Topological superconductors and Majorana fermions – an introduction and review

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Abstract. Topological superconductivity is an exciting new field of condensed matter. In very recent years, significant effort has been focused on the applications of novel superconductivity to quantum computing. The peculiar topological nature of topological superconductors gives rise to a new quantum state of condensed matter, hosting zero-energy fermionic states bound to vortices at the surface of a superconductor. The existence of these fermionic particles, the elusive Majorana fermion, has been questioned for decades, with recent studies detecting possible signatures of the never-before-seen quasiparticle. Scanning-tunnelling spectroscopy and specially designed Josephson junctions have detected possible zero-energy bound states, potentially confirming the existence of intrinsic topological superconductivity.

1. Introduction and background

1.1. BCS model and Bogoliubov quasiparticles

Superconductors are a state of matter exhibiting different electrical properties from that of regular conductors, such that at sufficiently low temperatures, their resistance to the flow of electrical current is zero [1]. The processes behind superconductivity in type I superconductors are well described by the Bardeen-Cooper-Schrieffer (BCS) model, which predicts the existence of Cooper pairs: two electrons of opposite spins bound together by a phonon-induced attraction [1]. Cooper pairs are responsible for the transfer of twice the elementary electron charge, $2e$, across the superconducting energy gap of size, Δ, as electrons are forbidden from traversing the gap due the Pauli exclusion principle [1]. Cooper pairs are created from electrons above the fermi energy and holes below, with charge difference $2e$ [2]. Holes are quasiparticles equivalent to the absence of an electron, containing the same mass as electrons, m_e, but opposite charge, $+e$. BCS theory predicts the existence of Cooper pairs through the inclusion of an electron-electron attraction potential in the superconductor Hamiltonian [1]. The BCS mean-field Hamiltonian for a conventional, singlet pairing, s-wave superconductor can therefore be expressed as (up to a constant):

$$H = \sum_{\boldsymbol{k},\sigma} \left(\frac{p^2}{2m} - \mu \right) C^{\dagger}_{\boldsymbol{k}\sigma} C_{\boldsymbol{k}\sigma} + \frac{1}{2} \sum_{\boldsymbol{k}} \Delta_{\boldsymbol{k}} \quad C^{\dagger}_{\boldsymbol{k}\uparrow} C^{\dagger}_{-\boldsymbol{k}\downarrow} + C_{-\boldsymbol{k}\uparrow} C_{\boldsymbol{k}\uparrow} \quad , \tag{1}$$

where p is the particle momentum, m the particle mass, μ the chemical potential ($\equiv$ Fermi energy at zero kelvin) and $C^{\dagger}_{k\sigma}$ and $C_{k\sigma}$ the electron creation and annihilation operators, respectively, for wave vector $\boldsymbol{k}$ and spin σ ($= \uparrow, \downarrow$). $\Delta_{\boldsymbol{k}}$ is the gap function, related to the electron-electron interaction potential and the superconducting energy gap [3]. In the superconducting regime, an electron can be transformed into a hole through forming the Cooper pair, thus, it is natural to consider superposition excitations when describing the superconductor state [1]. Thus, the Hamiltonian is diagonalised using the Bogoliubov-Valatin transformation, a linear transformation between the fermionic basis operators, to superpositions of these operators [4]. These operators are defined as:

$$\gamma_{\boldsymbol{k}\uparrow} = u^*_{\boldsymbol{k}} C_{\boldsymbol{k}\uparrow} - \upsilon^*_{\boldsymbol{k}} C^{\dagger}_{-\boldsymbol{k}\downarrow}$$
$$\text{and} \quad \gamma^{\dagger}_{-\boldsymbol{k}\downarrow} = u_{\boldsymbol{k}} C^{\dagger}_{-\boldsymbol{k}\downarrow} + \upsilon_{\boldsymbol{k}} C_{\boldsymbol{k}\uparrow} \tag{2}$$

