = 0A$

$\therefore\ i(0^+) = 0A$

2

$i(0^+) =$ 0A

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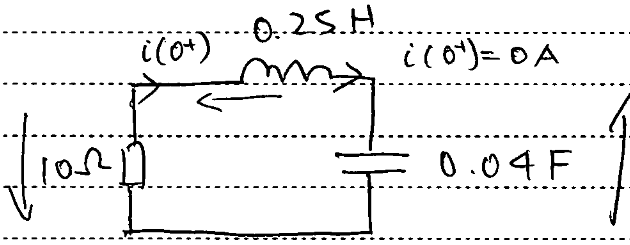
(iii) Find the characteristic equation of the circuit in Fig. 3.2. (2 marks)



$v_c(0^+) = 60V$

2

~~10i~~

$i\cancel{(0^+) =} 0A$

$10i$

$\cancel{-10i +} V_c + L\frac{di}{dt} + \cancel{10i(t)} = 0$

$L\frac{di}{dt} + v_c = 0$

$i = C\frac{dv_c}{dt}$

$\cancel{L\frac{di}{dt} + \int \frac{i}{c}\,dt = 0}$

$\int \frac{i}{c} = \int \frac{dv_c}{dt} = v_c$

$10i + L\frac{di}{dt} + \frac{1}{c}\int i \cdot dt + v_c(0) = 0$

differentiate

$10\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{c} = 0$

$\frac{10}{L}\frac{di}{dt} + \frac{d^2i}{dt^2} + \frac{i}{Lc} = 0$

$\cancel{s^2 + \frac{1}{Lc} = 0}$

$s^2 + \cancel{100} = 0$

$s^2 + 40s + 100 = 0$

The characteristic equation is: $\cancel{s^2 + 100 = 0}$ $s^2 + 40s + 100 = 0$

(iv) What type of damping does the circuit in Fig. 3.2 exhibit? Why? (2 marks)

$2\alpha = 40$ $\omega_0^2 = 100$, $\omega_0 = 10$

$\alpha = 20$ $\Delta = b^2 - 4ac$

$\Delta = 1600 - 400 > 0$

$\alpha > \omega_0$ distinct, real, roots

So we have overdamping

2

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QUESTION 4 (20 marks)

In the circuit shown in Fig. 4.1,

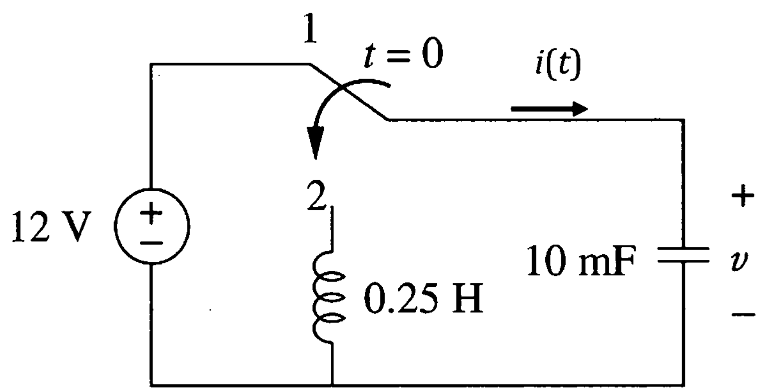


Fig. 4.1: Circuit 4-1.

Find the following:

(a) Prove from the first principle that the s-domain mapping of a capacitor having an initial voltage $v(0)$ is represented by Fig. 4.2. (4 marks)

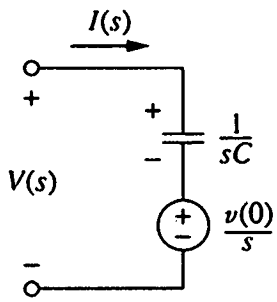


Fig. 4.2: s-mapping of capacitor.

