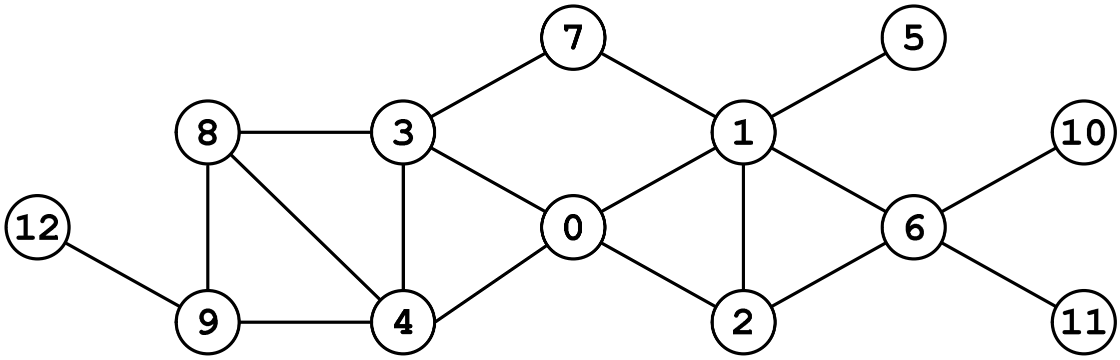
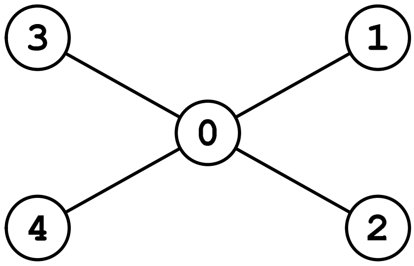
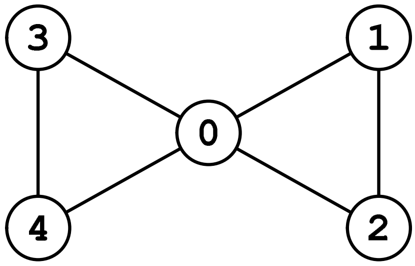


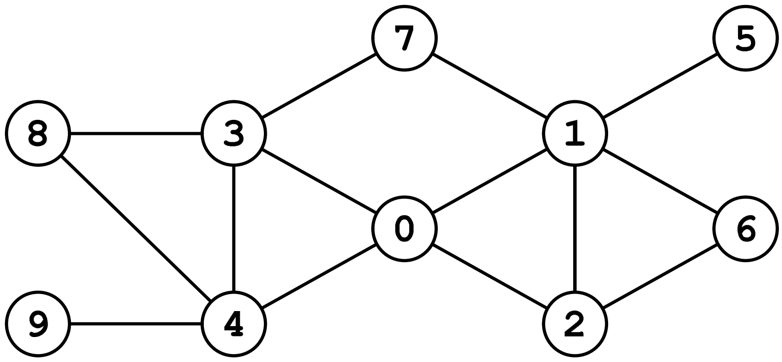
A given network



Π_0



Ω_0



X_0

	1	2	5	6
3	1/2	1/1	1/2	2/3
4	1/1	1/1	1/1	2/2
8	2/3	2/2	2/3	4/5
9	1/1	1/1	1/1	2/2

● Shortest paths between the node 7 and other nodes do not pass through the node 0

$$C_B(0) = \frac{\sum_{s \neq t \in N_i \cup N_i^2, s<t} \frac{\sigma_{st}(0)}{\sigma_{st}}}{9 \times 8/2} = \frac{13.801}{36} = 0.383$$

