# Studying optical and mechanical properties of a nematic liquid crystal

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# Contents

# Properties of Liquid Crystals

## 1 Abstract

The viscosity, $\eta$, and the splay elastic constant, $k_{11}$, of a nematic liquid crystal device are estimated through considering electro-optic effects and optical properties such as birefringence. The change in liquid crystal structure with applied voltage is studied to determine its effect on the polarisation of laser light. The best result for the viscosity, $\eta = (0.3 \pm 0.05)\mathrm{Pas}$, agrees somewhat with values provided in the experimental script [1]. Other values calculated range between $\eta = 1.3\mathrm{Pas}$ and $\eta = 1.5\mathrm{Pas}$, and differ significantly from expected results. $k_{11} = (7.6 \pm 0.4)\mathrm{pN}$ was found to agree somewhat with expected values.

## 2 Introduction

### 2.1 Liquid Crystals

Liquid crystals (LCs) are materials with properties of both pure liquids and crystalline solids and may be described as “ordered fluids” [1]. LCs are incredibly important materials to study; for example, understanding of LCs allowed the invention of the liquid crystal display. Consideration of the structure of LCs and the arrangement of the molecules is vital in making predictions about the electro-optic effects. A simple two-dimensional model of the structure of LCs is displayed in *figure 1* below.

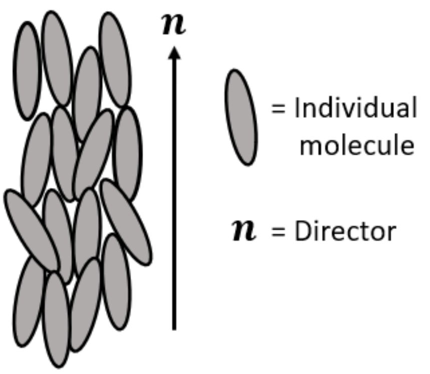


***Figure 1:*** *The arrangement of rod-like molecules within a liquid crystal [1].*

*Figure 1* demonstrates that LCs may be thought of as a semi-symmetric arrangement of anisotropic molecules. LCs have a preferential direction known as the *director*, $\boldsymbol{n}$, which determines the average direction of the molecules’ dipoles. The director can be influenced by the shape of the LC or externally applied fields. The presence of the director is referred to as *directional order* of an LC; the nematic phase. For the purposes of this experiment, an NLC device was used (enclosed LC between sheets of glass) to study how birefringence affects the polarisation of incoming laser light. *Birefringence* refers to a property of materials which causes a difference in light propagation speeds dependent on the direction of travel through the material. It is dependent on dielectric permittivity constants, which determine the strength of molecular alignment to an applied electric field. Light tends to be retarded more in one direction than another, and this causes a phase difference between outgoing waves [2,3]. The birefringence, $\Delta n$, can be expressed in terms of the ordinary (parallel to $\boldsymbol{n}$) and extraordinary (orthogonal to $\boldsymbol{n}$) refractive indices:

$$\Delta n = n_e - n_o \qquad (1)$$

$\Delta n$ is AC voltage-dependent and equation (1) only holds for voltages below a threshold voltage, $V_c$ [1,4]. Through measuring $V_c$, one can deduce the effect of applying voltages across an NLC and mechanical constants can be calculated.

### 2.2 Polarisation of light

Light is known to be *polarised* if the electric field vector of a light wave, $\boldsymbol{E}_0$, is restricted to a specific direction or set of directions [5]. It is useful to study the effects of passing light through crossed polarising filters and the NLC device oriented at various angles.

