A Haar wavelet-based perceptual similarity index for image quality assessment

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Outline

Gradient Map and Phase Congruency Map

- For a grayscale image $f \in R^2$, the gradient map is defined by,

$$G_f[x] = \sqrt{((g^{hor} * f)[x])^2 + ((g^{ver} * f)[x])^2} \tag{1}$$

where g^{hor} and g^{ver} denote horizontal and vertical gradient filters, and $*$ denotes the two-dimensional convolution operator.

- The phase congruency map of a grayscale image f is given by

$$PC_f[x] = \frac{|\sum_n (g_n^c * f)[x]|}{\sum_n |(g_n^c * f)[x]|} \tag{2}$$

where g_n^c denotes differently scaled and oriented wavelet filters

Local Feature Similarity Map

- The local feature similarity map for two grayscale images $f_1, f_2 \in R^2$ is defined by

$$FS_{f_1,f_2} = S(G_{f_1}[x], G_{f_2}, C_1)^{\beta} \cdot S(PC_{f_1}[x], PC_{f_2}, C_2)^{\gamma} \qquad (3)$$

where $C_1, C_2 > 0$ and $\beta, \gamma > 0$,S denotes the similarity measure function, which is defined by

$$S(a, b, C) = \frac{2ab + C}{a^2 + b^2 + C}$$

The Feature Similarity Index(FSIM)

Eventually, the feature similarity index is computed by taking the weighted mean of all local feature similarities, where the phase congruency map is used as a weight

$$FSIM_{f_1,f_2} = \frac{\sum_x FS_{f_1,f_2}[x] \cdot PC_{f_1,f_2}[x]}{\sum PC_{f_1,f_2}[x]} \tag{4}$$

The Haar Wavelet Filters

The 2-dimensional orthogonal Haar wavelet filters are

$$g_1^0 = \begin{bmatrix} 1/4 & 1/4 & 0 \\ 1/4 & 1/4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \; g_1^1 = \begin{bmatrix} -1/4 & 1/4 & 0 \\ -1/4 & 1/4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$g_1^2 = \begin{bmatrix} 1/4 & 1/4 & 0 \\ -1/4 & -1/4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \; g_1^3 = \begin{bmatrix} 1/4 & -1/4 & 0 \\ -1/4 & 1/4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where the subscript of $g_j^k (j \in N, k \in \{0, 1, 2, 3\})$ denotes the scale and the superscript is the type of the filter.

The Local Similarity Measure

The local similarity measure used to compute the HaarPSI is based on the first two stages of a two-dimensional discrete Haar wavelet transform and given by

$$HS^k_{f_1,f_2}[x] = l_\alpha \left(\frac{1}{2} \sum_{j=1}^{2} S\Big(|(g^k_j * f_1)[x]|, |(g^k_j * f_2)[x]|, C \Big) \right) \tag{5}$$

where $l_\alpha = \frac{1}{1+e^{-\alpha x}}$ is the logistic function with $\alpha > 0$ and $k \in \{1, 2\}$.

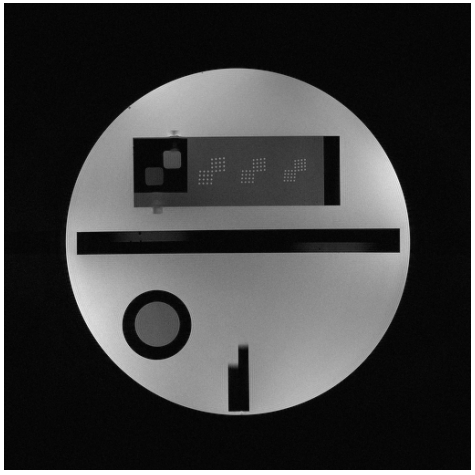
HaarPSI

The Haar-wavelet based perceptually similarity index for two grayscale images f_1, f_2 is eventually given as the weighted average of the local similarity map $HS^k_{f_1,f_2}$ that is

