CS HW

1. unusual. How many unique subsets of S?

u	n	s	a	l
3	1	1	1	1

\# of unique subsets w/ 1 u (4 other letters)
    +
\# of unique subsets w/ 2 u's (3 other letters)
    +
\# of unique subsets w/ 3 u's (2 other letters)

$= \binom{4}{4} + \binom{4}{3} + \binom{4}{2} = 11$

How many different strings? Set u's as stars and other letters as bars, for a given number of u's

1 u: $5!$
2 u's: $4 \cdot 3 \cdot 2 \cdot \binom{5}{2}$
3 u's: $4 \cdot 3 \cdot \binom{5}{3}$

$$\cdot 5! + 4 \cdot 3 \cdot 2 \cdot \binom{5}{2} + 4 \cdot 3 \cdot \binom{5}{3} = 480$$
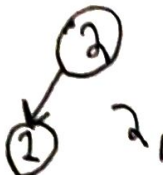
2. Choose 2 pairs from all possible pairs, and multiply by the remaining cards
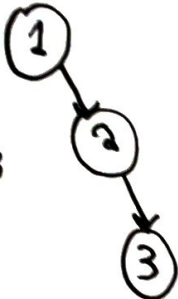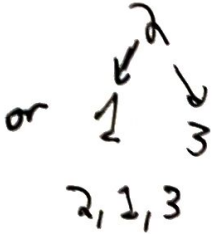All possible pairs: $\binom{4}{2} \cdot 13$
                                        ← remaining cards
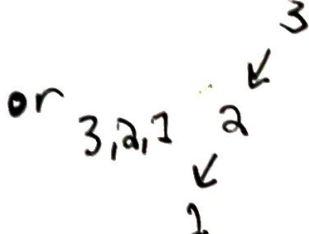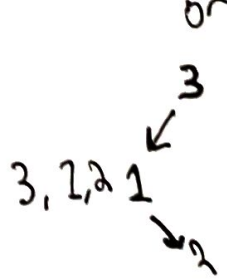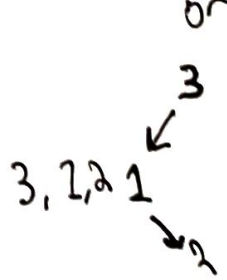
Answer: $$\binom{\binom{4}{2} \cdot 13}{2} \cdot (11 \cdot 4) = 132132$$

3. 16 indis. items, 7 boxes. BUT, one box takes AT MOST 1 item. So, do 15 indis items and 6 boxes, and then multiply by 7 (take into account the last item that's allowed to go into the last box)

Answer: $\boxed{\binom{20}{15} \cdot 7 = 108528}$

4.

2 nodes:

1,2 or 2,1   2 options

3 nodes:

1,2,3   or   2,1,3   or   3,2,1

1,3,2   or   2,3,1   or   3,2,1

5 options

4 nodes:

9 options

5 nodes: $9 + 5 + 2 \cdot 2 + 5 + 9 = 32$

4 (cont.).

Actual problem:

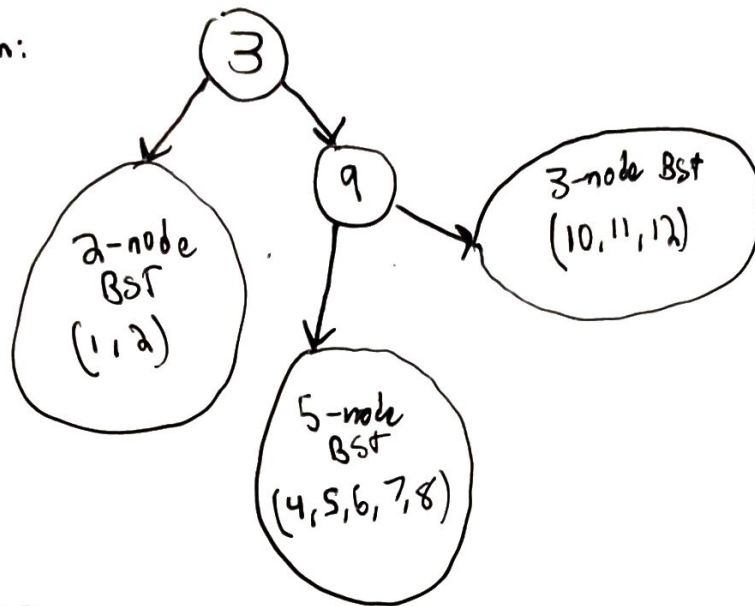

The tree diagram shows:
- Root node: 3
- Left child: 2-node BST (1,2)
- Right child: 9
  - Left child of 9: 5-node BST (4,5,6,7,8)
  - Right child of 9: 3-node BST (10,11,12)

$$2 \cdot 32 \cdot 5 = 320$$

5. 3: $(1, 1, 1, 0)$ 1
   4: $(1, 1, 1, 1)$
      $(2, 1, 1, 0)$ 2
   5: $(2, 1, 1, 1)$
      $(3, 1, 1, 0)$
      $(2, 2, 1, 0)$ 3
   6: $(3, 1, 1, 1)$
      $(3, 2, 1, 0)$
      $(4, 1, 1, 0)$
      $(2, 2, 2, 0)$
      $(2, 2, 1, 1)$ 5
   7: $(5, 1, 1, 0)$ $(2, 2, 2, 1)$
      $(4, 2, 1, 0)$ 7
      $(4, 1, 1, 1)$
      $(3, 3, 1, 0)$
      $(3, 2, 2, 0)$
      $(3, 2, 1, 1)$

   8: $(6, 1, 1, 0)$ 9: $(7, 1, 1, 0)$
      $(5, 2, 1, 0)$ $(6, 2, 1, 0)$
      $(5, 1, 1, 1)$ $(6, 1, 1, 1)$
      $(4, 3, 1, 0)$ $(5, 3, 1, 0)$
      $(4, 2, 2, 0)$ $(5, 2, 2, 0)$
      $(4, 2, 1, 1)$ $(5, 2, 1, 1)$
      $(3, 3, 2, 0)$ $(4, 3, 2, 0)$
      $(3, 3, 1, 1)$ $(4, 3, 1, 1)$
      $(3, 2, 2, 1)$ $(4, 2, 2, 1)$
      $(2, 2, 2, 2)$ $(3, 3, 2, 1)$
         10               $(3, 2, 2, 2)$
                             11

5 (cont.).

10: (7, 2, 1, 0)
   (7, 1, 1, 1)
   (6, 3, 1, 0)
   (6, 2, 2, 0)
   (6, 2, 1, 1)
   (5, 4, 1, 0)
   (5, 3, 2, 0)
   (5, 3, 1, 1)
   (5, 2, 2, 1)
   (4, 4, 2, 0)
   (4, 4, 1, 1)
   (4, 3, 3, 0)
   (4, 3, 2, 1)
   (4, 2, 2, 2)
   (3, 3, 3, 1)
   (3, 3, 2, 2) Total: 16

Final total = 1 + 2 + 3 + 5 + 7 + 10 + 11 + 16 = $\boxed{55}$