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Fourier-based optimal excitation trajectories for the dynamic identification of robots

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SUMMARY

This paper describes a new approach to the parameterization of robot excitation trajectories for optimal robot identification. The trajectory parameterization is based on a combined Fourier series and polynomial functions. The coefficients of the Fourier series are optimized for minimal sensitivity of the identification to measurement disturbances, which is measured as the d-optimality criterion, taking into account motion constraints in joint and Cartesian space. This parameterization satisfies both the guarantees of convergence by adding terms and the matching of the boundary conditions. Application of the method for the identification of the CRS A465 industrial robot proves the validity of the proposed approach.

KEYWORDS: Dynamic identification; Excitation trajectory; Fourier coefficients; D-optimality.

I. INTRODUCTION

The need for robot manipulator is increasing in order to raise the productivity and improve the quality of products in manufacturing industry. In this respect, off-line programming supported by simulation, and accurate motion control have become necessary. Accurate robot control and realistic robot simulation require an accurate dynamic robot model. The design of an advanced robot controller is based on the robot model, and its performance depends directly on the model accuracy. Experimental robot identification is the only efficient way to obtain accurate robot models as well as indications on their accuracy, confidence and validity. The dynamic model parameters provided by robot manufacturers are insufficient, inaccurate, or often non-existing, especially those dealing with friction and compliance characteristics. Direct measurement of the physical parameters is unrealistic, because of the complexity of most robots.

A typical experimental identification procedure consists of the following three steps: (1) the generation of an identifiable dynamic robot model; (2) the generation of optimized excitation trajectories; and (3) the estimation of the model parameters. For each of these steps the user has to make an optimal choice between different options, depending on the application of the model.

With respect to the model generation, several methods have been designed in the last decade. They can be divided into two categories according to the models and the type of sensors they use. The parameters are estimated from motion data(joint encoder signals) and actuator torques(actuator current measurements), both measured by "internal" measurement devices, when we are using an internal model.[1,2] An alternative approach makes use of the so-called reaction or external model of the robot.[3,4] This model relates the motion of the robot to the reaction forces and moments on its base plate and is, therefore, totally independent from internal torques such as joint friction torques.

It is well recognized that reliable, accurate, and efficient robot identification requires specially designed experiments. When designing an identification experiment for a robot manipulator, it is essential to consider whether (1) the excitation is sufficient to provide accurate and fast parameter estimation in the presence of disturbances such as measurement noise and actuator disturbances, and (2) the processing of the resulting data is simple and yields accurate and consistent results.

Swevers et al.[5] present a robot identification method using optimized excitation trajectories based on finite Fourier series. This approach guarantees periodic excitation, which is advantageous because it allows time domain data averaging which improves the signal to noise ratio of the measurements. Furthermore, this approach also allows calculation of the joint velocities and accelerations in the frequency domain from the measured position response. Frequency domain calculation of the time derivative of a signal is correct since the discrete Fourier transform does not introduce leakage errors[6] due to the periodicity of the considered signals. In addition, this allows filtering of the signal in the frequency domain, by windowing the spectrum, yielding an important noise reduction without introducing phase distortions.

This approach, however, has the following disadvantages: (1) mismatch of the required boundary conditions; (2) no guarantee of the convergence of the joint derivatives, i.e. joint velocities and accelerations; (3) slowness of the rate of convergence of the Fourier series. This paper presents a new approach toward the design of robot excitation trajectories that is based on a combined Fourier series and polynomial functions, to overcome the drawbacks mentioned above. Section II describes the generation of dynamic robot model (internal model), and formulation of ML (Maximum Likelihood) method and WLS (Weighted Least Squares) method to be applied to identify the robot parameters.

Section III presents a new approach toward the design of optimal robot excitation trajectories. We discuss the application of the presented techniques for the experimental identification of the first three axes of a CRS A465 industrial robot in Section IV.

II. ESTIMATION OF ROBOT PARAMETERS

II.1. Model reduction

The following set of n different equations describes the dynamic behavior of an n degree of freedom rigid robot[7]

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{c}(\boldsymbol{q}) = \boldsymbol{\tau}_d \tag{1}$$

where $\boldsymbol{q}$ is the n-vector of the joint angles, $\boldsymbol{M}(\boldsymbol{q})$ is the $(n \times n)$ inertial acceleration-related matrix (which is a function of the joint angles), $\boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ the $(n \times 1)$ nonlinear Coriolis and centrifugal force vector and $\boldsymbol{c}(\boldsymbol{q})$ the $(n \times 1)$ gravity loading force vector. Also $\boldsymbol{\tau}_d$ is the vector of the generalized forces associated with $\boldsymbol{q}$, i.e. the torques applied at the joints by the actuators.

The friction torque $\boldsymbol{\tau}_f$ can be expressed according to one of several models proposed in the literature, taking into account the different friction components, e.g., stiction, Coulomb, and viscous friction[8]

$$\boldsymbol{\tau}_f = \boldsymbol{F}(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{\beta}_f \tag{2}$$

where the expression of the function $\boldsymbol{F}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ and the choice of the parameter vector $\boldsymbol{\beta}_f$ $(n_f \times 1)$ depend on the considered friction model. Note that if only Coulomb and viscous friction are included in the model, then $\boldsymbol{F}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is a function of velocity $\dot{\boldsymbol{q}}$ only.