We know that

~~$i_c =$~~

$$i = C\frac{dv}{dt}$$

Take the laplace transform

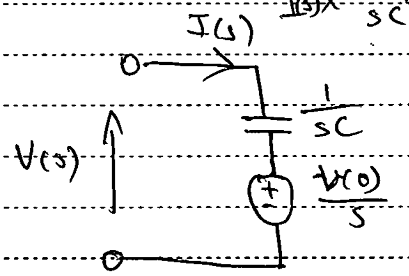
$$I(s) = C(sV(s) - v(0))$$

Rearrange for V(s)

$$\frac{I(s)}{C} + v(0) = sV(s)$$

$$V(s) = \underbrace{\frac{I(s)}{sC}}_{I(s) \times \frac{1}{sC}} + \frac{v(0)}{s}$$

→ ohm's law ⇒ V = IZ (Z: impedance)



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(b) Find the s-mapping of the circuit shown in Fig. 4.1 for $t > 0$. (4 marks)

inductor

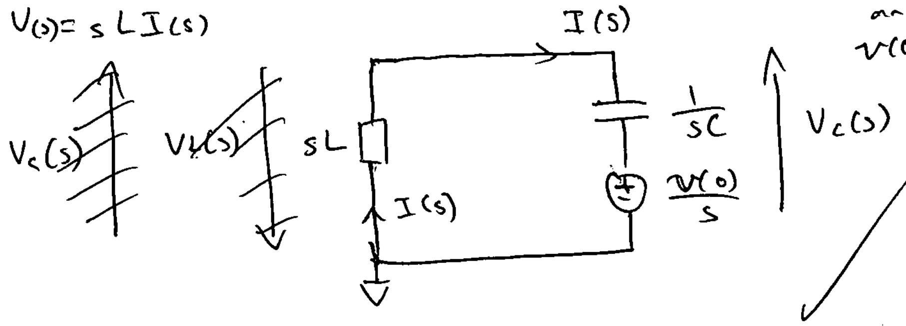
$V = L\frac{di}{dt}$

$V(s) = L(sI(s) - i(0))$

$V(s) = sLI(s)$

but $i(0^-) = i(0^+) = 0A$

where $C = 10mF$ and $L = 0.25H$ and $v(0) = 12V$



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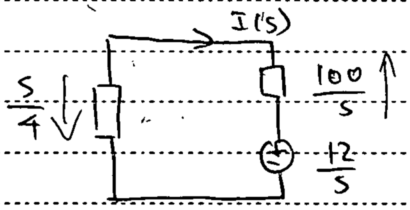
(c) Find the current $I(s)$ at $t > 0$ of the circuit shown in Fig. 4.1. (4 marks)

We know $v_c(0^+) = 12V$



Shorthand

$I(s) = I$

Using KVL

~~I(s)~~ $\frac{sI}{4} + \frac{100I}{s} + \frac{12}{s} = 0$

Simplify expression

$s^2I + 400I + 48 = 0$

$I(s^2+400) = -48$

$I = \frac{-48}{s^2+400}$ A.s

$I(s) = \frac{-48}{s^2+400}$ A.s

4

$I(s) = \frac{-48}{s^2+400}$ A.s

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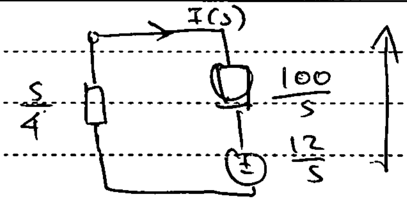
(d) Find the voltage $V(s)$ across the capacitor at $t > 0$ of the circuit shown in Fig. 4.1. (4 marks)



Found earlier $I(s) = \frac{-48}{s^2+400}$

$V(s) = \frac{-100 I(s)}{s} + \frac{12}{s}$

$V(s) = \frac{-100 I(s) + 12}{s}$

$V(s) = \frac{\frac{-4800}{s^2+400} + 12}{s} \times \left(\frac{s^2+400}{s^2+400}\right)$

$V(s) = \frac{-4800 + 12(s^2+400)}{s(s^2+400)}$

$V(s) = \frac{12s^2}{s(s^2+400)}$ V.s $\Rightarrow \frac{12s}{s^2+400}$ V.s

$V(s) = \frac{12s^2}{s(s^2+400)}$ V.s

3

(e) Find the voltage $v(t)$ across the capacitor at $t > 0$ of the circuit shown in Fig. 4.1. (4 marks)