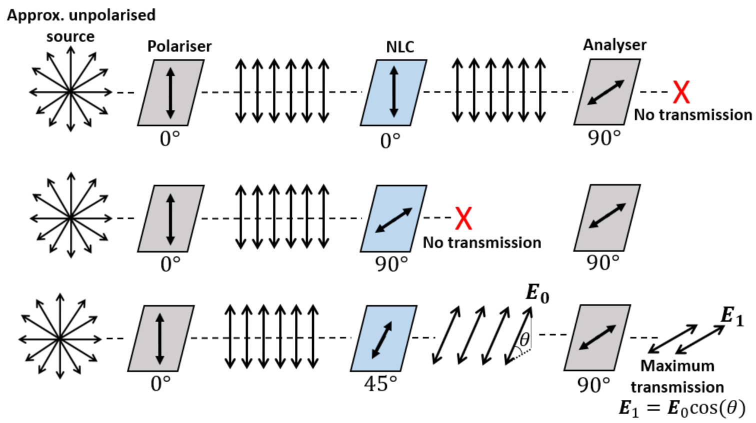


***Figure 2:*** *A geometrical representation of the electric field components of light waves for various orientations of $\boldsymbol{n}$. The output intensity is proportional to $|\boldsymbol{E}_1|^2$.*

Polarising filters are considered *crossed* if the atomic sheets, which define their preferred direction, lie perpendicular to each other, resulting in no transmission of the electric field vector of an incoming light wave. Due to the birefringent nature of NLCs, maximum intensity is expected when the director of the NLC device is oriented at $45°$ to the crossed polarisers. This occurs due to the NLC device 'splitting' the electric field vector into orthogonal components of equal, maximum size. This is outlined in *figure 2* above. One can study this maximum intensity as a function of the applied voltage across the NLC device to determine physical constants such as the elastic constants and viscosity.

## 2.3 Elastic constants and viscosity

*Elastic constants* are values which quantify the response a material will have to stress, be it elastic or inelastic deformation (e.g. Hooke's law spring constant). *Viscosity* refers to a material's ability to resist flow; defined as the ratio between shear stress and shear rate (velocity gradient). Elastic constants and viscosity are useful quantities for making predictions about the mechanical behaviour of LCs. Knowledge of this behaviour can facilitate the creation of highly useful materials such as LCD displays and synthetic biological structures which can enhance properties of existing biological mechanisms [6]. For an NLC device, the deformation is characterised as an increase in energy due to the director leaving its equilibrium position [1,7].

Considering the application of an electric field, there are three possible eigendeformations of an NLC device. These deformations are referred to as *splay*, *twist*, and *bend*; and differ in the direction and manner of deformation.

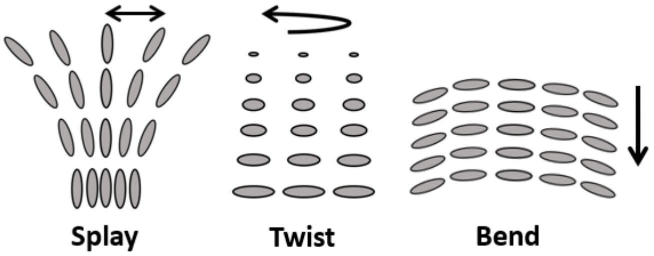


***Figure 3:*** *The 3 eigendeformations of an NLC device produced through application of an electric field.*

The associated elastic constants are $k_{ii}$, the diagonal components of a three-dimensional stress tensor, where $i = 1,2,3$ [7]. The manner in which the NLC device is deformed is significant, as the associated elastic constant can be used to estimate the viscosity of the NLC device.

A direct relationship between the threshold voltage, $V_c$, and splay elastic constant, $k_{11}$, [1,4] is derived through consideration of the free energy of the nematic LC:

$$V_c = \pi\sqrt{\frac{k_{11}}{\varepsilon_0 \Delta\varepsilon}} \tag{2}$$

Equation (2) allows the splay elastic constant to be calculated by measuring the threshold voltage. Measuring $V_c$ requires applying slowly varying voltages across the NLC device to build up characteristic voltage-intensity curves with well-defined features. Once these curves are produced and can be easily recreated, voltages can be applied suddenly to the NLC device to measure the response times. *Response times*, $\tau_{\text{on}}$ and $\tau_{\text{off}}$, quantify the time it takes for $\boldsymbol{n}$ to orient itself into a stable position and the time it takes to return to equilibrium at $0V$.

