

Probability Problems

$$1. \frac{\binom{15}{8}}{15^8} \approx 2.51 \times 10^{-6}$$

2. Total number of integers that meet the criteria
 $\hookrightarrow 5 \cdot 4 \cdot 5 + 5 \cdot 4 \cdot 5 \cdot 7 + 5 \cdot 4 \cdot 5 \cdot 7 \cdot 6 \approx 5000$

Total Integers: 100,000

Chance of picking 1: $\frac{5000}{100,000}$

Number of ways to generate 8 numbers with 5 being meeting criteria: $\binom{8}{5}$

Chance of generating 8 with 5 meeting criteria in one specific order:

$$\left(\frac{5000}{100,000}\right)^5 \cdot \left(\frac{95,000}{100,000}\right)^3$$

Final answer: $\left(\frac{5000}{100,000}\right)^5 \cdot \left(\frac{95,000}{100,000}\right)^3 \cdot \binom{8}{5} \approx 1.50 \times 10^{-5}$

3. $P(A \cap B) = \frac{1}{2}$ (444, 555, 666 / 111, 222, 333)

$$P(B) = \left(\frac{1}{6}\right)^3 \cdot 6 = \frac{1}{36}$$

$$P(A) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \binom{3}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

$$P(B) \cdot P(A) \neq P(A \cap B) \quad \text{Not independent}$$

4. chance of getting a flush:

$$\frac{13}{52} \cdot \frac{12}{52} \cdot \frac{11}{52} \cdot \frac{10}{52} \cdot 4 \approx .009388$$

$$1 / .009388 \approx 106.52$$

106.52 hands on average

5. Prob of winning 4/5 games = Prob of winning 4/5 ^{given} superstar

4/5 given superstar

$$(.7)^4 \cdot .3 \cdot \binom{5}{4}$$

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Prob of having superstar

+
Prob of winning 4/5 given no superstar

*
Prob of no superstar

4/5 given no superstar

$$(.5)^4 \cdot .5 \cdot \binom{5}{4}$$

Prob of winning 4/5

$$.75 \cdot (.7)^4 \cdot \binom{5}{4}$$

$$+ .25 \cdot (.5)^5 \cdot \binom{5}{4}$$

Given they won 4/5, prob that superstar played:

Prob that superstar played AND they won 4/5

Prob they won 4/5

$$\frac{.75 \cdot (.7)^4 \cdot .3 \cdot \binom{5}{4}}{.75 \cdot (.7)^4 \cdot \binom{5}{4} + .25 \cdot (.5)^5 \cdot \binom{5}{4}} \approx .8737$$