Such that $\gamma_{\boldsymbol{k}\uparrow}/\gamma^{\dagger}_{-\boldsymbol{k}\downarrow}$ is the annihilation/creation operator for the transformed particles with wave vector $\boldsymbol{k}/-\boldsymbol{k}$ and spin $\uparrow/\downarrow$, satisfying the usual fermionic anticommutation relations, yet squaring to 1. $u_{\boldsymbol{k}}$ and $\upsilon_{\boldsymbol{k}}$ are complex numbers which satisfy the relation $|u_{\boldsymbol{k}}|^2 + |\upsilon_{\boldsymbol{k}}|^2 = 1$. Physically, these operators can be used to describe the *Bogoliubov quasiparticle*, an intermediary quasiparticle between electrons and holes [1]. The parameters $u_{\boldsymbol{k}}$ and $\upsilon_{\boldsymbol{k}}$ control how electron-like or hole-like the quasiparticle is; when the quasiparticle is equally electron-like and hole-like, the charge vanishes [1]. This mathematical formulation predicts the existence of charge-less quasiparticles present at the edge of the Fermi sea, capable of occupying states within the superconducting energy gap [3], so-called *edge states* [1].

1.2. Features of topological superconductors

Bogoliubov quasiparticles are unique to any superconductor, due to the suppression of real solutions to the complex Dirac wave equation [3][5]. The emergence of a new quantum state of matter requires some symmetry to be broken. In the case of topological superconductors (TS), particle-hole symmetry (Hamiltonian remains the same upon conjugating the electron operators) cannot be avoided, but other symmetries such as time-reversal-symmetry (dynamics are equivalent under transforming $t \rightarrow -t$) and spin-rotation symmetry can be broken [6]. It has been predicted that time-reversal-invariant TSs exist however [3][7][8]. Such symmetry breaking reveals real solutions to the complex superconductor wave function; predicted by Majorana (1937), these solutions are represented as quasiparticles termed *Majorana fermions* (MF) [5][9]. MFs bound to defect sites or vortices, *Majorana bound states* (MBS), of a superconducting surface have zero charge and are simultaneously their own antiparticle [2][10]. MFs are a result of real solutions to the Dirac equation, producing real wave equations, in the hope that these particles can be directly observed without the need for particle/antiparticle distinction [5]. *Majorana zero modes* (MZMs) are defined by their peculiar zero-energy nature, where the Bogoliubov annihilation/creation operators are equal, $\gamma_{\boldsymbol{k}\uparrow} = \gamma^{\dagger}_{\boldsymbol{k}\uparrow}$ [3][11], such that these quasiparticle excitations can annihilate with themselves [2]. A TS may be defined as a material which hosts MZMs (zero-energy states comprised of a superposition of electron and hole states) bound to flux vortices at the edges of the superconductor, which are known to behave neither like fermions nor bosons, in the sense that they are described by *non-Abelian statistics* [12][13]. Figure 1 demonstrates the emergence of MZMs in the dispersion relation of a superfluid (zero viscosity fluid) superconductor [14].

Superconductors are defined as topological if they can be assigned a non-zero *topological invariant* [15]. The topology of an object is defined by features such as the number of holes it contains (an invariant), and two objects are considered topologically equivalent (homeomorphic) if they can be smoothly deformed into one another, without making discontinuous separations [16][17]. In the context of superconductivity, topological symmetries are defined according to the symmetries in the energy levels and Hamiltonian describing the system [8]. The physical analogue of a 'number of holes' characterising the topology of a superconductor is the *Chern number*; a topological invariant (does not