These quantities are related to the viscosity, $\eta$, of the NLC device [4]:

$$\tau_{\text{on}} = \frac{\eta d^2}{\varepsilon_0 \Delta\varepsilon (V^2 - V_c^2)} \tag{3}$$

$$\tau_{\text{off}} = \frac{\eta d^2}{\varepsilon_0 \Delta\varepsilon V_c^2} \tag{4}$$

Within (2), (3), (4) and (5), $V_c$ and $k_{11}$ have their previously mentioned meanings, $\Delta\varepsilon = \varepsilon_{\parallel} - \varepsilon_{\perp}$, the dielectric anisotropy, $d$ = device thickness and $V$ = applied rms voltage. Equation (3) corresponds to linear proportionality between $\frac{1}{\tau_{\text{on}}}$ and $V_c^2$:

$$\frac{1}{\tau_{\text{on}}} = \frac{\varepsilon_0 \Delta\varepsilon}{\eta d^2}(V^2 - V_c^2) \tag{5}$$

Therefore, multiple measurements of $\tau_{\text{on}}$ can be used to produce a curve with gradient $\frac{\varepsilon_0 \Delta\varepsilon}{\eta d^2}$, and thus, the viscosity can be evaluated. Equations (4) and (5) are the methods used to calculate the viscosity of the NLC device.

Ultimately, the goal of this experiment was to consider the effectiveness of the above equations and how best to utilise them to obtain the most accurate results for $\eta$. Experimental apparatus is used to calibrate the system, to measure the response times, then used to generate multiple estimates for $\eta$.

# 3 Experimental Design

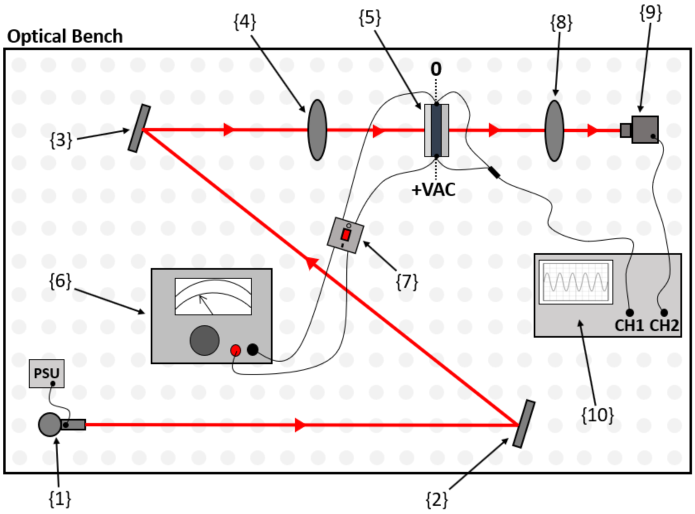


***Figure 4**: A schematic diagram of the arrangement of the experimental apparatus.*

The setup of the experiment is displayed in *figure 4*. A $635\text{nm}$ laser source {1} is shone directly onto two highly reflective mirrors {2}{3} which reflect the laser light at $\sim 45°$. Each mirror had three screws, to allow fine adjustment of beam direction. Light passes through the initial polariser (P1) {4} oriented in such a way to produce maximum transmission (laser light contains some small inherent polarisation). The NLC device {5} is placed after the P1, oriented at $45°$ with respect to P1, and an analogue signal generator {6} is used to apply a 5kHz AC voltage across the device, controlled with a toggle switch {7}. Light then passes through the second polarising filter (analyser) {8} which is oriented orthogonal to P1. The analyser functions to filter the remaining polarisation direction of the light after it passes through the NLC device. Finally, photons are detected using a photodiode {9}, which records the intensity of the incident as a proportional voltage. All measurements are taken using an oscilloscope {10}, which records and displays the AC signal applied across the NLC device and the DC signal generated by the photodiode.