By introducing vector $\boldsymbol{\tau}$, containing the n torques applied to the joints Eqs. (1) and (2) can be rearranged in the form

$$\boldsymbol{\tau} = \bar{\boldsymbol{\Phi}}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})\boldsymbol{\beta}_m \tag{3}$$

which is linear with respect to the vector $\boldsymbol{\beta}_m(m \times 1)$ which contains all the inertial parameters of the manipulator, i.e., the masses m_i, the inertia moments(six for each link), and the first-order moments(three for each link) of the robot links and the n_f friction parameters. $\bar{\boldsymbol{\Phi}}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$, which is a nonlinear function of joint position, velocity, and acceleration vectors, is then an $n \times m$ matrix with $m = 10n + n_f$.

It is well known that only some of the inertial parameters of the robot really affect its dynamics and are then identifiable by identification procedures. It is possible to determine the set of minimum number of inertial parameters, i.e., the base parameter set containing only the p identifiable dynamic parameters collected in $\boldsymbol{\beta}_p(p \times 1)$, by means of the reduction procedure proposed by Gautier and Khalil,[9] based on the energy mode of the robot. Equation (3) can be then rewritten as

$$\boldsymbol{\tau} = \boldsymbol{\Phi}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})\boldsymbol{\beta} \tag{4}$$

in which $\boldsymbol{\beta}(r \times 1) = [\boldsymbol{\beta}_p^T \boldsymbol{\beta}_f^T]^T$ with $r = p + n_f$, and the $n \times r$ regression matrix $\boldsymbol{\Phi}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$ is properly defined. Relation (4), which is linear with respect to $\boldsymbol{\beta}$ can be used for the estimation of $\boldsymbol{\beta}$ by collecting the values of $\boldsymbol{\tau}$, $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$, and $\ddot{\boldsymbol{q}}$.

II.2. Least-squares parameter estimation

Robot identification deals with the problem of estimating the model parameters from the data measured during a robot excitation experiment. In most cases, the data are sequences of joint angles and motor currents, from which a sequence of joint velocities, accelerations, and motor torques are calculated. Actuator torques are assumed to be proportional to the actuator current.

In system identification based on a statistical framework, it is common to assume that the measured joint angles $q_m(k)$ and actuator torques $\tau_m(k)$, for $k = 1, N$, are corrupted by independent zero-mean Gaussian noise $n_q(k)$, $n_\tau(k)$, i.e.,

$$\begin{aligned} q_m(k) &= q(k) + n_q(k) \\ \tau_m(k) &= \tau(k) + n_\tau(k) \end{aligned} \tag{5}$$

where N is the number of data samples. The noise-free joint angles $q(k)$ and actuator torques $\tau(k)$ satisfy equation (4).

Considering a linear relation between the motor current and torque is a first order approximation, which may result in modeling errors. Extension of the robot model with a model describing the relation between the motor torque and the motor current eliminates these modeling errors, and yields a robot model to which the trajectory design and maximum-likelihood estimation methods which are presented below, are still applicable.

The maximum-likelihood estimate $\boldsymbol{\beta}_{ml}$ of the parameter vector $\boldsymbol{\beta}$ is given by the value $\boldsymbol{\beta}$ which maximizes the likelihood of the measurement $q_m(k)$ and $\tau_m(k)$. Since the noise on the different measurements is independent and Gaussian, this corresponds to minimizing the following quadratic cost function:[5]

$$K(\boldsymbol{q}_m, \tau_m | \theta) = \frac{1}{2} \sum_{k=1}^{N} \sum_{i=1}^{n} \left(\frac{n_{q_i}^2(k)}{\sigma_{q_i}^2} + \frac{n_{\tau_i}^2(k)}{\sigma_{\tau_i}^2} \right) \tag{6}$$

with $n_{q_i}(k)$ the noise on the measured joint angle $q_i(k)$, and $n_{\tau_i}(k)$ the noise in the measured actuator torque $\tau_i(k)$. $\sigma_{q_i}^2$ and $\sigma_{\tau_i}^2$ are their corresponding variances. It is assumed that these variances are constant and known. The minimization of criterion equation (6), taking into account equation (4), is a nonlinear least squares optimization problem. Its practical implementation requires that $n_q(k)$ and $n_\tau(k)$ be calculated for every estimate of $\boldsymbol{\beta}$ given the measured data $q_m(k)$ and $\tau_m(k)$.

The maximum-likelihood parameter estimation simplifies significantly if the measured joint angles are free of noise. This assumption can be justified by the fact that the noise level on the joint angle measurement is much smaller than the noise level on the actuator torque measurements. Under this assumption, maximum-likelihood parameter estimation reduces to the weighted linear least squares estimate for which the weighting function is the reciprocal of the standard deviation of the noise on the measured actuator torque values (Σ^{-1}).[10]

$$\boldsymbol{\beta}_{ml} = (\boldsymbol{F}^t \boldsymbol{\Sigma}^{-1} \boldsymbol{F})^{-1} \boldsymbol{F}^t \boldsymbol{\Sigma}^{-1} \boldsymbol{b} \tag{7}$$

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with

$$F = \begin{bmatrix} \Phi(q_m(1), \dot{q}_m(1), \ddot{q}_m(1)) \\ \vdots \\ \Phi(q_m(N), \dot{q}_m(N), \ddot{q}_m(N)) \end{bmatrix} \tag{8}$$

and

$$b = \begin{bmatrix} \tau_m(1) \\ \vdots \\ \tau_m(N) \end{bmatrix} \tag{9}$$

and Σ is the diagonal covariance matrix of the measured actuator torques. The covariance matrix of the maximum-likelihood β_{ml} is equals to

$$\left(F^t \Sigma^{-1} F\right)^{-1} \tag{10}$$

This simplification is applied here based on the results of Olsen et al.[11] In that paper, the simulated identification experiments showed that the maximum-likelihood performs better than a weighted least squares method. Nevertheless, using data measured on a real robot, no difference between the weighted least squares method and the maximum-likelihood method was observed, which was due to the fact that the variances on the measured positions was very small.