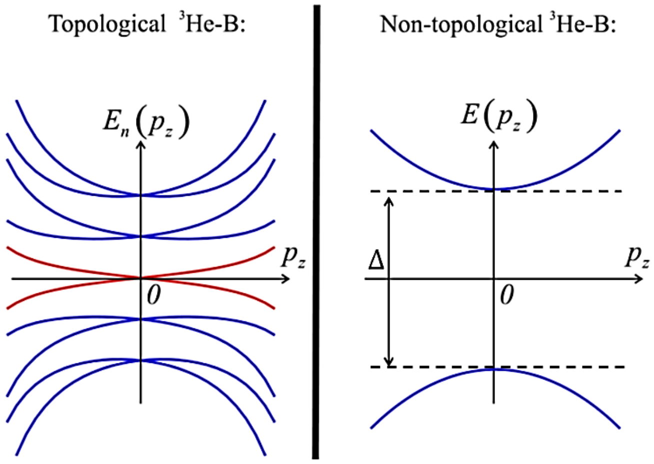

Figure 1. Emergence of zero-energy bound states in the centre of the superconducting energy gap; superconductor dispersion relation (energy vs momentum). The topological phase of ^{3}He-B superfluid hosts Majorana zero modes, while the trivial phase does not [14].

change under smooth transformations) [15]. The Chern number is related to the quantised Hall conductance, such that any material realising the quantum Hall effect has a non-zero Chern number (i.e. is topological) [15]. Thus, measurement of the conductance across a superconductor surface (~ Chern number) and demonstration of its quantised nature is an indication of topological superconductivity.

2. Topological superconductors in literature

2.1. Motivations for the search for Majorana fermions

It is important to note that TSs are extremely rare if not non-existent in nature as they are predicted to arise from p-wave, spin-triplet superconductivity [18][19]. Conventional s-wave superconductors are well described by BCS theory, where the quantum state has angular momentum quantum number $l = 0$. The electrons within the Cooper pairs of an s-wave superconductor have opposing spins. In contrast to this, p-wave superconductors have a single angular momentum quantum (i.e., $l = 1$), producing different symmetries in the form of the superconducting energy gap. The electrons within the Cooper pairs of an s-wave superconductor have opposing spins, while in the p-wave superconductor, the electrons have parallel spins [18][19]. An s-wave superconductor in close proximity to a topological insulator, a material which behaves as a bulk insulator but exhibits conducting ability at the surface, can host MBSs at the interface [17][19][20][21]. This process relies on the mechanism of *spin-orbit coupling* [22]. This was first suggested by Sato (2003) as a method of producing non-Abelian statistics in 2D systems [23]. This was the first emergence of the process behind topological superconductivity, fuelling a push to observe MBSs and an intrinsic TS. Properties exhibited are similar properties to that of topological insulators, although they do not require the use of the superconducting proximity effect [2][24]. TSs are generally believed to exist intrinsically in the form of p-wave superconductors, a rare, unconventional form of superconductor [11][18][25].

The majority of research into topological insulators and superconductors focuses on the fundamental nature of the MF, and the applications of topological superconductivity in novel quantum computing [18][26]. MBSs exhibit topological behaviour, such that information can be stored in the states globally. This allows the states to be stable against disruptions from noise and perturbation, making them strong candidates for reliable storage of information [4][15]. The property which allows MZMs to be used in quantum computing is their intrinsic violation of Abelian statistics [4]. As explained previously, MZMs bound to a two-dimensional surface, such as the surface of a topological insulator or a defect on the surface of a TS are defined as non-Abelian *anyons* – particles restricted to a 2D surface obeying neither Fermi-Dirac (FD) nor Bose-Einstein (BE) statistics [1][2][4]. Appearing within the superconducting energy gap, they cost zero energy to appear, creating multiple degenerate ground states [13]. The exchange of indistinguishable particles in these ground states projects the current state of the system onto another state, modified by some phase factor $e^{i\theta}$. This phase factor is identically -1 (Pauli exclusion) for fermions or +1 for bosons, owing to their antisymmetric and symmetric exchange properties exclusively [26]. For anyons, however, this phase factor can take any value between. This can be summarised as follows:

$$|\psi_1\psi_2\rangle = \alpha|\psi_2\psi_1\rangle, \quad \alpha = \begin{cases} -1 & ; \quad \textit{Fermion} \\ +1 & ; \quad \textit{Boson} \\ \exp(i\theta) & ; \quad \textit{Non-Abelian anyon} \end{cases} \tag{3}$$

where $|\psi_1\psi_2\rangle$ indicates the state of a two bound-state system on a 2D surface, and $|\psi_2\psi_1\rangle$ is the particle-exchanged quantum state [2][13]. The final state of the system is controlled by the order in which the exchange takes place, which is distinct from how the exchange of fermions and bosons modifies the quantum state [13][15]. In general, the final state of the system after the exchange has taken place is **not** the same as the initial state, in contrast to FD (same state) and BE (same state, opposite sign) statistics. The interchanging of MBSs is known as *braiding*, as it produces knot-like patterns [27]. It is precisely this topological braiding behaviour that MBSs exhibit which makes them a useful candidate for applications such as topological quantum computing [2][11][21][28]. Topological braids 'remember' their configuration upon creation of knots; this property can be used to store information. Such properties of the exchange statistics are utilised in forming the basic quantum logic gate, an important and essential component of any quantum circuit (just as the classical logic gate, transistor networks, are the most fundamental building blocks of classical computing) [12]. For example, Lee (2011) outlines a model for the creation of a quantum computational gate, utilizing a 2D topological, p-wave, superconductor [4].

2.2. Previous findings

It is speculated that chiral p-wave superconductors are good candidates for harbouring MBSs at their edges. Kitaev (2000) first demonstrated the possibility of realising MBSs on the edges of a 1D topological superconducting wire [25]. MZMs are predicted to be bound to vortices within the Abrikosov lattice of topological, chiral p-wave superconductors [29]. Chiral p-wave superconductors (those with $p_x + ip_y$ orbital symmetry [2][5][12][13][20]) are characterised by their odd-parity, such that the orbital angular momentum wave function is odd (since the two-particle spin wave function is the even triplet state). This requires that the total fermionic wave function is antisymmetric upon exchange of fermions [28]. Though MZMs have still eluded observation, they are predicted to emerge on the edges of topological superconducting wires [30]. Tunneling (STM) can reveal Majorana modes at the boundaries of a superconducting material [16][30]; the appearance of such modes influences the process of Andreev reflection between a superconducting tip and sample surface. Characteristic features, such as sharp peaks at zero DC bias are predicted to appear when Majorana modes are present [16]. Through deposition of a thin film ferromagnetic insulator onto a gold nanowire surface, Manna et. al. (2020) observes the characteristic (predicted) signatures of MZMs. The density of states of a conventional s-wave superconductor can be related to the differential conductance, dI/dV at the interface of the material, thus, one would expect the fermionic density of states in the superconducting energy gap to be zero in the superconducting state,. MZMs are predicted to exist on the boundaries of TSs, effectively modifying the density of states, as they can exist between the occupied and unoccupied energy levels. Figure 2 demonstrates the changes in conductance spectra which hint at the existence of MZMs. One can observe that high magnetic fields are helpful, if not essential, for detection of MZMs [30][31].

Apparent MZMs should be treated with care, as it has been noted [10][15][30] that trivial occurrences can produce similar signatures in conductance spectra, and other mechanisms can noticeably modify the spectrum [10][30]. Due to the sensitive nature of superconductivity, zero-bias peaks can appear if the