# 4 Methodology

## 4.1 Initial measurements

Initial aims of this experiment considered confirming the predictions laid out by the theory. The intensity of light passing through the NLC device was initially measured with no voltage applied, to consider the effect the birefringence has on the polarisation of the light. With P1 and the analyser at fixed, crossed orientations, the NLC device was rotated and intensity was recorded as a function of the rotation angle. This was performed to evaluate where the peaks in intensity lied in the range of rotation angles, so that this orientation could be chosen for later measurements. As outlined above, it was expected intensity peaks would occur every $45°$.

## 4.2 Measuring the threshold voltage

The signal generator is used to apply sinusoidal voltages across the NLC device. Due to the structure of LCs outlined in *figure 1*, one would expect that the anisotropic molecules would respond to an applied electric field by changing their orientation [8]. Therefore, it was predicted that the birefringent retardance (the phase difference between orthogonal electric field components) would with applied voltage. Five sets of intensity measurements were taken, varying the voltage each time in steps of $0.2\text{V}$. Peak-to-peak voltages in the range $0\text{V} \leq V_{\text{pp}} \leq 10\text{V}$ were applied, to ensure all features of the intensity-voltage curve would be resolved to acceptable accuracy. From these five separate curves, constant intensity is expected until the voltage reaches the threshold voltage, $V_c$ [4]. The voltage at which changes in photodiode intensity were significant was determined to be $V_c$. It was difficult to pinpoint this location precisely however, as the curve showed asymptotic behaviour towards the region of no NLC response. Horizontal lines were thus superimposed onto the curves to determine exactly when the curve deviated. An average value was then taken across the five values along with its associated uncertainty.

Using measured values for the threshold voltage, the splay elastic constant, $k_{11}$, was calculated directly from equation (2). The uncertainty in this result arose from uncertainty in the dielectric anisotropy, $\Delta\varepsilon$, and the measurement uncertainty in $V_c$. The uncertainty in $V_c$ was evaluated from the standard deviation in the five measurements, as this proved large enough to cover the variation in the measured values. The permittivity of free space, $\varepsilon_0$, was assumed to have negligible uncertainty.

### 4.3 Measuring response times

The switching-on response time of an NLC, $\tau_{\text{on}}$, was measured by applying a large voltage (between 3 and 7V) suddenly to the device. The intensity would then change by following the exact traces which were observed when measuring $V_c$. Thus, using the trigger and capture features of the oscilloscope, the trace could be recorded, and the time window could be measured. Due to the asymptotic behaviour previously mentioned, it was difficult to determine precisely when the NLC began responding, and when it reached a stable position.

To solve this issue, the voltage difference, $\Delta V$, between the off and on state was recorded, and $\tau_{\text{on}}$ was defined as the time between points 10% of $\Delta V$ **past** the constant intensity portions. This was also to ensure anomalous effects of the start-up and end approach to constant intensity were disregarded. *Figure 5* demonstrates this measurement process.