$v(t) = \mathcal{L}^{-1}\{V(s)\}$

$v(t) = \mathcal{L}^{-1}\left[\frac{12s}{s^2+400}\right]$

$v(t) = 12\,\mathcal{L}^{-1}\left[\frac{s}{s^2+20^2}\right]$

$v(t) = 12\cos(20t)$ V

4

$v(t) =$ 12 cos(20t) V

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QUESTION 5 (20 marks)

(a) A voltage signal $v(t) = v_1(t) + v_2(t)$ consists of a sinusoidal component $v_1(t) = V_A \cos\left(2\pi \frac{1}{T} t\right)$ and a DC component $v_2(t) = V_B$.

(i) State the RMS value of $v_1(t)$ and $v_2(t)$. (2 mark)

$V_{1(\text{RMS})} = \frac{V_A}{\sqrt{2}}$ (V_{rms})	$V_{2(\text{RMS})} = V_B$ (V_{rms})

2

(ii) Apply the definition of RMS to show that the RMS value of $v(t)$ is

$$V_{\text{RMS}} = \sqrt{V_{1(\text{RMS})}^2 + V_{2(\text{RMS})}^2} \text{ volts.}$$

where $V_{1(\text{RMS})}$ and $V_{2(\text{RMS})}$ denote the RMS value of $v_1(t)$ and $v_2(t)$ respectively. (3 marks)

Hint: It is not necessary to work out what $V_{1(\text{RMS})}$ and $V_{2(\text{RMS})}$ are.

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (v_1+v_2)^2 \cdot dt}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T \left(V_A\cos\left(\tfrac{2\pi}{T}t\right) + V_B\right)^2 \cdot dt}$$

$$V_{rms} = \sqrt{\frac{1}{T}\left[\underbrace{\int_0^T V_A^2\cos^2\left(\tfrac{2\pi}{T}t\right) dt}_{\text{RMS of } V_1 \times T} + \underbrace{\int_0^T V_B^2\, dt}_{\text{RMS of } V_2 \times T} + 2\int_0^T V_A V_B \cos\left(\tfrac{2\pi}{T}t\right) dt\right]}$$

$$V_{rms} = \sqrt{V_{1\,rms}^2 + V_{2\,rms}^2 + \frac{2}{T}\int_0^T V_A V_B \cos\left(\tfrac{2\pi}{T}t\right) dt}$$

$$V_{rms} = \sqrt{V_{1\,rms}^2 + V_{2\,rms}^2 + \frac{1}{T}(V_A V_B)\left[\cancel{2\pi\sin} \frac{T\sin\left(\frac{2\pi}{T}t\right)}{2\pi}\right]_0^T}$$

$$V_{rms} = \sqrt{V_{1\,rms}^2 + V_{2\,rms}^2 + \frac{1}{T}(V_A V_B)\left[\underbrace{\frac{T\sin 2\pi}{2\pi}}_{0} - \underbrace{\frac{T\sin 0}{2\pi}}_{0}\right]}$$

$$V_{rms} = \sqrt{V_{1\,rms}^2 + V_{2\,rms}^2 + 0 + 0}$$

$$V_{rms} = \sqrt{V_{1\,rms}^2 + V_{2\,rms}^2} \text{ volts} \qquad \text{Q.E.D}$$

3

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(iii) If $v(t)$ is applied across a load resistor $R_L = K\ \Omega$ (K is a real number), apply the definition of average power to prove that the average power dissipated by the resistor is

$$P_{\text{average}} = \frac{V_A^2}{2K} + \frac{V_B^2}{K} \text{ Watts.}$$ (3 marks)

Hint: You may find Q5(a)(i) and Q5(a)(ii) useful.