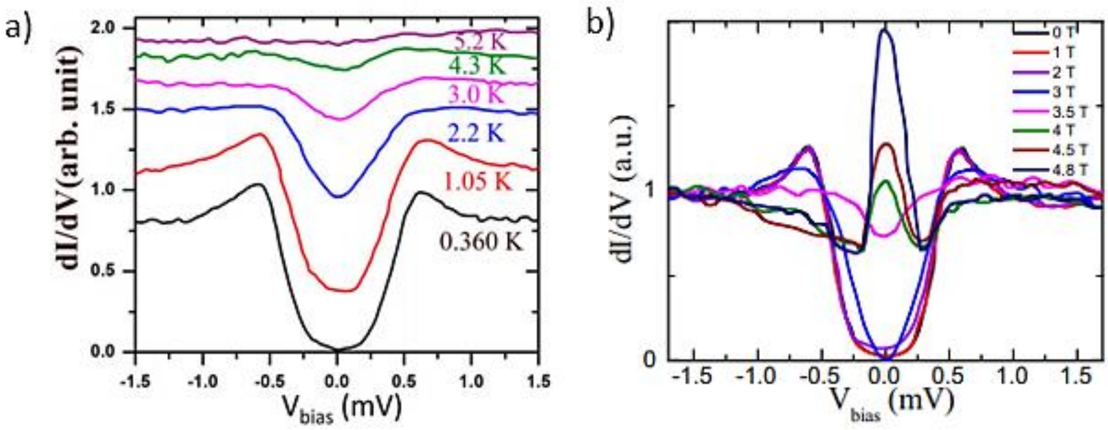

Figure 2. a) conductance spectra for varying temperatures, demonstrating the opening of the superconducting energy gap at lower temperatures. b) conductance spectra for varying magnetic flux density, the centre peaks (so-called *zero-bias peaks*) grow at a field of the order 4T, indicating the existence of states within the superconducting energy gap, a possible observation of the Majorana zero modes [30].

critical current, the maximum current through a superconductor at which it operates in the zero-resistance state, is exceeded. Sasaki et. al. (2011) remark that this can be caused by thermal effects at the tip of the measurement probe, locally forcing the current higher than the critical current, breaking the superconducting state and increasing the conductance [10]. It is noted that the zero-bias peak in the conductance spectrum due to these effects would not change in magnitude, as this effect is purely a result of overcoming a fixed threshold. Figure 2b clearly shows a variation in the height of the zero-bias peak, contrasting the appearance predicted from the above argument, arguing in favour of the true observation of MZMs. This result is corroborated by other studies using conductance measurements [10][19][21]. Kezilebieke et. al. (2020) successfully measure the existence of Majoranas, bound to the edges of a superconducting heterostructure ($CrBr_3$-$NbSe_2$). Zero-bias peaks are shown to appear when measuring at the edges of the structures, but importantly **not** in the bulk, a characteristic feature of the 2D nature of MZMs on the surface of a TS [19][29]. Ménard et. al. (2019) support this observation with zero-bias conductance peaks appearing on a superconducting lead surface. Das et. al. (2012) support the claim that zero-bias conductance peaks may be created by MBSs, with tunnelling studies and numerical simulations. The simulations and behaviour of the zero-bias peaks strongly support the conclusion that MZMs were observed, however, the results are still treated with care, as several other processes may reproduce the same behaviour [32]. For example, trivial *Andreev bound states* (ABSs – zero-energy states arising from Andreev reflection) can produce similar signatures in conductance spectra to MZMs, previously thought to appear due to variations in the chemical potential, μ. More recently, it has been shown that ABSs are frequently indistinguishable from MZMs when observed as zero-bias peaks [7][10][21][32]. Zhang et. al., (2018) conclude, in their experimentation with tunnelling and Andreev reflection, that MZMs were detected. This was however later falsified by Frolov, demonstrating that non-topological ABSs can produce extremely similar signatures. Such states are trivial and non-topological, thus, not useful to the field of topological superconductivity [33]. The filtering of such bound states requires clean contact between probe and sample, as well as extremely fine-tuning of experimental parameters [29][30]. Manna et. al. (2020) conclude that their observed zero-bias peaks are undoubtedly MZMs, as ABSs can appear as tunnelling conductance peaks between the edges of a sample and the measurement probe. Scanning-tunnelling spectroscopy minimises this tunnelling contact, using atom sized contacts, ruling out the possibility of these strong coupling effects [30]. It is thus important to consider the positioning of sample and probe before taking measurements and making any definitive conclusions.