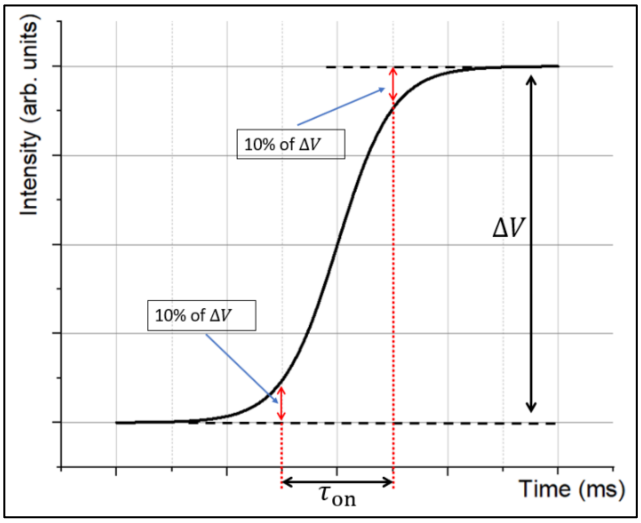


***Figure 5:*** *The method employed when measuring time differences (exaggerated for clarity).*

The switching-off response time, $\tau_{\text{off}}$, was measured in a similar manner and 10 measurements of $\tau_{\text{on}}$ and $\tau_{\text{off}}$ were taken for four different applied voltages. Values for $\eta$ were calculated separately from $\tau_{\text{on}}$ and $\tau_{\text{off}}$, using equations (4) and (5) respectively. The process of measuring of $\tau_{\text{on}}$ and $\tau_{\text{off}}$ was very consistent and thus it was expected the experimental uncertainty would be small. The uncertainty in the calculated values for $\eta$ arose from uncertainties in $d$, $\Delta\varepsilon$, $V_c$, $\tau_{\text{on}}$ and $\tau_{\text{off}}$.

## 5 Results

The relationship between the rotation angle of the NLC placed between crossed polarisers and photodiode intensity is shown in *figure 6*.

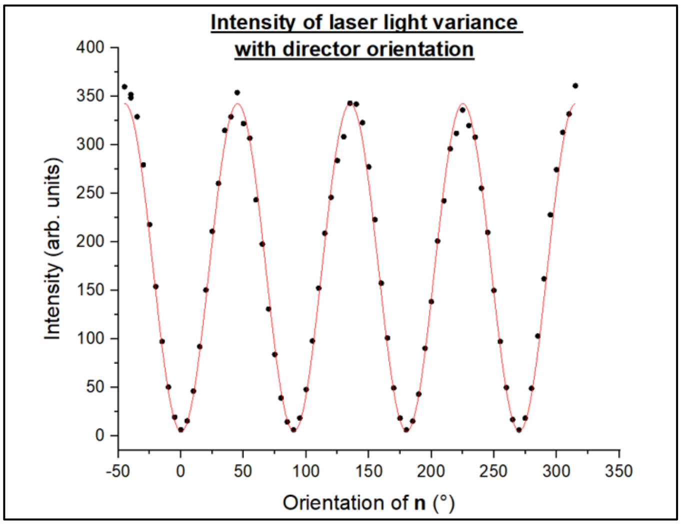


***Figure 6:*** *Light-intensity measurements for different orientations of **n**.*

Maximum transmission occurred when the director of the NLC device was oriented at $45°$ to both polarisers. The error in this result is negligible, as the peaks lie within $< 1°$ of multiples of $45°$. Characteristic intensity-voltage curves consistently displayed the same features, and the threshold voltages were reliable. The curves display all predicted peaks, based on the changing birefringence, $\Delta n$, with voltage. The NLC device can thus be modelled as a voltage-dependent waveplate.