III. OPTIMAL ROBOT EXCITATION TRAJECTORIES

The generation of an optimal robot excitation trajectory involves nonlinear optimization with motion constraints (i.e., constraints on joint angle, velocities and accelerations, and on the robot end effector position in the Cartesian space in order to avoid collision with the environment and with itself). Several approaches have been presented. They all used a different trajectory parameterization. These parameters are the degrees of freedom of the optimization problem. Armstrong et al.[8] described an approach in which the degrees of freedom are the points of a sequence of joint accelerations. This approach is the most general one, but it results in a large number of degrees of freedom, such that optimization is cumbersome. The optimization is done by maximizing the minimum singular value of the matrix F^tF.

Gautier and Khalil[12] optimized a linear combination of the condition number and the equilibrium of the set of equations that generate the parameters, i.e. of matrix F. The degrees of freedom are a finite set of joint angles and velocities separated in time. The actual trajectory is continuous and smooth, and is calculated by interpolating a fifth-order polynomial between the optimized points, assuming zero initial and final acceleration. Only a very small part, namely the finite set of joint angles and velocities, of the final trajectory is optimized. As a result, the total smooth trajectory cannot be guaranteed to satisfy all motion constraints nor to be optimal with respect to the condition number or the covariance matrix criterion.

Swevers et al.[5] presented a robot excitation that is periodic. The excitation trajectory for each joint is a finite Fourier series. This approach guarantees (1) improving the quality of the measured signals by time-domain averaging, (2) estimating the noise characteristics without performing additional measurements, and (3) calculating accurate and noise-free estimates of the joint velocities and accelerations, which are required to calculate the identification matrix Φ (equation (4)).

This approach, however, has the following disadvantages:

(1) Convergence is guaranteed only in (0, t_f), where t_f is duration time. To satisfy the requirements on arbitrary conditions, convergence should be extended from (0, t_f) to [0, t_f].
(2) Although the joint angle converges to the optimal solution, there is no guarantee that the derivatives of joint angle, i.e., joint velocity and acceleration, will converge to the derivatives of the optimal solution.
(3) The rate of convergence of the Fourier series depends on the optimal solution. This can be quite slow.

This section presents a new approach toward the design of robot excitation trajectories that is based on a combined Fourier series and polynomial, to overcome the disadvantages mentioned above.

III.1. Parameterization of the robot excitation trajectory

The angular position q_i for joint i of a n degree of freedom robot are written as

$$q_i(t) = \lambda_i(t) + \delta_i(t) \tag{11}$$

where

$$\lambda_i(t) = \sum_{j=0}^{5} \lambda_{ij} t^j, \quad \delta_i(t) = \sum_{m_f=1}^{M} a_{im_f} \cos \frac{m_f \pi}{t_f} t \tag{12}$$

with t_f the duration time, i.e. the period. This Fourier series specifies a periodic function with fundamental pulsation $\omega_f = 2\pi/t_f$. The fundamental pulsation is common for all joints, in order to preserve the periodicity of the overall robot excitation. Here the constant term of the Fourier series has been included in the fifth order polynomial $\lambda_i(t)$.

The boundary condition requirements can be written as

$$\begin{aligned} q_i(0) &= \lambda_i(0) + \delta_i(0), & q_i(t_f) &= \lambda_i(t_f) + \delta_i(t_f) \\ \dot{q}_i(0) &= \dot{\lambda}_i(0) + \dot{\delta}_i(0), & \dot{q}_i(t_f) &= \dot{\lambda}_i(t_f) + \dot{\delta}_i(t_f) \\ \ddot{q}_i(0) &= \ddot{\lambda}_i(0) + \ddot{\delta}_i(0), & \ddot{q}_i(t_f) &= \ddot{\lambda}_i(t_f) + \ddot{\delta}_i(t_f) \end{aligned} \tag{13}$$

These equations can be used to determine the coefficients of the polynomial in terms of the coefficients of the Fourier series and the boundary values. Solving the above boundary condition equations gives the following closed-form expression of the six coefficients:

$$\lambda_{ij} = (a_{im_f}, \text{ boundary values of } q_i, \dot{q}_i, \ddot{q}_i) \tag{14}$$

Therefore we can remedy the disadvantages of the Fourier series expansion, not including the number of terms to be designed. Each Fourier series contains M parameters, that constitute the degrees of freedom for the optimization problem: a_{im_f} for $m_f = 1$ to M, which are the amplitude of the cosine functions. Once the design variables are determined, the joint displacements, velocities and accelerations can be obtained by differentiating equation (10).

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This approach guarantees band-limited periodic trajectories and therefore allows:

(1) time-domain data averaging, which improves the signal-to-noise ratio of the experimental data. This is extremely important since motor current(torque) measurements are very noisy.
(2) Estimation of the characteristics of the measurement noise. This information is valuable in case of maximum-likelihood parameter estimation.
(3) Specification of the bandwidth of the excitation trajectories, such that excitation of the robot flexibility can be either completely avoided or intentionally brought about.
(4) Calculation of joint velocities and accelerations from the measured response in an analytic way. For this purpose, the measured encoder readings are first approximated, in a least squares sense, as a combined Fourier series and polynomial functions. This corresponds to taking the discrete Fourier transform of the encoder readings and selecting the main spectral lines. The Fourier transform does not introduce leakage errors because of the periodicity of the excitation. This frequency-domain approach toward the differentiation of time series is simple, efficient and accurate.
(5) Guarantees of convergence by adding Fourier series terms and exact satisfactions of boundary conditions, including velocity and acceleration boundary conditions, can be achieved by combining Fourier series and polynomial functions.