$P(t)_{average} = i(t) \cdot v(t) \times \frac{1}{2} = \frac{i(t)}{\sqrt{2}} \cdot \frac{v(t)}{\sqrt{2}}$ $\qquad \frac{v(t)}{\sqrt{2}} = V_{rms}$

$p(t) = \frac{V_{rms} \times V_{rms}}{R}$ $\qquad V = IR$

$I \Rightarrow \frac{V}{R}$, $I_{rms} = \frac{V_{rms}}{R}$

$= \frac{V_1^2 rms + V_2^2 rms}{R}$ $\qquad V_{rms} = \sqrt{V_1^2 rms + V_2^2 rms}$

$p(t) = \frac{\left(\frac{V_A}{\sqrt{2}}\right)^2}{K} + \frac{V_B^2}{K}$

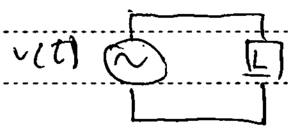
$P_{average} = \frac{V_A^2}{2K} + \frac{V_B^2}{K}$ W $\qquad$ Q.E.D

2

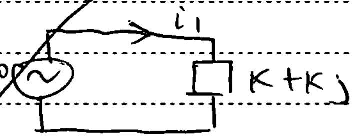
(iv) If $v(t)$ is applied across a load of impedance of $Z_L = (K + jK)\ \Omega$ at the frequency of input (K is a real number), use superposition to show that the steady-state load current $i_L(t)$ is

$$i_L(t) = \frac{V_A}{\sqrt{2}K}\cos\left(2\pi\frac{1}{T}t - 45^\circ\right) + \frac{V_B}{K} \text{ amperes.}$$ (3 marks)

i)



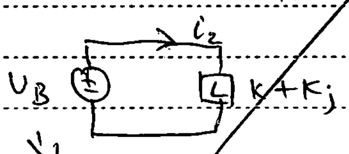
$\Rightarrow$ Turn V_2 off



3

$i_1 = \frac{V_A \angle 0^\circ}{\sqrt{2} K \angle 45^\circ} = \frac{V_A}{\sqrt{2} K} \angle -45^\circ$

Now turn off V_1



~~i_2~~ Ohm's law to find i_2

$\Rightarrow \frac{V_B}{K}$

Note V_B is a D.C signal so $\omega \Rightarrow 2\pi f$ frequency $= 0$ rad s^{-1}

So the ~~aspect~~ reactance $Kj) \Rightarrow 0$

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~~$i_1 =$~~ We found

$i_1 = \frac{V_A}{\sqrt{2}K} \angle -45°$ } same sinusoid as source voltage

rewrite as

$i_1 = \frac{V_A}{\sqrt{2}K} \cos(2\pi \frac{1}{T} t - 45°)$

$i_2 = \frac{V_B}{K}$

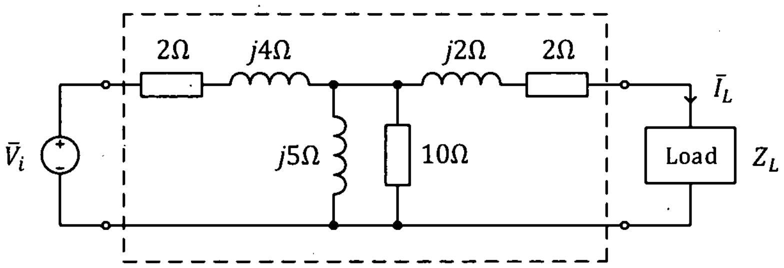
$i_L(t) = i_1 + i_2$

Add them up to get $i_L(t)$

$i_L(t) = \frac{V_A}{\sqrt{2}K} \cos\left(2\pi \cdot \frac{1}{T} t - 45°\right) + \frac{V_B}{K}$

Q.E.D

(b) Fig. 5.1 shows the equivalent circuit of a practical transformer connected in between an AC excitation $\bar{V}_i = 12\angle 45°$ V(**rms**) with a frequency of $\omega_i = 2$ rad/s and a load with an impedance of $Z_L = 3 + j2\ \Omega$ at this particular frequency.