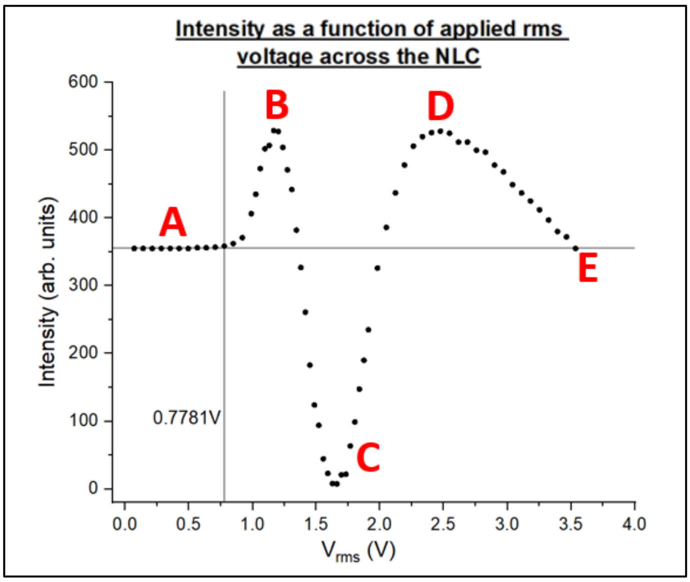


***Figure 7:*** *An example of an intensity-voltage curve. Peaks and troughs occur due to waveplate characteristics.*

The five curves produced an average threshold voltage:

$$\boxed{V_c = (0.78 \pm 0.02)\text{V}_{\text{rms}}}$$

Using equation (2), the splay elastic constant was calculated:

$$\boxed{k_{11} = (7.6 \pm 0.4)\text{pN}}$$

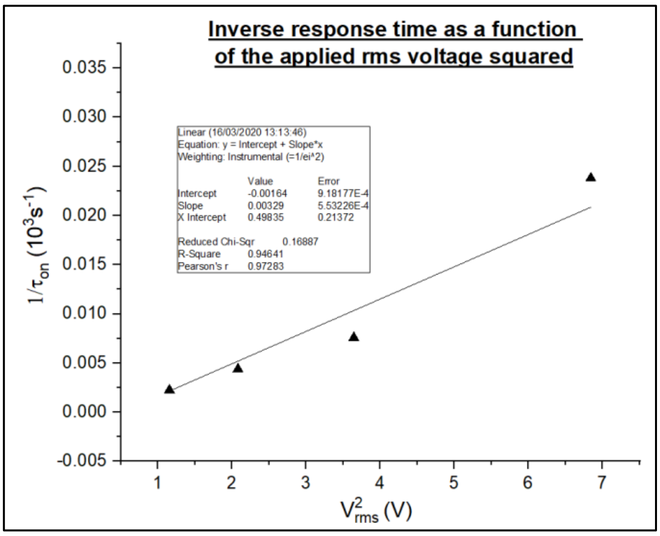


***Figure 8:*** *A linear plot of equation (5). Each point is the average of a set of 10 measurements.*

Response times remained consistent within each set of 10 measurements. $\tau_{\text{on}}$ was found to grow with increasing voltage as shown in *figure 8*. In contrast, $\tau_{\text{off}}$ varied with changing voltage.

Viscosities were calculated from the measurements of $\tau_{\text{on}}$, $\tau_{\text{off}}$ and $V_c$:

$$\eta = (0.3 \pm 0.05)\text{Pas} \qquad \text{from (4)}$$

$$\eta = (1.5 \pm 0.3)\text{Pas} \qquad \text{from (5)}$$

These results differ significantly from one another. The large uncertainty in the result from equation (5) is primarily due to the error in the linear best fit.

## 6 Discussion

*Figure 6* confirms the relationship between rotation of the director and intensity of transmitted light. The data points fit a $\sin^2(2x)$ curve exceptionally well, hence the maxima occurring at multiples of $45°$. The NLC was oriented at one of these locations for the duration of the experiment. *Figure 7* outlines the characteristic features expected for the intensity-voltage relationship. These features were predicted by theory: considering the NLC device as a voltage-dependent waveplate. One can confirm the behaviour of the NLC through observation of the curve. As expected, the birefringent retardance, $d\Delta n$, changes with voltage, and has well-defined values at the peaks and troughs. This is outlined in equation (6).

$$d\Delta n = \begin{cases} \left(m + \frac{1}{2}\right)\lambda & ; \text{ at B, D} \\ m\lambda & ; \text{ at C} \\ 0 & ; \text{ at E} \\ d(n_e - n_o) & ; \text{ at A} \end{cases} \qquad (6)$$

$d\Delta n$ can be evaluated for the remaining sections of the curve, however this was not performed in the experiment. The threshold voltage average was remarkably close to the accepted value of $V_c \approx 0.77V_{\text{rms}}$. $V_c$ did vary to a degree, however, and this may be attributed to the subjectivity in the measurement method employed. It was difficult to determine precisely where the photodiode began to respond, solely by looking at the intensity-voltage data. It may be beneficial to repeat these measurements, whilst recording significantly more data around the expected value for $V_c$. This may result in higher resolution and hence improved ability to locate where the curve deviates from the horizontal.