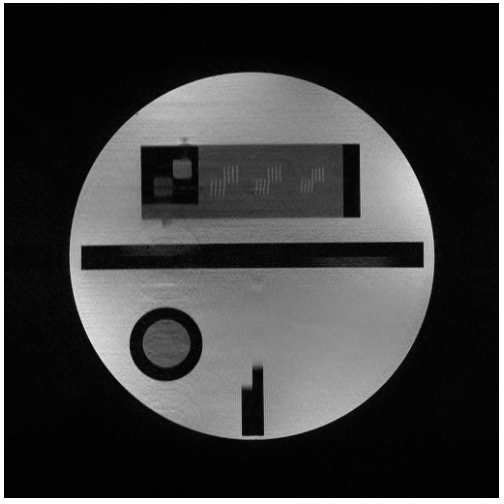
$$HaarPSI_{f_1,f_2} = l_\alpha^{-1}\left(\frac{\sum_x \sum_{k=1}^{2} HS^k_{f_1,f_2}[x] \cdot W^k_{f_1,f_2}[x]}{\sum_x \sum_{k=1}^{2} W^k_{f_1,f_2}[x]}\right)^2 \tag{6}$$

where $W^k_{f_1,f_2}[x] = max(W^k_{f_1}[x], W^k_{f_2}[x]),\ W^k_f[x] = |g^k_3 * f[x]|$