III.2. Optimization of the parameterized robot excitation trajectory

If the joint angle measurements are free of noise, and if the model parameters estimated according to maximum-likelihood criterion equation (7), the covariance matrix of the estimated model parameters equals equation (10). Equation (10) does not depend on the measurements or the estimated parameters. It depends on the exact joint angles, velocities and accelerations which are assumed to correspond to the designed excitation trajectory. As a result, the optimization of the model parameter covariance matrix as function of the trajectory parameters does not require the knowledge of the exact model parameter vector $\boldsymbol{\beta}$.

The covariance matrix of the parameter estimates cannot be calculated if the actuator torque and joint angle measurements are both corrupted by noise. However, the covariance matrix approaches the Cramér-Rao lower bound, i.e. the inverse of the Fisher information matrix, asymptotically if the parameters are estimated with an efficient estimator, for example a maximum-likelihood estimator. This suggests taking the Cramér-Rao lower bound on the covariance matrix instead of the covariance matrix itself as the robot excitation design criterion.

The covariance matrix or its Cramér-Rao lower bound cannot be optimized in matrix sense. They have to be replaced by a representative scalar measure. Ljung[6] presents some possible scalar measures. –log det $\boldsymbol{P}$, with $\boldsymbol{P}$ the covariance matrix or its Cramér-Rao lower bound, is called the *d-optimality* criterion and is the most appealing of these measures: 1) its minimum is independent of the scaling of the parameters and 2) it has a physically interpretation: the determinant of $\boldsymbol{P}$ is related to the volume of highest probability density region for the parameters. As the remaining noise on the measured joint angles can be neglected compared with the noise on the torques,[5] we can take $\boldsymbol{P}$ as the covariance matrix (equation (10)).

The minimization of the uncertainty on the estimated parameters is a complex nonlinear optimization problem with motion constraints. The motion constraints are limitations on the joint angles, velocities and accelerations, and on the robot end effector position in the Cartesian space in order to avoid collision with the environment and with itself. This last type of constraint involves forward kinematics calculations. All constraints are implemented as continuous functions which are negative if the constraint is satisfied and positive if it is violated.

IV. NUMERICAL EXAMPLE AND SIMULATION EXPERIMENT

The simulation uses a model of a CRS A465 robot. Only the first three axes are considered. Fig. 1 shows the robot, its base coordinate system (X_0, Y_0, Z_0), and coordinate systems for the first three links: (X_i, Y_i, Z_i), $i = 1,2,3$. The inertial parameters of the links are related to their coordinate systems.

(1) I^i_{xx}, I^i_{yy}, I^i_{zz} are the moments of inertia of link i about the X_i, Y_i and Z_i axis, respectively, $i = 1, 2, 3$.
(2) I^i_{xy}, I^i_{yz}, I^i_{xz} are the inertia products of link i, $i = 1, 2, 3$.
(3) $(\bar{x}_i, \bar{y}_i, \bar{z}_i)$ is the position of the center of mass of link i in (X_i, Y_i, Z_i), $i = 1, 2, 3$.
(4) l_i is the position of the joint of link i in $(X_{i-1}, Y_{i-1}, Z_{i-1})$, $i = 1, 2, 3$.
(5) m_i is the mass of link i, $i = 1, 2, 3$.

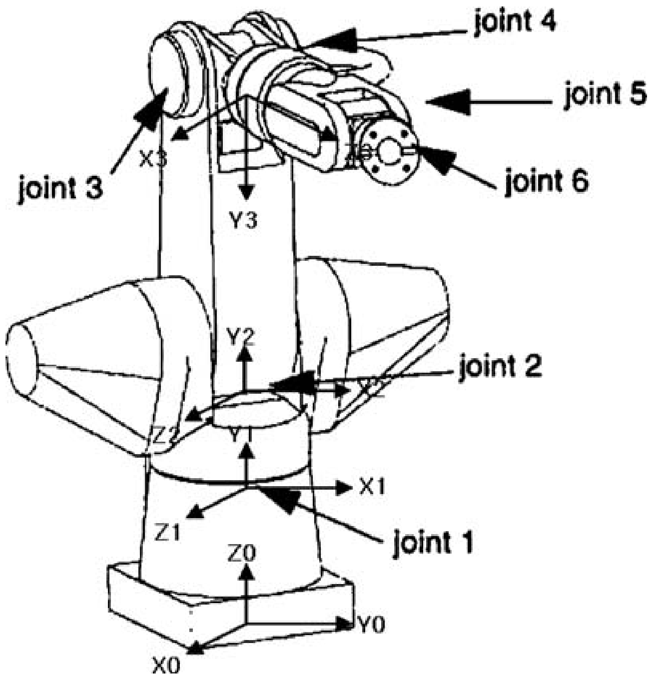

Fig. 1. The link coordinate system for CRS A465 robot.

Table I. Data for the simulation experiments.