MBSs have also been predicted to appear in Josephson junctions comprised of a trivial superconductor and a TS, contributing to the total Josephson current across the junction [8][34][35][36]. Zazunov et. al. (2018) study the supercurrent through a Josephson junction consisting of a quantum dot placed between trivial and TS nanowires. The quantum dot plays the role of a magnetic defect, as no Josephson current is predicted to flow across a defect-less junction alone [8][36]. It is concluded that the Majorana contribution to the total supercurrent is negligible, due to the adherence to time-reversal-symmetry [6]. Liu et. al. (2021) consider the case where time-reversal-symmetry is broken, freeing the necessary restrictions which forbade observation of Majorana-induced supercurrents. It is concluded that the DC Josephson effect can be used effectively to demonstrate the effect of MBSs, and thus indirectly confirm their existence [35]. A globally applied magnetic field can be utilised to increase the measured Josephson current, predicted to be due to MZMs.

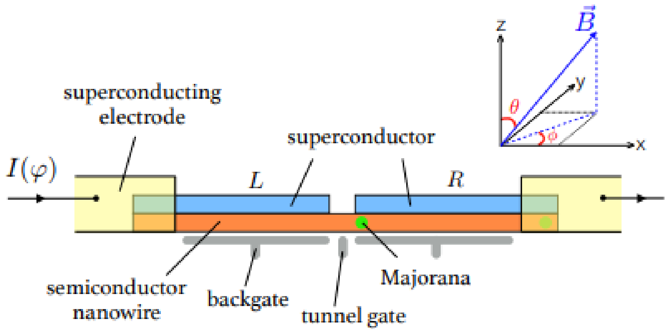

Figure 3. A schematic diagram of the superconductor-topological superconductor Josephson junction. The Majorana zero mode is found to appear on the interface between the topological superconductor (R) and the semiconductor beneath. This mode enhances the supercurrent across the tunnel boundary [35].

This result is corroborated by a study of a 1D TS system by Soori (2020). Two Kitaev chains (nanowires hosting Majorana modes on the edges) attached to normal conductors, form a Josephson across their interface. It is shown that the Kitaev chains enhance Andreev reflection, increasing the total electron tunnelling across the junction [37]. It can be argued that another physical process is responsible for the enhancing of supercurrent however, as MZMs have only ever been detected indirectly.

Signatures of MZMs should also be rigorously tested in such proximity-induced topological superconductivity experiments. Chang (2017) confirmed the detection of MZMs in superconductor-quantum hall insulator configurations. However, results were scrutinised; the same quantised behaviour of the conductance was found to occur regardless of the strength of the applied magnetic field. Thus, it was concluded that the superconductor in the setup used by Chang acted as an electrical short, showing no variance in the data measured – MZMs were **not** detected [38].

Possible signatures of MZMs have been detected in graphene (a single layer of graphite, a very strong and highly conductive carbon allotrope). San-Jose et. al. (2015) successfully demonstrate that coupling the zero Landau level (lowest of the quantised energy levels in terms of the cyclotron frequency) of graphene to a trivial superconductor gives rise to bound states upon the edges of the now TS. Fraunhofer patterns in the supercurrent across a graphene Josephson junction have been used to demonstrate intrinsic topological order. The lack of decay in the patterns indicate the presence of MZMs [22]. This is generally a well-supported study, indicating graphene as a useful tool in the realisation of MZMs.