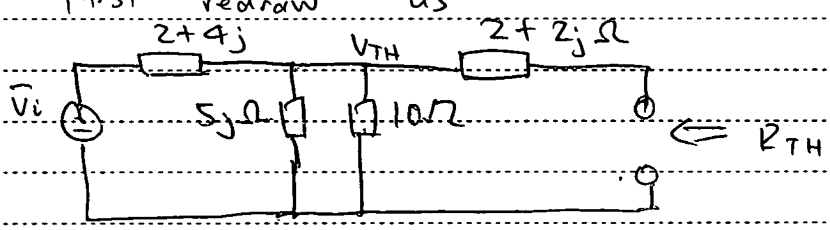


Practical Transformer

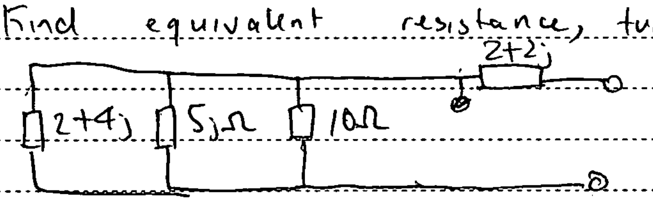
Fig. 5.1: Practical transformer equivalent circuit.

(i) Find and sketch the Thevenin equivalent circuit of the transformer circuit as seen by the load at steady-state. (3 marks)

First redraw as



i) Find equivalent resistance, turn off voltage source



2.5

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$((2+4j) \,//\, (5j) \,//\, (10)) + 2+2j = R_{TH}\ (Z_{TH})$

$\frac{(2+4j)5j}{2+9j} \,//\, 10 \quad + 2+2j = Z_{TH}$

$\frac{-20+10j}{2+9j} \,//\, 10 \quad + 2+2j = Z_{TH}$

Some calculations later :-

$Z_{TH} = \cancel{2.236 \angle 6.06^\circ} \quad + 2+2j$

$Z_{TH} = 2.236 \angle 63.43^\circ + 2+2j$

$Z_{TH} \Rightarrow 3+4j$

Find V_{TH} — since $2+2j$ impedance doesn't get current for V_{TH}

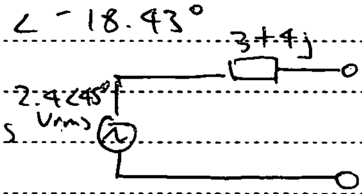
$i_{source} = \frac{\bar{V}_i}{2.236\angle 63.43^\circ}$

$i_{source} = \frac{12 \angle 45^\circ}{2.236 \angle 63.43^\circ} \Rightarrow 5.366 \angle -18.43^\circ$ A rms

$\Rightarrow$ current divider

$V_{TH} = \cancel{+0 \times i_{s}} \frac{5j}{10+5j} \times 5.366 \angle -18.43^\circ$

$V_{TH} \Rightarrow 2.4 \angle 45^\circ$ V_{TH} rms



(ii) Determine the time-domain expression of the steady-state load current $i_L(t)$. (2 marks)

$\bar{I}_L = \frac{V_{TH}}{Z_{TH}} = \frac{2.4 \angle 45^\circ}{5 \angle 53.13^\circ} \Rightarrow 0.48 \angle -8.13^\circ$ Arms

$i_L(t) = 0.48\,(w_i t - 8.13^\circ)$ — w_i is given to be 2 rad s^{-1}

$i_L(t) = 0.48\,(2t - 8.13^\circ)$ A rms

0.5

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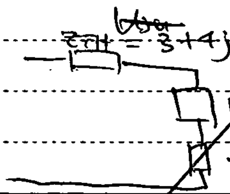
(iii) Explain what can be done in order to maximise the average power that the load Z_L can receive. (2 marks)

$Z_{TH} = 3+4j$ $Z_L = 3+2j$ @ $w_i = 2\ rad\,s^{-1}$

To maximise the average power load Z_L receives we must make $|Z_{TH}| = |Z_L|$, but minimise the reactive power. To do this Z_L must be the conjugate of Z_{TH}

Hence Z_L should be 3-4j.