$$A = \begin{pmatrix} 0&1&1&1&1&0&0&0&0&0 \\ 1&0&1&0&0&1&1&1&0&0 \\ 1&1&0&0&0&0&1&0&0&0 \\ 1&0&0&0&1&0&0&1&1&0 \\ 1&0&0&1&0&0&0&0&1&1 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&1&1&0&0&0&0&0&0&0 \\ 0&1&0&1&0&0&0&0&0&0 \\ 0&0&0&1&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \end{pmatrix} \quad A^2 = \begin{pmatrix} 4&1&1&1&1&1&2&2&2&1 \\ 1&5&2&2&1&0&1&0&0&0 \\ 1&2&3&1&1&1&1&1&0&0 \\ 1&2&1&4&2&0&0&0&1&1 \\ 1&1&1&2&4&0&0&1&1&0 \\ 1&0&1&0&0&1&1&1&0&0 \\ 2&1&1&0&0&1&2&1&0&0 \\ 2&0&1&0&1&1&1&2&1&0 \\ 2&0&0&1&1&0&0&1&2&1 \\ 1&0&0&1&0&0&0&0&1&1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 4&10&7&9&8&1&2&2&2&1 \\ 10&4&7&2&3&5&7&7&3&1 \\ 7&7&4&3&2&2&5&3&2&1 \\ 9&2&3&4&7&2&3&6&6&2 \\ 8&3&2&7&4&1&2&3&6&4 \\ 1&5&2&2&1&0&1&0&0&0 \\ 2&7&5&3&2&1&2&1&0&0 \\ 2&7&3&6&3&0&1&0&1&1 \\ 2&3&2&6&6&0&0&1&2&1 \\ 1&1&1&2&4&0&0&1&1&0 \end{pmatrix} \quad A^4 = \begin{pmatrix} 34&16&16&16&16&10&17&19&17&8 \\ 16&36&21&23&16&4&11&6&5&3 \\ 16&21&19&14&13&7&11&10&5&2 \\ 16&23&14&28&21&2&5&6&11&7 \\ 16&16&13&21&25&3&5&10&11&4 \\ 10&4&7&2&3&5&7&7&3&1 \\ 17&11&11&5&5&7&12&10&5&2 \\ 19&6&10&6&10&7&10&13&9&3 \\ 17&5&5&11&11&3&5&9&12&6 \\ 8&3&2&7&4&1&2&3&6&4 \end{pmatrix}$$

For a node i, let u and w be the number of nodes N_i and N_i^2, respectively.

$$A = \begin{pmatrix} [A]_{00} & [A]_{01} & [A]_{02} \\ [A]_{10} & [A]_{11} & [A]_{12} \\ [A]_{20} & [A]_{21} & [A]_{22} \end{pmatrix} = \begin{pmatrix} 0 & \overline{1} & \overline{0} \\ \overline{1}^T & [A]_{11} & [A]_{12} \\ \overline{0}^T & [A]_{21} & \tilde{0} \end{pmatrix} \text{ where}$$

- $[A]_{00}$ is the 1×1 matrix of the entry zero.
- $[A]_{01} = \overline{1}$ is $1 \times u$ row vector with all entries one.
- $[A]_{02} = \overline{0}$ is $1 \times w$ row vector with all entries one.
- $[A]_{22} = \tilde{0}$ is $w \times w$ matrix with all entries zero (because the links among the 2-hop neighbors are not included in an expanded ego network).

Let $[A^k]_{ab}$ denote the sub-matrix on indices a and b of the matrix A's kth power (i.e., A^k) where $a, b \in \{0,1,2\}$. The values 0, 1 and 2 of a (or b) mean that the path starts from (or ends up with) the node i, a node in N_i and a node in N_i^2, respectively. Then, each entry of the sub-matrix $[A^k]_{ab}$ contains the number of paths of length k on the different pairs of nodes according to the values of a and b.

Let $I = \overline{1}^T \cdot \overline{1}$ be $u \times u$ matrix with all entries one.

$$[A^2]_{11} = I + ([A]_{11})^2 + [A]_{12} \cdot [A]_{21}$$
$$[A^3]_{12} = [A^2]_{11} \cdot [A]_{12} = (I + ([A]_{11})^2 + [A]_{12} \cdot [A]_{21}) \cdot [A]_{12}$$
$$[A^4]_{22} = A_{21} \cdot [A^2]_{11} \cdot A_{12} = A_{21} \cdot (I + ([A]_{11})^2 + [A]_{12} \cdot [A]_{21}) \cdot [A]_{12}$$