CSCE 104 HW 1

Q3

a) $T(n) = \sum_i \Theta(1) = \sum_{k=1}^{?} \Theta(1) \to \sum_{k=0}^{\log\log n} \Theta(1) = \boxed{\Theta(\log\log n)}$

k	1	2	3	4
i	2	4	16	256
	2^1	2^2	2^4	2^8

$i = 2^{2^{(k-1)}}$

stop when $i = n = 2^{2^{(k-1)}}$

$\log_2 n = 2^{(k-1)}$

$\log\log n = k-1$

$k = 1 + \log\log n$

b) $T(n) = \sum_{i=1}^{n} \Theta(1) + O\left(\sum_{i=1}^{n} \sum_{k=0}^{i^3} \Theta(1)\right)$

$= \Theta(n) + \sum_i \Theta(i^3) \to \sum_{i=1}^{n^{1/2}} \Theta\left(c^3 n^{\frac{3}{2}}\right) = \dots$

c	1	2	3	…
i	$\sqrt{n}$	$2\sqrt{n}$	$3\sqrt{n}$	

stop when $i = n = c\sqrt{n}$

$i = c\sqrt{n}$ $\quad n^{1/2} = c$

$= \Theta(n) + \Theta\left(\dots n^{\frac{3}{2}} \left(n^{\frac{1}{2}}\right)^4\right)$

$= \Theta(n) + \Theta\left(n^{\frac{3}{2}} n^{\frac{4}{2}}\right) = \boxed{\Theta\left(n^{\frac{7}{2}}\right)}$

c) $T(n) = \sum_{i=1}^{n} \sum_{k=1}^{n} O\left(\sum_m \Theta(1)\right) = \sum_{i=1}^{n} \sum_{k=1}^{n} O\left(\sum_{c=0}^{\log n} \Theta(1)\right) = \sum_{i=1}^{n} \sum_{k=1}^{n} O(\log n)$

c	1	2	3	4
m	1	2	4	8

$m = 2^{c-1}$ stop when $m = n = 2^{c-1}$

$1 + \log n = c$

$T(n) = \sum_{i=1}^{n} \sum_{k=1}^{n} \Theta(1) + \sum_{i=1}^{n} O\left(\sum_{k=1}^{n} \log n\right)$

In the worst case the if statement will only ~~run n~~ be true n times. There are n elements in a, all of...

possible worst case	iterations
$\{1, 2, 3, \dots, n-1, n\}$	once every i (n times)
$\{1, 1, 1, 1, \dots\}$	n times when i = 1, 0 times otherwise so n times total

them checked to be equal to i=1, 2, 3, ... only once. Thus, on each iteration of the outer loop, A[k]==i is true for the number of i-values in the array (let's call this n_i). Since i never repeats, (i is unique for each iteration), then $n_1 + n_2 + \ldots + n_n \leq n$ where n is the total elements in A.

$$T(n) = \theta(n^2) + \theta(n\log n) = \theta(n^2)$$

d) $T(n) = \theta(n) + O\left(\sum_{i=0}^{n-1}\sum_{j=0}^{size}\theta(1) + \sum_{i=0}^{n-1}\theta(1)\right)$ $\quad \sum_i \theta(i) + \sum_i \theta(1)$

i	10
size	10

size=i (in the internal for loop)

K	0	1	2	3	4
i	10	15	22	33	49

$i = 10 \cdot \left(\frac{3}{2}\right)^k$

stops at $i = n-1 = 10 \cdot \left(\frac{3}{2}\right)^k$

$k = \log\left(\frac{n-1}{10}\right)$

$$T(n) = \theta(n) + \sum_{k=0}^{\log\left(\frac{n-1}{10}\right)} \theta\left(\cancel{10} \cdot \left(\frac{3}{2}\right)^k\right) + \cancel{\sum_{k=0}^{\log(n-1/10)} \theta(1)}$$

$$= \theta(n) + \theta\left(\left(\frac{3}{2}\right)^{\log n}\right)$$

$$= \theta(n) + \theta(n)$$

$$\boxed{T(n) = \theta(n)}$$