We can add an impedance of $-6j$, IN SERIES WITH THE LOAD so that $Z_L - 6j \Rightarrow 3+2j-6j = 3-4j = Z_{TH}^*$ conjugate.

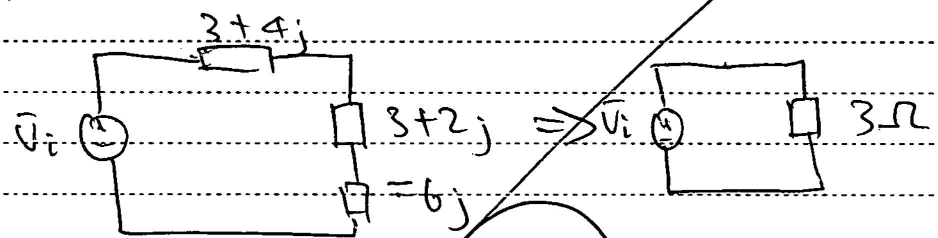


← will probably be a capacitor.

2

(iv) Find the maximum average power that a load connected to this transformer circuit can receive. (2 marks)

Max power aveage is when $|Z_{TH}| = |Z_L|$ and load is $3-4j$ as found earlier so.



$$P_{max} = \frac{(V_{TH})^2}{4\times3}$$

$$= \frac{2\cdot4^2\,V_{rms}^2}{12}$$

$$P_{max} = 0.48\ W$$

0

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QUESTION 6 (20 marks)

(a) Fig. 6.1 shows a simplified model of a power transmission-distribution network containing two loads. It is known that Load 1 has an impedance of $634.4 - j475.8\ \Omega$, and Load 2 was measured to absorb an average power of 15 W at a lagging power factor of 0.8. The

$\Rightarrow \phi_v - \phi_i$

$\phi_i < \phi_v$

current lags voltage.

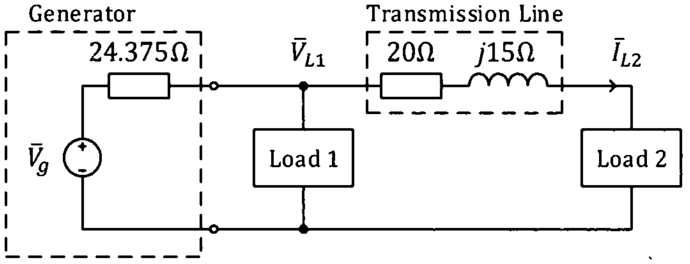

Fig. 6.1: Power transmission-distribution network.

(i) State whether each load is resistive, capacitive, or inductive. (2 marks)

Load 1 is: Capacitive	Load 2 is: Inductive

2

(ii) Determine the phasor current $\bar{I}_{L2}$ through Load 2. (3 marks)

~~P_{L2} = Pave = t~~ $P_{ave\,L2} = 15W$

~~1st~~ $P_{ave} = |s| \times \cancel{\angle(\phi_v - \phi_i)}\ \underbrace{\cos(\phi_v - \phi_i)}_{0.8}$ with lagging

$\cos(\phi_v - \phi_i) = 0.8$

$\phi_v - \phi_i = 36.86°$ but $\phi_v = 0°$

Hence $\phi_i = -36.86°$

Now find the magnitude of $\bar{I}_{L2}$

$P_{ave} = |s| \times 0.8$

$|s| = 18.75\ W$

$S = \frac{1}{2} V_m . I_m \angle(\phi_v - \phi_i)$

$S = V_{rms} . I_{rms}$

$I_{rms} = \frac{18.75}{120} \Rightarrow 0.15625\ A_{rms}$

$\bar{I}_{L2} = 0.15625 \angle -36.86°\ A\ rms$

3

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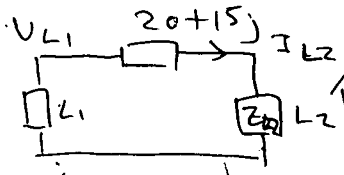
(iii) By means of a phasor diagram, explain why the voltage $\bar{V}_{L1}$ across Load 1 is in phase with the voltage $\bar{V}_{L2}$ across Load 2, **and** find the phasor voltage $\bar{V}_{L1}$. (4 marks)