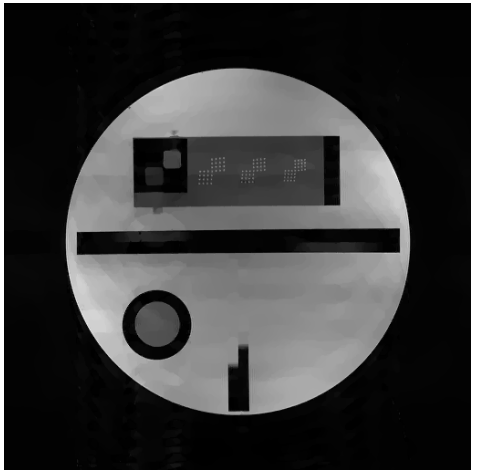
Experiments result



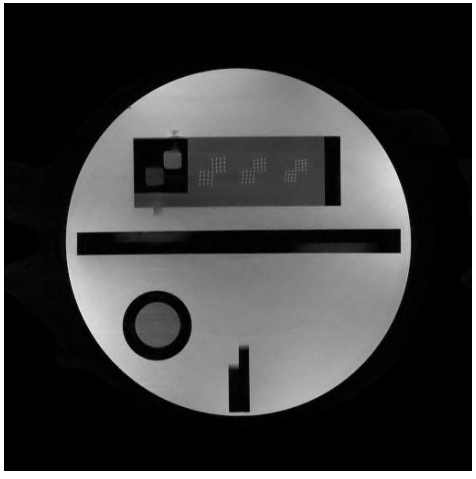
(a) reference



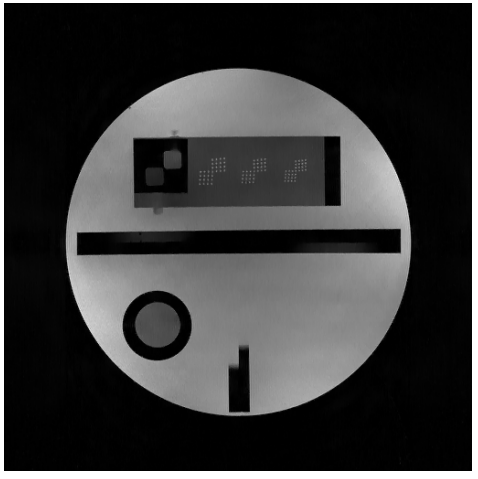
(b) ALOHA



(c) FADHFA



(d) ESPIRiT



(e) 3D-US

The comparison of the SSIM and HaarPSI