3. Conclusions

Topological superconductivity is a long sought-after form of superconductivity, due to peculiar and novel properties such as the hosting of charge-less, non-Abelian quasiparticles, Majorana fermions

(MFs). Predicted to exist by Majorana in 1937, such particles exhibit exchange statistics in stark contrast to the statistics of Fermions and Bosons in our 3D universe. Topological order is predicted to appear in chiral p-wave superconductors, superfluids, superconductor-semiconductor heterostructures, superconductor-topological insulator structures and more. Such topological order has been observed directly, such as in point-contact conductance spectra and in the Fraunhofer patterns of the critical current across a Josephson junction. MFs are thought to be extremely useful to the field of topological quantum computing, due to their peculiar topological properties and anyonic braiding. It is difficult to conclude the existence of MFs based on recorded data alone, as other phenomena may be responsible – for example trivial ABSs and local heating. However, the area of research is very well scrutinised, such that there exist many studies which have demonstrated the real possibility of creating MFs.

References

[1] Gastiasoro, M., 2012. *Bogoliubov-de Gennes studies of Fe-based superconductors*. MSc Thesis. University of Copenhagen.

[2] Beenakker, C., 2013. Search for Majorana Fermions in Superconductors. *Annual Review of Condensed Matter Physics*, **4**(1), pp.113-136.

[3] Bernevig, B. and Hughes, T., 2013. *Topological insulators and topological superconductors*. Princeton: Princeton University Press.

[4] Lee, J., 2011. *Scalable Gate-Defined Majorana Fermions in 2D p-Wave Superconductors*. College of Nanoscale Science and Engineering, SUNY-Polytechnic Institute, Albany, NY 12203, United States

[5] Beenakker, C. and Kouwenhoven, L., 2016. A road to reality with topological superconductors. *Nature Physics*, **12**(7), pp.618-621.

[6] Chew, A., Mross, D. and Alicea, J., 2020. Time-Crystalline Topological Superconductors. *Physical Review Letters*, **124**(9).

[7] Ando, Y. and Fu, L., 2015. Topological Crystalline Insulators and Topological Superconductors: From Concepts to Materials. *Annual Review of Condensed Matter Physics*, **6**(1), pp.361-381.

[8] Qi, X. and Zhang, S., 2011. Topological insulators and superconductors. *Reviews of Modern Physics*, **83**(4), pp.1057-1110.

[9] Majorana, E. 1937. A SYMMETRIC THEORY OF ELECTRONS AND POSITRONS. *Nuovo Cim*, **14**, pp.171–184

[10] Sasaki, S., Kriener, M., Segawa, K., Yada, K., Tanaka, Y., Sato, M. and Ando, Y., 2011. Topological Superconductivity in CuxBi2Se3. *Physical Review Letters*, **107**(21).

[11] Leijnse, M. and Flensberg, K., 2012. Introduction to topological superconductivity and Majorana fermions. *Semiconductor Science and Technology*, **27**(12), p.124003.

[12] Alspaugh, D., Sheehy, D., Goerbig, M. and Simon, P., 2020. Volkov-Pankratov states in topological superconductors. *Physical Review Research*, **2**(2).

[13] Alicea, J., Oreg, Y., Refael, G., von Oppen, F. and Fisher, M., 2011. Non-Abelian statistics and topological quantum information processing in 1D wire networks. *Nature Physics*, **7**(5), pp.412-417.

[14] Silaev, M. and Volovik, G., 2014. Andreev-Majorana bound states in superfluids. *Journal of Experimental and Theoretical Physics*, **119**(6), pp.1042-1057.

[15] Sato, M. and Fujimoto, S., 2016. Majorana Fermions and Topology in Superconductors. *Journal of the Physical Society of Japan*, **85**(7), p.072001.

[16] Sato, M. and Ando, Y., 2017. Topological superconductors: a review. *Reports on Progress in Physics*, **80**(7), p.076501.

[17] Moore, J., 2010. The birth of topological insulators. *Nature*, **464**(7286), pp.194-198.

[18] Trang, C. et. al., 2020. Conversion of a conventional superconductor into a topological superconductor by topological proximity effect. *Nature Communications*, **11**(1).