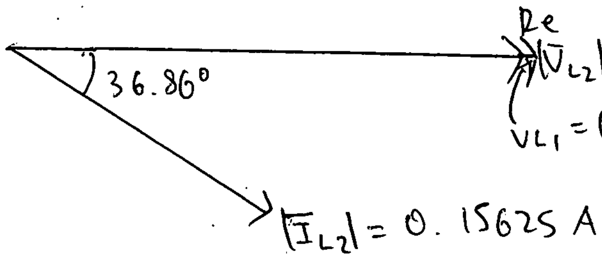


$\bar{Z}_{L1} = 634.4 - \cancel{475.8}\,\Omega$

$\frac{|V_1|}{|Z_{L1}|} = |I_{L1}|$

$\hookrightarrow V_{L1} = (\bar{Z}_{L2} + 20 + 15j) \times I_{L2}$

V_{L1} is calculated to be in phase with V_{L2}, since Z_{L2} and $(Z_{L2} + 20 + 15j)$ have the same phase angle.



$V_{L1} = (Z_{L2} + 20 + 15j) \times I_{L2} = 123.906\angle 0°$ Vrms

$\bar{Z}_{L2} = \frac{120\angle 0°}{0.15625\angle -36.86°}$

$\bar{Z}_{L2} = 614.4 + 460.8j\ \Omega$

also $\Rightarrow 768\angle 36.86°\ \Omega$

~~$\bar{Z}_{L2} = 634.4 + 475.8\,\Omega$~~

AND

$Z_{L2} + 20 + 15j \Rightarrow Z_{L1}$ conjugate

but $(Z_{L2} + 20 + 15j) I_{L2} = V_{L1}$

$V_{L1} = (634.4 + 475.8j) \times I_{L2} \Rightarrow 123.906\angle 0°$

4

(iv) Using your answer from Q6(a)(iii), calculate the complex power of Load 1. (2 marks)

$S = \frac{1}{2} \bar{V} . (\bar{I})^*$

$S = \frac{1}{2} \bar{V}_{L1} (\bar{I}_1)^*$

$= \cancel{\frac{1}{2}}\ V_{rms} \times (I_{rms}^*)$

$= 123.906\angle 0° \times \left(\frac{V_{rms}}{Z}\right)^*$

$S = \frac{123.906^2}{Z^*} \Rightarrow \frac{123.906^2}{634.4 + 475.8j} = \cancel{0.1562}$

$c\phi_v - \phi_i = 0.8$

leading.

$S = 19.36\,W$

1.5

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(v) It is known that the generator in this scenario does not produce any reactive power, what can you say about the equivalent impedance of the network seen by the generator? (1 marks)

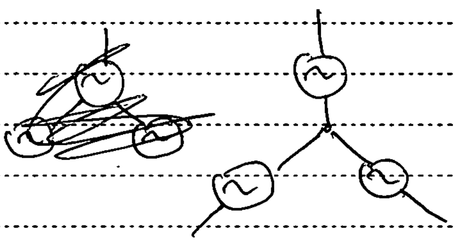
If the generator does not produce reactive power, the equivalent impedance of the network seen by the generator is said to have 0 reactance. We can also say this is purely resistive.

(b) A balanced three-phase Y-connected generator with a **negative** (acb) phase sequence has an impedance of $3 + j1.5\ \Omega$ and an internal voltage of 230 V(rms) per phase. The generator feeds a balanced three-phase Y-connected load having an impedance of $21.5 - j27\ \Omega$ per phase via a transmission line with an impedance of $0.5 + j0.5\ \Omega$ per phase.