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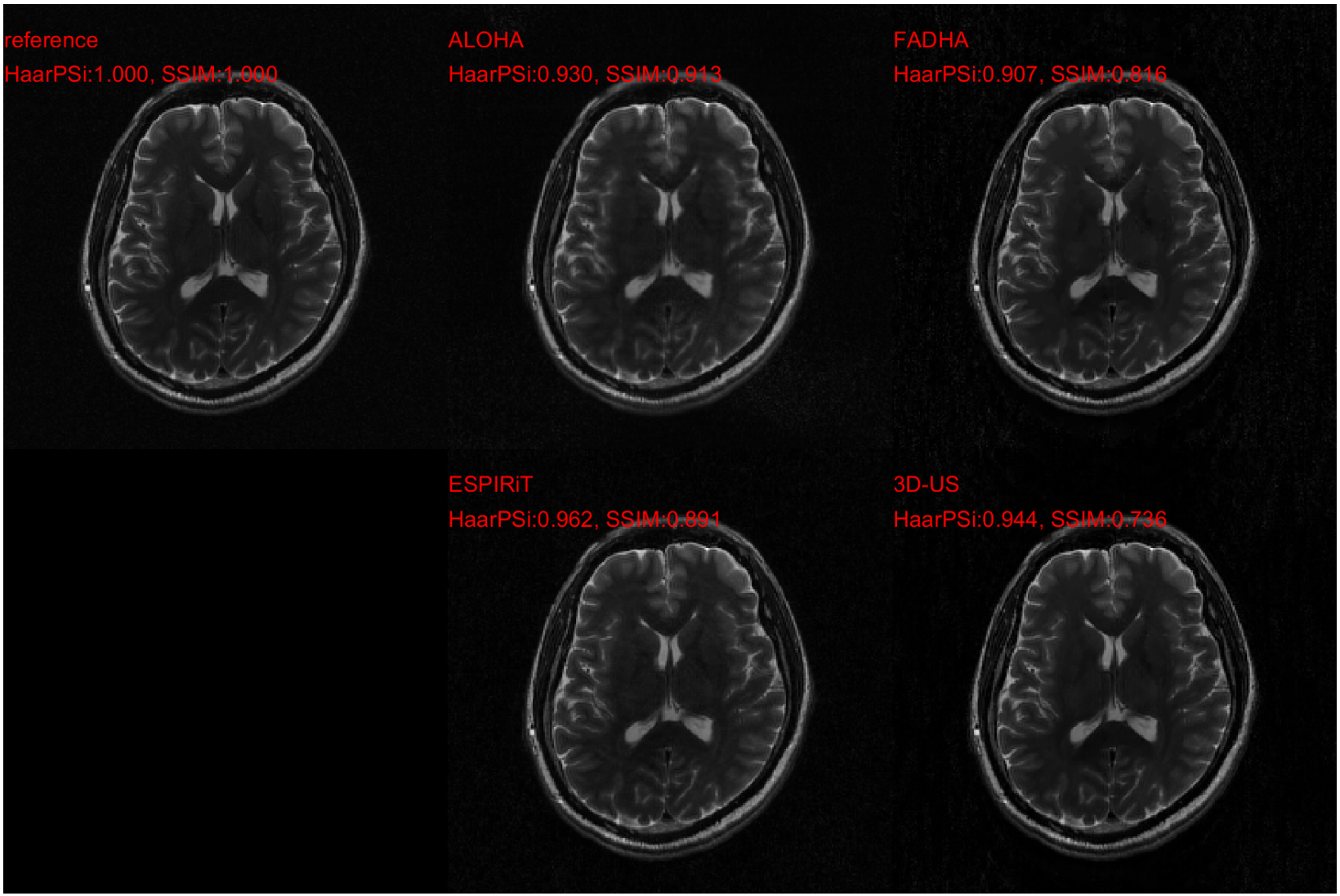
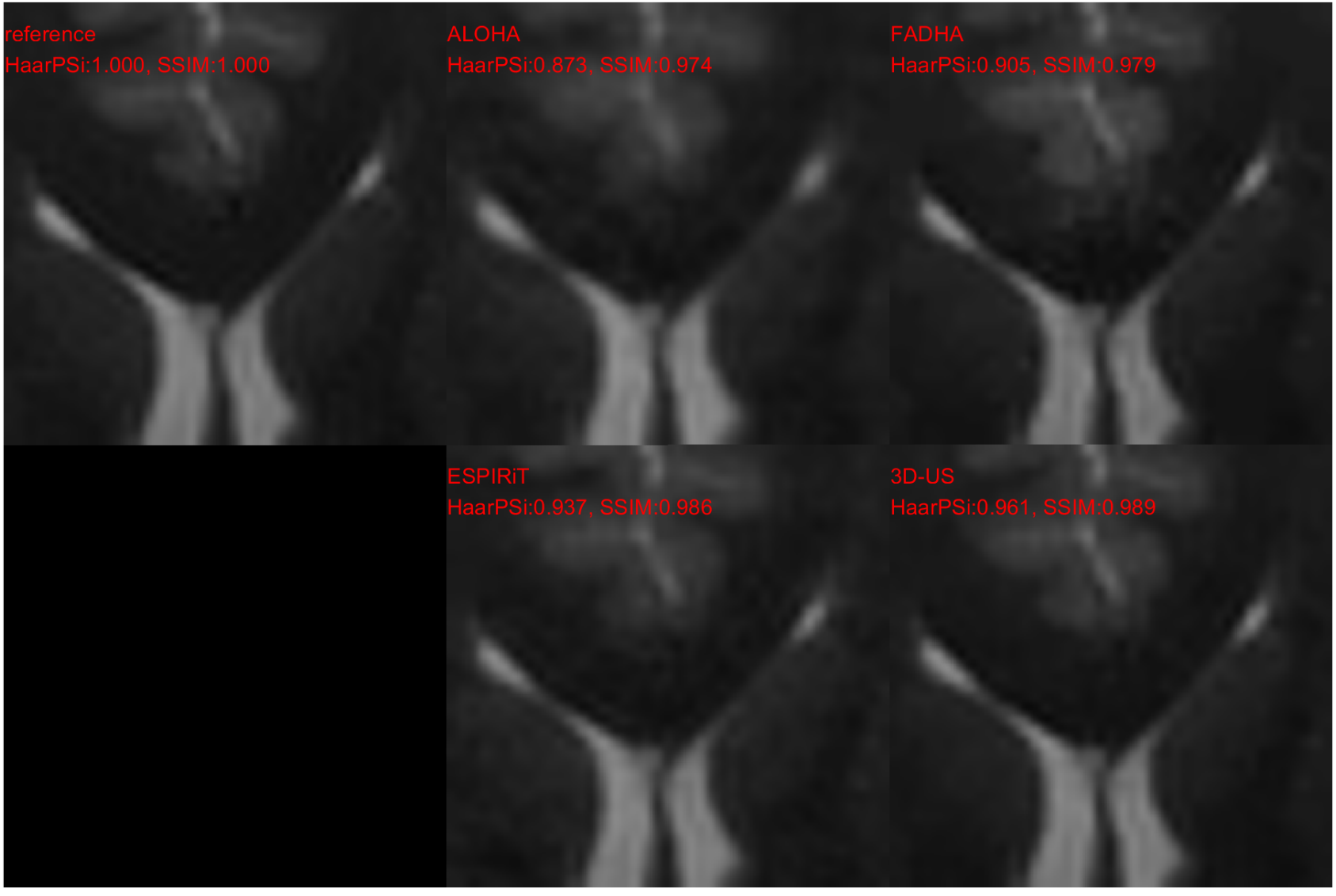
Algorithm	SSIM	HaarPSI
ALOHA	0.765	0.818
FADHFA	0.735	0.836
ESPIRiT	0.520	0.873
3D-US	0.629	**0.920**