Results for the splay elastic constant agreed somewhat with the expected value of $(11 \pm 1)\text{pN}$ [1]. The discrepancy may be a result of the large uncertainty in the calculated result. It may have been wise to perform more measurements of $V_c$ in the manner outlined above, to reduce the measurement uncertainty and increase the reliability of the calculated value for $k_{11}$.

Results for the viscosity were somewhat inconsistent. From measurements of $\tau_{\text{on}}$, the viscosity was found to disagree significantly with the expected result. Most of the measurements seemed to fit the linear model outline in equation (5) well, so there may have been some systematic error that skewed the results. If time was permitted, $\tau_{\text{on}}$ measurements would be made again to a larger degree, to observe whether the skew was still present. $\tau_{\text{off}}$ measurements provided a more reliable result for $\eta$, coming close to the expected value [1]. Although, it was more difficult to determine when the NLC had begun responding. The response produced a very sudden spike in intensity, which considerably decreased the viability of the '10% of $\Delta V$' method.

## 7 Conclusion

The aim of this experiment was to investigate the optical and mechanical properties of a nematic liquid crystal device. The behaviour of birefringence with regards to changing the polarisation of light was studied, as well as elastic properties such as viscosity.

Using a comprehensive set of optical components, the intensity of light passing through polarisers and the NLC device could be measured. This provided insight into the molecular structure of the NLC, and how this might affect the properties of electromagnetic waves passing through. It was observed that intensity was a maximum when the director of the NLC was oriented equally between crossed polarisers, i.e. at $45°$ to both. This was predicted by the theory.

Through applying a voltage across the NLC, intensity changes can be measured. The intensity of light was found to change in a characteristic way when the voltage was increased. Before the director of the NLC responded, the intensity remained constant. This allowed the measurement of a threshold voltage $V_c$. This voltage is constant for a specific NLC. $V_c = (0.78 \pm 0.02)\mathrm{V_{rms}}$ was found to agree exceptionally well with the accepted value, with a difference of only $1\%$. Using equation (2), the splay elastic constant was calculated to be $k_{11} = (7.64 \pm 0.4)\mathrm{pN}$. This calculation agreed, to an extent, with expected values. Equations (3), (4), (5) were then used, along with measurements of the response time of the NLC, to estimate the viscosity, $\eta$. Values for $\eta$ were inconsistent and deviated far from expected results.

Repeats of the measurements of $\tau_{\mathrm{on}}$ and $\tau_{\mathrm{off}}$ are necessary to improve the accuracy and reliability of the calculations. A peculiar inconsistency was noticed with $\tau_{\mathrm{off}}$. Equation (4) predicts $\tau_{\mathrm{off}}$ is constant for a specific NLC device. This was **not** observed; $\tau_{\mathrm{off}}$ changed significantly when increasing the voltage. The result for $\eta$ was also derived from an average $\tau_{\mathrm{off}}$ value, hence it may not be valid or reliable. A better process would be to repeat $\tau_{\mathrm{off}}$ measurements altogether and compare to what is expected, to eliminate any anomalous results. Working backwards from equation (3) and using the given values for $k_{11}$ and $\eta$, $\tau_{\mathrm{off}} \approx 60\mathrm{ms}$. One should then observe small changes, if not none at all, in $\tau_{\mathrm{off}}$.

## 8 References

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