

Assignment 5

due Wednesday, May 22, 2019

1. (exercise 15.2-1, p 378 from CLRS) Find an optimal parenthesization of a matrix-chain product instance whose sequence of dimensions is $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$. Show the final m and s tables. [8 points]
2. For this dynamic programming problem and the next two, be sure to
 - (a) describe the subproblem
 - (b) give a recurrence for the subproblem
 - (c) provide pseudo-code showing how a table for the subproblems is filled
 - (d) give the time and space requirements of your method

The residents of the underground city of Zion defend themselves through a combination of kung-fu, heavy artillery, and efficient algorithms. Recently, they have become interested in automated methods that can help fend off attacks by swarms of robots.

Here's what one of these robot attacks look like:

- A swarm of robots arrives over the course of n seconds; in the i^{th} second, x_i robots arrive. Based on remote sensing data, you know this sequence x_1, x_2, \dots, x_n in advance.
- You have at your disposal an electromagnetic pulse (EMP), which can destroy some of the robots as they arrive; the EMP's power depends on how long it's been allowed to charge up. To make this precise, there is a function $f(-)$ so that if j seconds have passed since the EMP was last used, then it is capable of destroying up to $f(j)$ robots.
- So, specifically, if it is used in the k^{th} second, and it has been j seconds since it was previously used, then it will destroy $\min[x_k, f(j)]$ robots. (After this use, it will be completely drained.)
- We will also assume that the EMP starts off completely drained, so if it is used for the first time in the j^{th} second, then it is capable of destroying up to $f(j)$ robots.

Given the data on robot arrivals $X = (x_1, x_2, \dots, x_n)$, and given the recharging function $f(-)$, the problem is to determine the maximum number of robots that can be destroyed by activating the EMP at certain points in time.

For example, suppose $n = 4$, $X = (1, 10, 10, 1)$ and $f(-) = (1, 2, 4, 8)$. The best solution would be to activate the EMP in the 3rd and 4th seconds. In the 3rd second, the EMP has gotten to charge for 3 seconds, and so it destroys $\min(10, 4) = 4$ robots. In the 4th second, the EMP has only gotten to charge for 1 second since its last use, and it destroys $\min(1, 1) = 1$ robot. This is a total of 5. [10 points]

3. Suppose we have two transmitters, each of which sends out repetitions of some short string. For example, transmitter 1 may repeat string $x=101$ over and over, so what we will hear from it will be a prefix of x^k - that is, x concatenated to itself k times, possibly with a few bits

chopped of the end (as in *10110110*). Transmitter 2 repeats another string, y . Our job is to determine if a sequence s that we have heard is an interleaving of these two transmissions.

For example, suppose transmitter 1 repeats $x=101$ and transmitter 2 repeats $y=01$. The sequence *010111010101* can be unraveled into x and y : positions 1, 5, 9, and 12 contain *0101*, a repetition of y , while the remainder of the string contains *10110110*, a repetition of x .

Describe an efficient algorithm which takes a sequence s of length n , and two strings x and y , and determines if s is an interleaving of repetitions of x and y . **[10 points]**

4. (exercise 6.23, p 196, from DPV) A mission-critical production system has n stages that have to be performed sequentially; stage i is performed by machine M_i . Each machine M_i has a probability r_i of functioning reliably and a probability $1 - r_i$ of failing (and the failures are independent). Therefore, if we implement each stage with a single machine, the probability that the whole system works is $r_1 \cdot r_2 \cdots r_n$. To improve this probability we add redundancy by having m_i copies of the machine M_i that performs stage i . The probability that all m_i copies fail simultaneously is only $(1 - r_i)^{m_i}$, so the probability that stage i is completed correctly is $1 - (1 - r_i)^{m_i}$ and the probability that the whole system works is $\prod_{i=1}^n [1 - (1 - r_i)^{m_i}]$. Each machine M_i has a cost c_i , and there is a total budget B to buy machines. (Assume that B and the c_i are positive integers.)

Given the probabilities r_1, r_2, \dots, r_n , the costs c_1, c_2, \dots, c_n , and the budget B , find the maximum reliability that can be achieved within budget B . **[10 points]**

Total: 38 points

Note(s):

- A problem very similar to problem 3 had been outlined during the midterm review. Pictures of notes from the board are here <https://classes.cs.uoregon.edu/19S/cis315/midterm/Photos/interleave/>