1	2		3	4	5
i	Physical meaning of parameter		*Exact*	*Estimated*	σ_i
1	$I^1_{yy} + I^2_{yy} + I^3_{xx}$	[kg·m^2]	0.2812	0.0660	0.1141
2	$m_2\bar{x}_2 + l_2 m_2 + l_2 m_3$	[kg·m]	4.3310	4.3209	0.0220
3	$m_2\bar{y}_2$	[kg·m]	−0.0910	−0.0732	0.0203
4	$I^2_{xx} - I^2_{yy}$	[kg·m^2]	−0.0898	0.1169	0.0986
5	I^2_{xy}	[kg·m^2]	−0.0028	0.0284	0.0516
6	$I^2_{xz} + l_2 m_3 \bar{y}_3 + l_2 m_2 \bar{z}_2$	[kg·m^2]	0.0458	0.0220	0.0478
7	I^2_{yz}	[kg·m^2]	0.0006	−0.0157	0.0407
8	I^2_{zz}	[kg·m^2]	0.1204	0.1416	0.0384
9	$m_3\bar{x}_3$	[kg·m]	−0.0500	−0.0743	0.0138
10	$m_3\bar{z}_3$	[kg·m]	−1.9800	−1.9705	0.0130
11	$I^3_{zz} - I^3_{xx}$	[kg·m^2]	−0.0951	−0.1614	0.0483
12	I^3_{xy}	[kg·m^2]	0.0003	0.0493	0.0296
13	I^3_{xz}	[kg·m^2]	−0.0016	0.0252	0.0263
14	I^3_{yy}	[kg·m^2]	0.1081	0.1144	0.0166
15	I^3_{yz}	[kg·m^2]	0.0016	−0.0059	0.0233
16	b_1	[N·m·s]	1.7760	1.3954	0.2029
17	b_2	[N·m·s]	3.3480	3.4604	0.1244
18	b_3	[N·m·s]	2.3110	2.4069	0.1149
19	c_1	[N·m]	3.5120	3.8686	0.2170
20	c_2	[N·m]	0.6350	0.3888	0.2619
21	c_3	[N·m]	1.4510	1.6514	0.1272

The joint friction model includes viscous and Coulomb friction, represented by constant coefficients, b_i and c_i, respectively.

IV.1. The robot model

The closed-form equations were derived for CRS A465 robot. Column 2 and 3 of Table I present the resulting minimal set of parameters after model reduction and the values are used for the simulation experiments, respectively. The minimal robot model is given by equation (4) for which $\boldsymbol{\tau}$ is a 3×1 column vector, $\boldsymbol{\beta}$ a 21×1 column vector, $\boldsymbol{\Phi}$ a 3×21 matrix. The exact actuator torques are calculated according to equation (4) using the model parameters presented in Table I and an optimized excitation trajectory. Section 4.2 discusses the trajectory optimization. Adding independent zero-mean Gaussian noise to the exact actuator torques simulates torque measurement noise. The variance of the noise is $25\,\mathrm{N^2m^2}$, $16\,\mathrm{N^2m^2}$ and $9\,\mathrm{N^2m^2}$ for the actuator torques of joint 1, 2 and 3, respectively. The joint angles are assumed to be free of noise.

IV.2. Trajectory optimization

The robot excitation trajectory is optimized according to a criterion $-\log\det(\boldsymbol{F}'\Sigma^{-1}\boldsymbol{F})$. $\boldsymbol{F}'\Sigma^{-1}\boldsymbol{F}$ is the information matrix related to the maximum-likelihood estimation parameters. It is equal to the inverse of the covariance matrix of the parameter estimates(equation (10)).

The motion constraints correspond to those of the CRS A465 in our laboratory environment.

(1) Joint angle limits (rad): $-3.0 < \theta_1 < 3.0$, $-1.6 < \theta_2 < 1.6$, $-1.9 < \theta_3 < 1.9$.
(2) Joint velocity limits(rad/s): $-3.14 < \dot{\theta}_i < 3.14$.
(3) Joint acceleration limits(rad/s^2): $-12.5 < \ddot{\theta}_i < 12.5$.
(4) The robot touches its base if $r_{ee} < 330$ mm where r_{ee} is the distance of the end effector from the first robot axis. It can be obtained from forward kinematics calculations.

The excitation trajectories are five-term Fourier series, yielding 15 parameters for three joints. The fundamental frequency of the trajectories is 0.2 Hz. The sampling rate for the simulation is 300 Hz. The length of the data sequence is 1500 data samples, i.e., one period of the trajectory. The constrained optimizations are performed using the "FMINCON" function of the Optimization Toolbox of Matlab. The unconstrained optimizations are also performed using the "FMINSEARCH" function of Matlab, to verify and to be compared with the constrained results.

Figure 2 shows the excitation trajectories optimized under constraints according to the criterion based on the determinant of the covariance matrix. The initial trajectory was chosen as the cycloidal motion, which is known to be very smooth types of motion, largely used in the design of cam profile. The iteration process was stopped after 5,000 iterations. We can see that the boundary values for velocity and acceleration as well as displacement exactly satisfy the required conditions. It can be shown from the results that the fluctuations of the excitation trajectories are larger than those of the cycloidal motion. This fact can be confirmed from Table II also, by observing the coefficients for higher harmonics become larger compared with the cycloidal trajectory.

Figure 3 shows the tip path both for the joint trajectory optimized with the constraints and without those. It can be seen