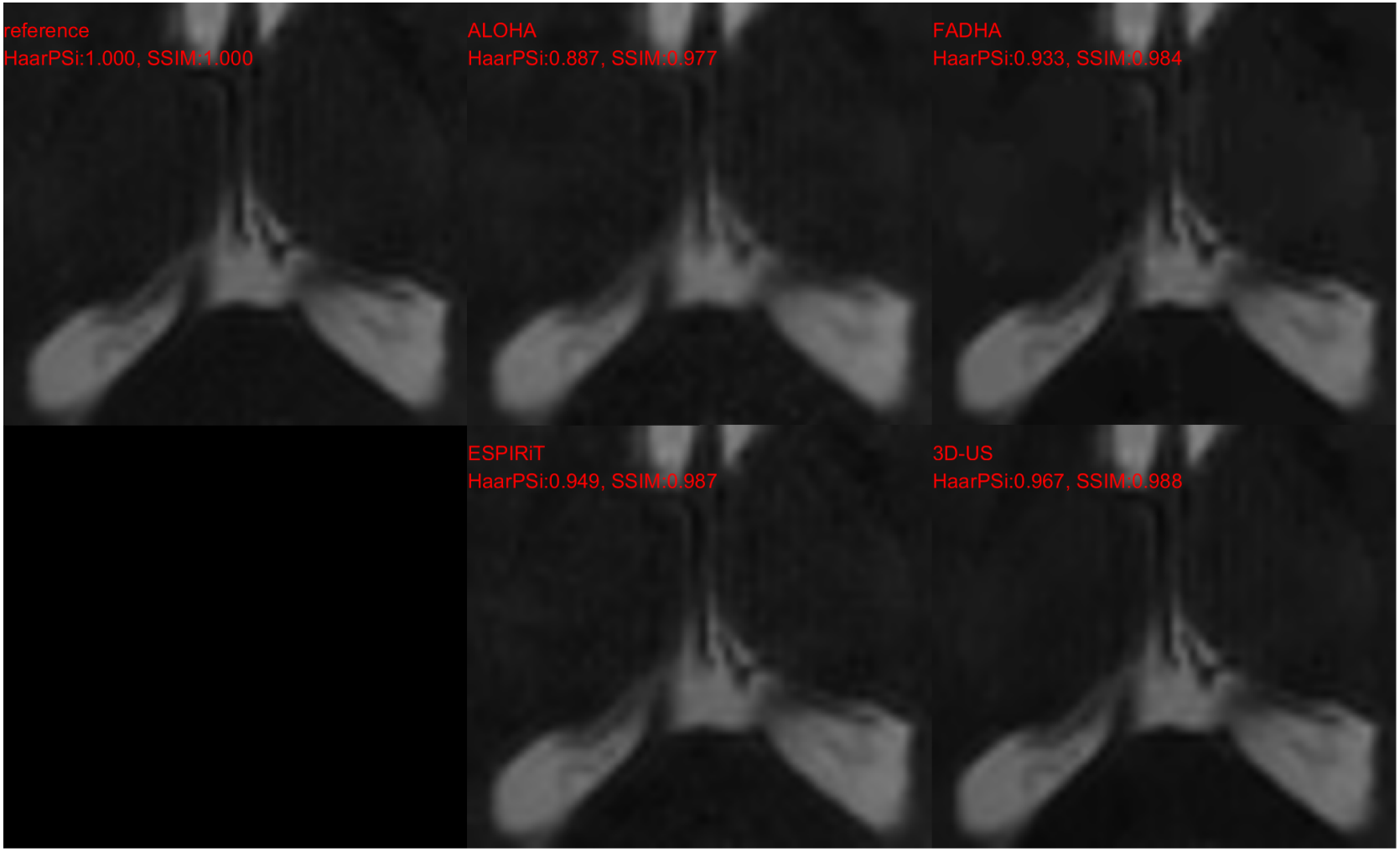
Comparison


reference
HaarPSi:1.000, SSIM:1.000
ALOHA
HaarPSi:0.930, SSIM:0.913
FADHA
HaarPSi:0.907, SSIM:0.816
ESPIRiT
HaarPSi:0.962, SSIM:0.891
3D-US
HaarPSi:0.944, SSIM:0.736

Comparison


reference
HaarPSi:1.000, SSIM:1.000
ALOHA
HaarPSi:0.873, SSIM:0.974
FADHA
HaarPSi:0.905, SSIM:0.979
ESPIRiT
HaarPSi:0.937, SSIM:0.986
3D-US
HaarPSi:0.961, SSIM:0.989

Comparison


reference
HaarPSi:1.000, SSIM:1.000
ALOHA
HaarPSi:0.887, SSIM:0.977
FADHA
HaarPSi:0.933, SSIM:0.984
ESPIRiT
HaarPSi:0.949, SSIM:0.987
3D-US
HaarPSi:0.967, SSIM:0.988