Report : Reconstruction Algorithm Based on 3D Patch

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1 Notation definition

1. $\boldsymbol{x} \in \mathbb{C}^{\boldsymbol{n}}$: the vecter form of the desired MRI image.
2. $\boldsymbol{P_i}$: the patch decompositon operator.
3. $\boldsymbol{b_i} \in \mathbb{C}^{\boldsymbol{L}\times\boldsymbol{L}}$: the $\boldsymbol{i}$ th patch can be expressed as $\boldsymbol{b_i} = \boldsymbol{P_i x}$
4. $\boldsymbol{v_j} = \{\boldsymbol{i_1}, \boldsymbol{i_2}, \ldots, \boldsymbol{i_Q}\}$: stores the index of patches.
5. $\boldsymbol{R_{v_j}}$: the grouping operator.
6. $\boldsymbol{R_{v_j} b_i}$: the $\boldsymbol{v_j}$ group of the image patches.
7. $\boldsymbol{W_{3D}}$: the 3D Tight framlet transfrom.

2 Patch-based nonlocal operator(PANO)

The nonlocal operator $\boldsymbol{A_j}$ is given by (1)

$$\boldsymbol{A_j} = \boldsymbol{W_{3D} R_{v_j} P_i} \tag{1}$$

If we perform $\boldsymbol{A_j x}$, this operator is mainly divided into three steps:

1. A patch $\boldsymbol{b_i} \in \mathbb{C}^{\boldsymbol{L}\times\boldsymbol{L}}$ can be get from $\boldsymbol{P_i x}$.
2. A similar patches of one group can be obtained by $\boldsymbol{R_{v_j} P_i x}$.
3. A 3D tight framelet transform is perfromed on the group $\boldsymbol{R_{v_j} P_i x}$.

The adjoint operator of $\boldsymbol{A_j}$ is $\boldsymbol{A_j^T} = \boldsymbol{P_i^T R_{v_j}^T W_{3D}^T}$, it satisfies

$$\begin{aligned}
\sum_{\boldsymbol{j}}^{\boldsymbol{J}} \boldsymbol{A_j^T A_j} &= \sum_{\boldsymbol{j}}^{\boldsymbol{J}} \boldsymbol{P_i^T R_{v_j}^T W_{3D}^T W_{3D} R_{v_j} P_i} \\
&= \sum_{\boldsymbol{j}}^{\boldsymbol{J}} \boldsymbol{P_i^T R_{v_j}^T R_{v_j} P_i} \\
&= \boldsymbol{diag}\{\boldsymbol{o_1}, \boldsymbol{o_2}, \cdots, \boldsymbol{o_n}, \cdots, \boldsymbol{o_N}\} \\
&= \boldsymbol{O}
\end{aligned}$$

where o_i is a counter indicating the times that the ith pixel is grouped into 3D patch arrays. Therefor the $\sum_j^J A_j^T A_j = O$, where O is a diagonal matrix with the ith diagonal element is o_i. The invertibility of O requires that each pixel must be contained in at least one group. The PANO coefficients α_j is given by

$$\alpha_j = A_j x \tag{2}$$

and the image can be estimated from PANO coefficients by

$$\hat{x} = O^{-1} \sum_j^J A_j^T \alpha_j \tag{3}$$

3 MRI reconstruction model using PANO

3.1 PANO reconstruction model

Based on the 3D tight framelet, we can propose a MRI reconstruction model using PANO by solving the following problem

$$\hat{x} = \underset{x}{argmin} \left\{ \frac{1}{2} \|PFSx - y\|_2^2 + \sum_j^J \lambda_j \|A_j x\|_1 + \|\Gamma W_{3D} F^{-1} (\tilde{P} FSx + y)\|_1 \right\} \tag{4}$$

where y denotes the measured k-space data, F denotes the Fourier transform, λ_j is a positive regularization parameter, $\tilde{P} = I - P$, Γ is diagonal matrix with the postive diagonal elements.

3.2 Structural texture similarity based on 2D tight framelet(STSIM-2DTF)

The conventional method for obtaining similar patches is to calculate the l_2-norm between the reference patch and candidate patches. $Q - 1$ candidates with the smallest distance are selected as similar patches, which called the patch match. In this paper, we use another method to measure the distance between the patches, called the Structural Texture Similarity based on 2D Tight Framelet(STSIM-2DTF).

$S = \lfloor (W - W_i)/2 \rfloor \{aa\}$