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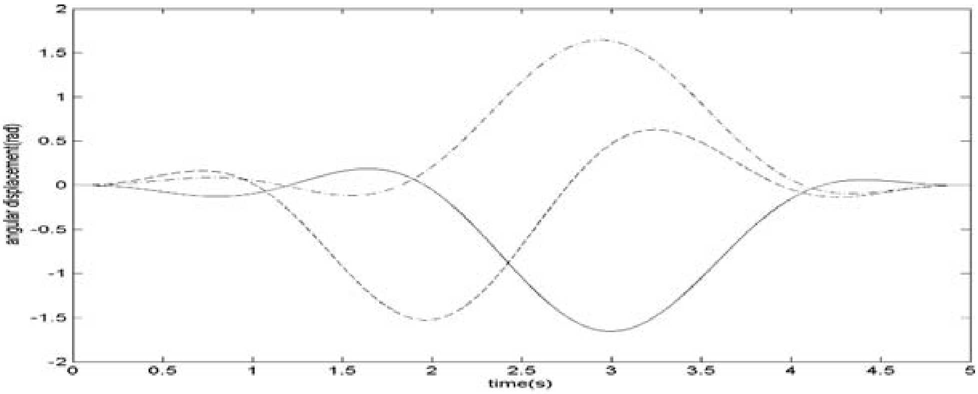
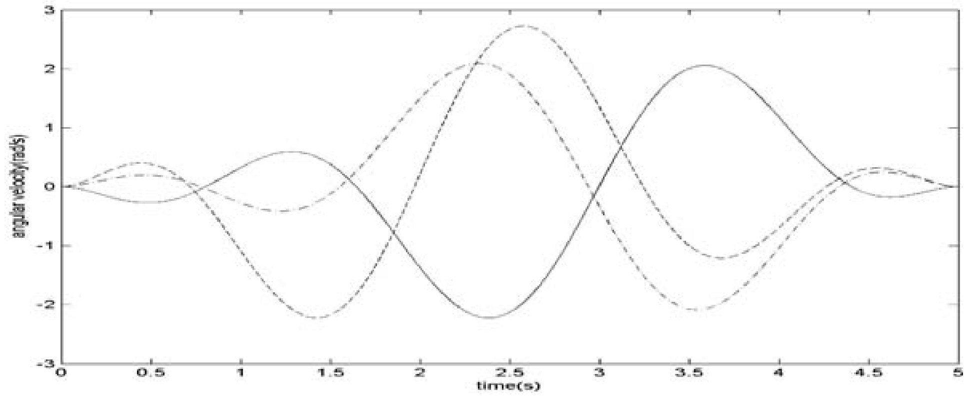
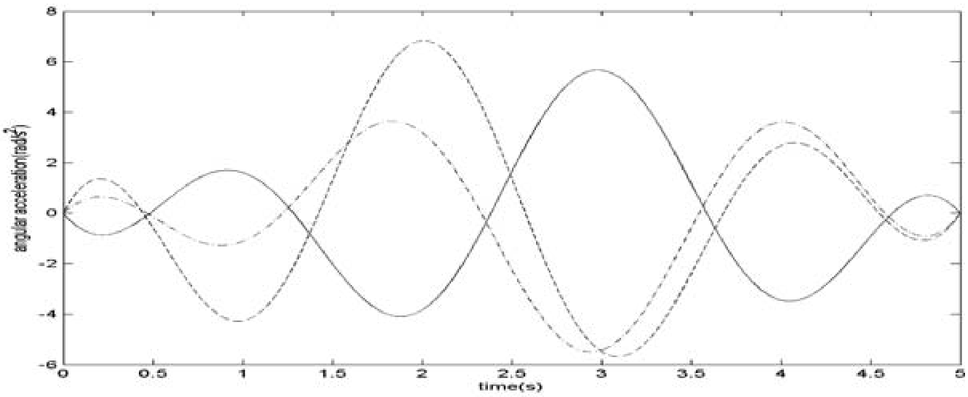


(a) angular displacement



(b) angular velocity



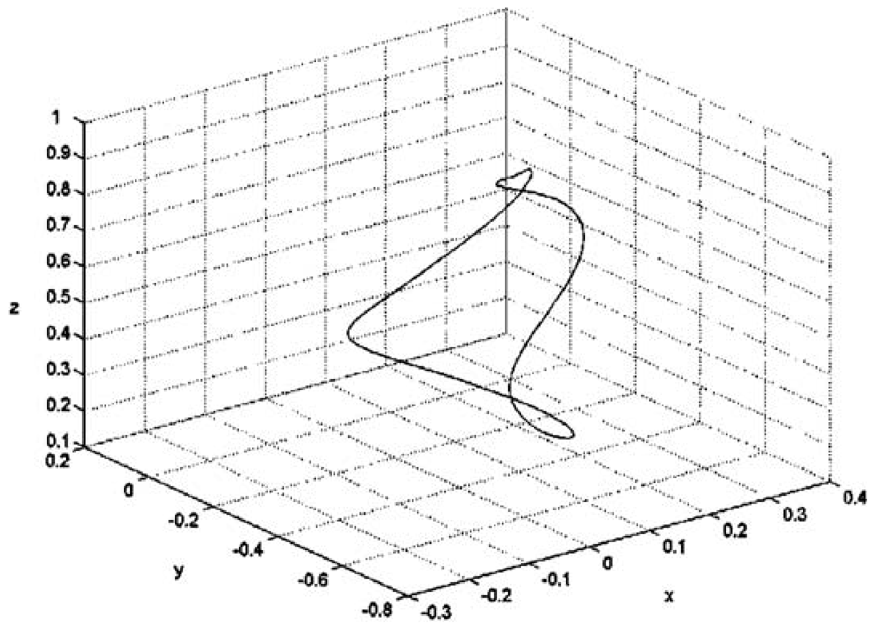
(c) angular acceleration

Fig. 2. Optimized excitation trajectory(solid line: axis 1, dashed line: axis 2, dash-dotted line: axis 3).