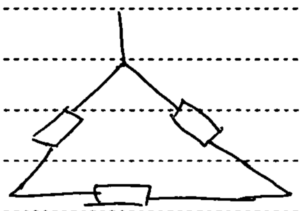
(i) Briefly explain what it is meant by a "balanced" three-phase circuit and why it is desirable for a three-phase system to be balanced. (3 marks)

~~The~~ A balanced three phase circuit is a circuit with 3 phase loads or generators in a delta or WYE connection

i.e



or



~~This usually~~ The balance allows the signal (voltage) to be mostly constant, so that the signal doesn't deviate too much. Too much perturbation in signal can lead to vibration, i.e in refridgerators with single phase systems, the fridge can get vibrate because of unbalance

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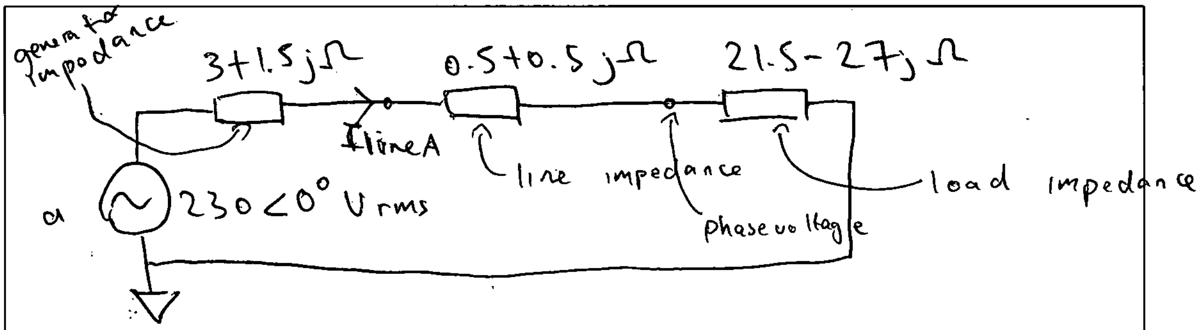
(ii) By taking the a-phase internal voltage of the generator as the reference, construct the single-phase equivalent circuit of the system using the a-phase. (1 mark)



By using the single-phase equivalent circuit in (ii), NEGATIVE SEQUENCE

(iii) Find the line-current in a-phase of the system. (1 mark)

$I_{line A} = \frac{V_a}{Z_a} = \frac{230\angle 0^\circ}{25-25j} = \frac{230\angle 0^\circ}{35.35\angle -45^\circ}$

$I_{line A} = 6.505 \angle 45^\circ$ A rms

(iv) Determine the phase voltage at the c-phase terminal of the load. (2 mark)

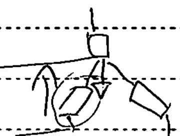
Same as line to neutral voltage.

Find line to neutral for phase A

$V_{line\ to\ neutral}(A) = 6.505 \angle 45 \times (21.5-27j)$

$V_{LN}(A) = I_{LINE A} \times 21.5-27j$

$V_{LN}(A) \Rightarrow 224.529 \angle -6.4698^\circ$



~~negative~~ phase C, −120°

$V_{phase}(c) \Rightarrow 224.53 \angle -126.47^\circ$ V rms

2

(v) Calculate the apparent power drawn by the three-phase load. (1 mark)

S = apparent power $\bar{V}_{rms}$ = line voltage

$\bar{I}_{rms}$ = line current

$S = 3 \times \bar{V}_{rms} \times \bar{I}_{rms}$

$S = 3 \times 224.53 \angle -6.46 \times 6.505 \angle 45^\circ$

$S = 3 \times 1460.65 \angle 38.53^\circ$

$S = 4381.96$ W

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APPENDIX

Some Laplace Transform formulae

$f(t)$	$F(s)$
$\delta(t)$	1
$\mu(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
$f(t)e^{-at}$	$F(s+a)$
$\frac{df(t)}{dt}$	$sF(s)-f(0^-)$
$\frac{t^{n-1}}{(n-1)!}e^{-at}\mu(t)$	$\frac{1}{(s+a)^n}$
$\sin \omega t$	$\frac{\omega}{(s^2+\omega^2)}$
$\cos \omega t$	$\frac{s}{(s^2+\omega^2)}$
$f(0^+)$	$\lim_{s\to\infty} sF(s)$
$f(\infty)$	$\lim_{s\to 0} sF(s)$

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