[19] Kezilebieke, S. et. al., 2020. Topological superconductivity in a van der Waals heterostructure. *Nature*, **588**(7838), pp.424-428.

[20] Fu, L. and Kane, C., 2008. Superconducting Proximity Effect and Majorana Fermions at the Surface of a Topological Insulator. *Physical Review Letters*, **100**(9).
[21] Frolov, S., Manfra, M. and Sau, J., 2020. Topological superconductivity in hybrid devices. *Nature Physics*, **16**(7), pp.718-724.
[22] San-Jose, P., Lado, J., Aguado, R., Guinea, F. and Fernández-Rossier, J., 2015. Majorana Zero Modes in Graphene. *Physical Review X*, **5**(4).
[23] Sato, M., 2003. Non-Abelian statistics of axion strings. *Physics Letters B*, **575**(1-2), pp.126-130.
[24] Zhang, P. et. al., 2018. Observation of topological superconductivity on the surface of an iron-based superconductor. *Science*, **360**(6385), pp.182-186.
[25] Kitaev, A., 2001. Unpaired Majorana fermions in quantum wires. *Physics-Uspekhi*, **44**(10S), pp.131-136.
[26] Nayak, C., Simon, S., Stern, A., Freedman, M. and Das Sarma, S., 2008. Non-Abelian anyons and topological quantum computation. *Reviews of Modern Physics*, **80**(3), pp.1083-1159.
[27] Beenakker, C., 2020. Search for non-Abelian Majorana braiding statistics in superconductors. *SciPost Physics Lecture Notes*.
[28] Liu, X., Zhou, Y., Wang, Y. and Gong, C., 2017. Characterizations of topological superconductors: Chern numbers, edge states and Majorana zero modes. *New Journal of Physics*, **19**(9), p.093018.
[29] Ménard, G., Mesaros, A., Brun, C., Debontridder, F., Roditchev, D., Simon, P. and Cren, T., 2019. Isolated pairs of Majorana zero modes in a disordered superconducting lead monolayer. *Nature Communications*, **10**(1).
[30] Manna, S., Wei, P., Xie, Y., Law, K., Lee, P. and Moodera, J., 2020. Signature of a pair of Majorana zero modes in superconducting gold surface states. *Proceedings of the National Academy of Sciences*, **117**(16), pp.8775-8782.
[31] Li, Y. and Xu, Z., 2019. Exploring Topological Superconductivity in Topological Materials. *Advanced Quantum Technologies*, **2**(9), p.1800112.
[32] Das, A., Ronen, Y., Most, Y., Oreg, Y., Heiblum, M. and Shtrikman, H., 2012. Zero-bias peaks and splitting in an Al–InAs nanowire topological superconductor as a signature of Majorana fermions. *Nature Physics*, **8**(12), pp.887-895.
[33] Castelvecchi, D., 2021. Evidence of elusive Majorana particle dies — but computing hope lives on. *Nature*, **591**, pp.334-335.
[34] Tanaka, Y., Yokoyama, T. and Nagaosa, N., 2009. Manipulation of the Majorana Fermion, Andreev Reflection, and Josephson Current on Topological Insulators. *Physical Review Letters*, **103**(10).
[35] Liu, C., van Heck, B. and Wimmer, M., 2021. Josephson current via an isolated Majorana zero mode. *Physical Review B*, **103**(1).
[36] Zazunov, A., Iks, A., Alvarado, M., Levy Yeyati, A. and Egger, R., 2018. Josephson effect in junctions of conventional and topological superconductors. *Beilstein Journal of Nanotechnology*, **9**, pp.1659-1676.
[37] Soori, A., 2020. Probing nonlocality of Majorana fermions in Josephson junctions of Kitaev chains connected to normal metal leads. *Solid State Communications*, **322**, p.114055.
[38] Kayyalha, M. et. al., 2020. Absence of evidence for chiral Majorana modes in quantum anomalous Hall-superconductor devices. *Science*, **367**(6473), pp.64-67.