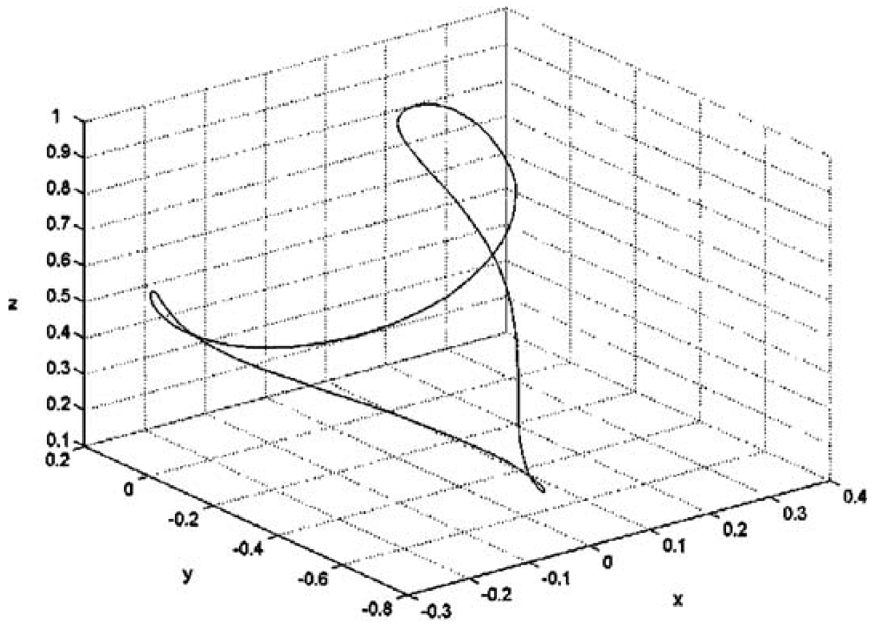
from the unconstrained path that the tip moves largely in the space. The tip path for the constrained tries to follow the unconstrained path but returns to the end point without completing the motion developed in the unconstrained path, due to the motion constraints. It can be confirmed from these results that the optimized trajectory is near optimal solution, although the obtained optima are not necessarily the global optima.

The parameters obtained from the weighted linear least squares estimate by using the excitation trajectory optimized under constraints are shown in column 4 of Table I. The column 5 of Table I shows the square root of the diagonal elements of the covariance matrix(σ_i). These elements are the standard deviations of the maximum-likelihood model parameter estimates. The average relative error of the estimated parameters in relation to the exact parameter, $\boldsymbol{\beta}^*$, defined as:

$$\varepsilon_{AV} = \frac{1}{21}\sum_{i=1}^{21}\left|\frac{\beta_i^* - \beta_i}{\beta_i^*}\right|$$



(a) Constrained optimization



(b) Unconstrained optimization

Fig. 3. Tip path for optimized joint trajectories.

is equal to 0.66. The maximum relative error of the estimated parameters, defined as:

$$\varepsilon_{MAX} = \max_{i=1}^{21} \left| \frac{\beta_i^* - \beta_i}{\beta_i^*} \right|$$

is equal to 3.26. Comparison of ε_{AV} and ε_{MAX} with the results in the other researches[5,11] shows that the estimation results are reasonably accurate.

Calculation of the actuator torques using the estimated model parameters and comparison of these estimates with the exact actuator torques provides an alternative means to validate the accuracy of the models. Figure 4 shows the exact and calculated torques. The RMS difference between these torques is defined as:

$$\varepsilon_{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(\tau_i^* - \tau_i^C\right)^2}$$

where τ_i^* and τ_i^C are the exact and the calculated actuator torques, respectively. The RMS torque prediction error for

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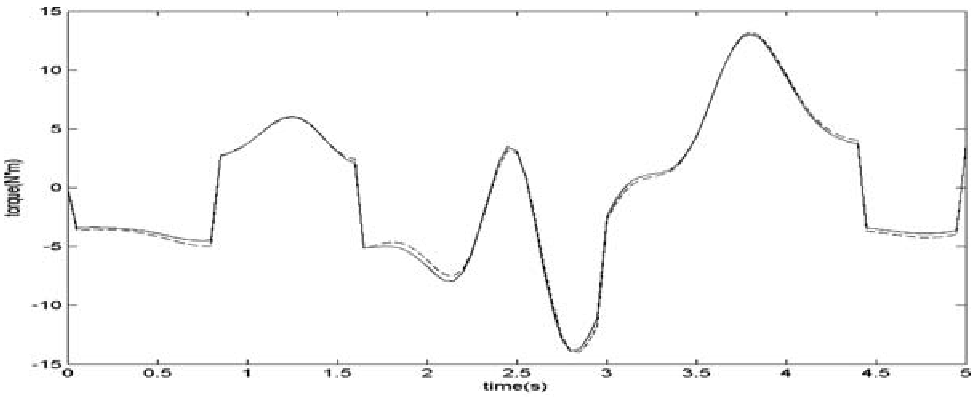
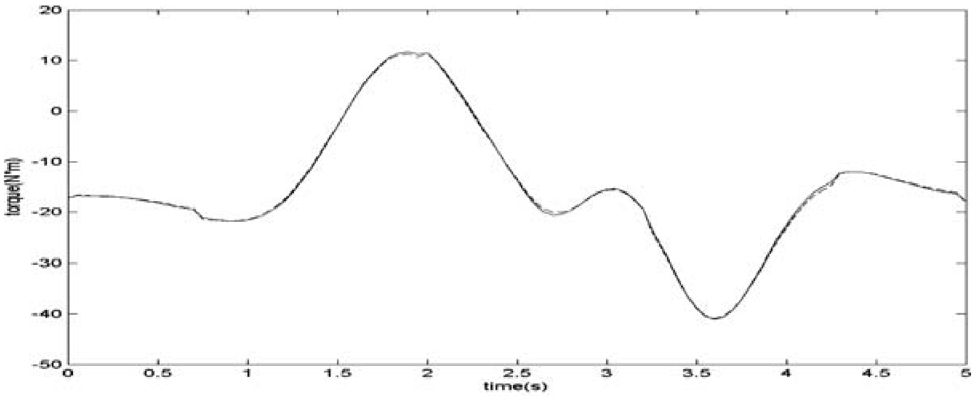
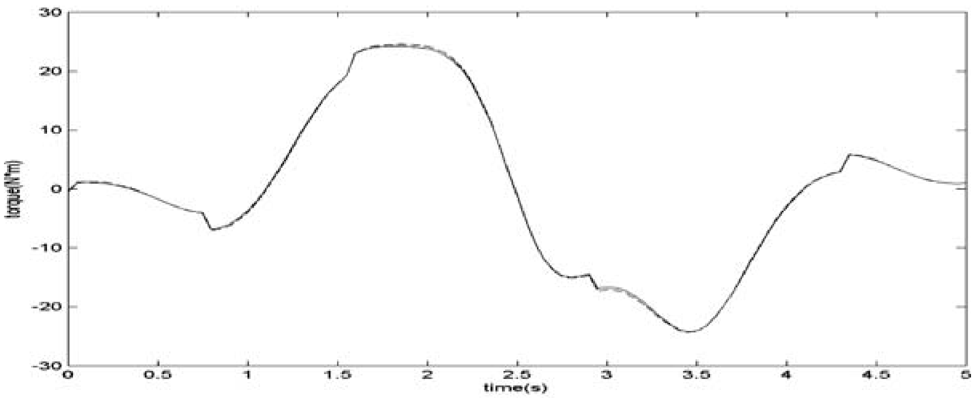


(a) Axis 1



(b) Axis 2



(c) Axis 3

Fig. 4. Exact torques (solid lines), estimated torques (dashed lines).

joint 1 is 5.4236, 2.5394 for joint 2 and 2.4632 for joint 3, respectively. From these results we can see that the estimated model parameters are comparatively accurate.

V. CONCLUSION

The robot excitation design method presented in this paper generates trajectories which aim at estimating the robot model parameters with minimal uncertainty. In addition, the trajectories are periodic and have a band-limited frequency contents. These attractive properties simplify the analysis and conditioning of the measurements, for example the estimation and improvement of the signal-to-noise ratio.

The excitation trajectory for each joint has been functionally developed by a combined Fourier series and polynomial functions. This expansion satisfies both the guarantees of convergence by adding terms and the matching of the boundary conditions.

The formulated maximum-likelihood estimation method takes into account actuator torque and joint angle

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Table II. Fourier series coefficients for the cycloidal and the optimized trajectory.

	Cycloidal	Optimized
Axis 1	−2.8294	−0.0136
	0	0.5298
	0.1886	−0.0498
	0	−0.1282
	0.0196	0.3746
Axis 2	−1.6976	−0.1601
	0	−0.1204
	0.1132	−0.0758
	0	−0.1580
	0.0098	−0.5085
Axis 3	−1.6976	−0.0352
	0	−0.0683
	0.1132	−0.0009
	0	0.2270
	0.0098	−0.3481

measurement noise, i.e. combines the estimation of exact joint angles, angular velocities and accelerations, with the estimation of the robot parameters. Simulation results show that the maximum-likelihood estimation method is asymptotically unbiased and efficient.

References

1. J. Swevers, C. Ganseman, J. De Schutter and H. van Brussel, “Experimental Robot Identification Using Optimized Periodic Trajectories”, *Mechanical Systems and Signal Processing* **10**(5), 561–577 (1996).
2. M. Gautier, “Identification of Robot Dynamics”, *Proceedings of IFAC Symposium on Theory of Robots*, Vienna (1986) pp. 351–356.
3. B. Raucent and J. C. Samin, “Modeling and Identification of Dynamic Parameters of Robot Manipulators”, *Robot Calibration* (Chapman & Hall, London, UK, 1993).
4. G. Liu, K. Iagnemma, S. Dubowsky and G. Morel, “A Base Force/Torque Sensor Approach to Robot Manipulator Inertial Parameter Estimation”, *Proceedings of IEEE International Conference on Robotics and Automation*, Leuven (1998) pp. 3316–3321.
5. J. Swevers, C. Ganseman, D. Bilgin, J. De Schutter and H. van Brussel, “Optimal Robot Excitation and Identification”, *IEEE Transactions on Robotics and Automation* **13**(5), 730–740 (1997).
6. L. Ljung, *System Identification: Theory for the User* (Prentice-Hall, Englewood Cliffs, N.J., 1987).
7. K. S. Fu, R. C. Gonzalez and C. S. G. Lee, *Robotics: Control, Sensing, Vision, and Intelligence* (Mc-Graw-Hill, New-York, USA, 1987) pp. 84–97.
8. B. Armstrong, P. E. Dupont and W. C. Canudas, “A Survey of Models, Analysis Tools and Compensation Methods for the Control of Machines with Friction”, *Automatica* **30**, 1083–1138 (1994).
9. M. Gautier and W. Khalil, “Direct Calculation of Minimum Set of Inertial Parameters of Serial Robots”, *IEEE Transactions on Robotics and Automation* **6**(3), 368–373 (1990).
10. M. Gautier and W. Khalil, “Exciting Trajectories for the Identification of Base Inertial Parameters of Robot”, *Int. J. Robotics Research* **11**(4), 362–375 (1992).
11. M. M. Olsen, J. Swevers and W. Verdonck, “Maximum-Likelihood Identification of a Dynamic Robot Model: Implementation Issues”, *Int. J. Robotics Research* **21**(2), 89–96 (2002).
12. G. Zak, B. Benhabib, R. G. Fenton and I. Saban, “Application of Weighted Least Squares Parameter Estimation Method to the Robot Calibration”, *Trans. ASME, Journal of Mechanism Design* **116**, 890–893 (1994).

https://doi.org/10.1017/S0263574706002712