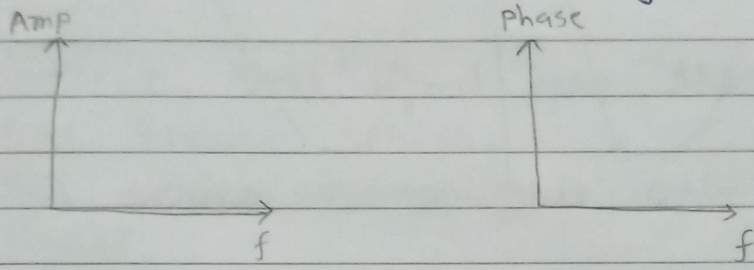
CHAPTER - 2 HMP sir

WAVEFORM SPECTRA

→ Graphical representation of signal is called waveform.

→ Specturum related to frequency.
→ Specturum means frequency band.



Amplitude spectrum phase spectrum

→ spectra = Amplitude spectrum + phase spectrum

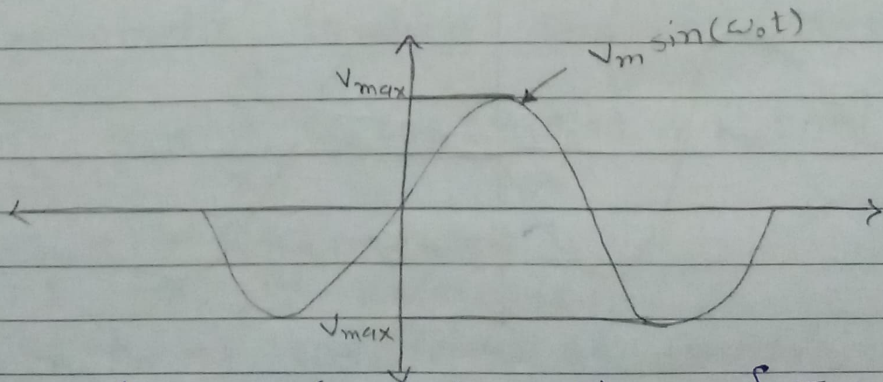
→ square signal → consist all component of cosine

time domain to frequency domain

→ fourier series → convert time domain to frequency domain

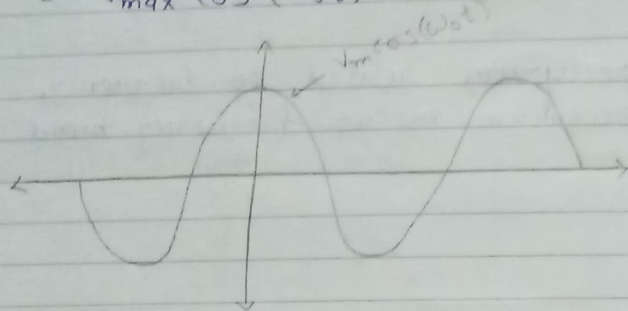
* For sine

$V(t) = V_{max} \sin(2\pi f_0 t)$
$= V_{max} \sin(\omega_0 t)$



$\therefore$ $V(t) = V_{max} \cos(2\pi f_0 (t - \tau))$ $\{ \tau = T_0/4 \}$

$v(t) = V_{max} \cos(2\pi f_0 t)$
$= V_{max} \cos(\omega_0 t)$



* for sine

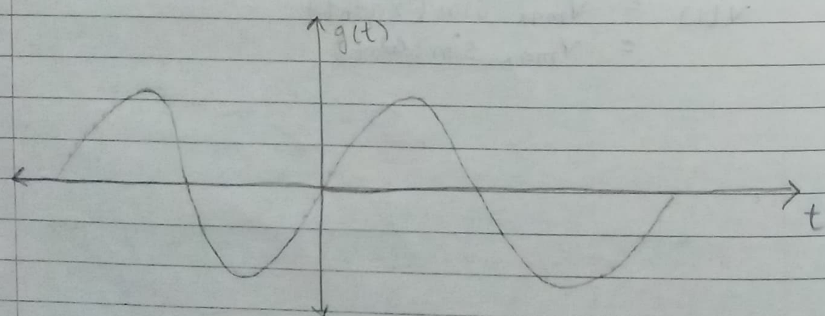
$\therefore$ $V(t) = V_{max} \cos(2\pi f_0 t + \phi)$

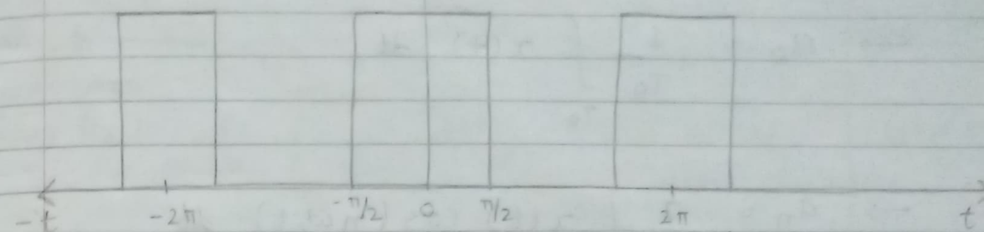
* $\boxed{\phi = -\frac{2\pi \tau}{T_0}}$ ← ϕ → $-ve$ → lagging signal; $+ve$ → leading signal

ϕ → phase angle

* Periodic Signal

→ $-\infty \leq t \leq \infty$ (t = time period)
→ $g(t) = g(t + T_0)$ (value for any time is same)
→ The shape of each cycle is same.





→ Convert time domain signal into frequency domain signal use fourier series.
→ This series is applicable only on periodic signal.

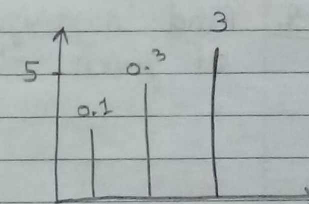
★ FOURIER TRANSFORM

$x(t) = 5 + 0.1 \sin(2\pi\, 100 t) + 0.3 \sin(2\pi\, 200 t) + 3 \sin(2\pi\, 300 t) + \dots$

(f_0 (fundamental frequency))

f_0 = fundamental frequency
(first term's frequency after D.C. component)

Harmonic components:
$2f_0$ → 2nd Harmonic
$3f_0$ → 3rd Harmonic
$\vdots$
nf_0 → n^{th} Harmonic



* Trigonometic fourier series (M-1)

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$n = 1$ → fundamentall
$n = 2$ → 2^{nd} Harmonic
a_0, a_n, b_n → coefficient of fourier series

$\rightarrow a_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt$ ← dc component (1st term)

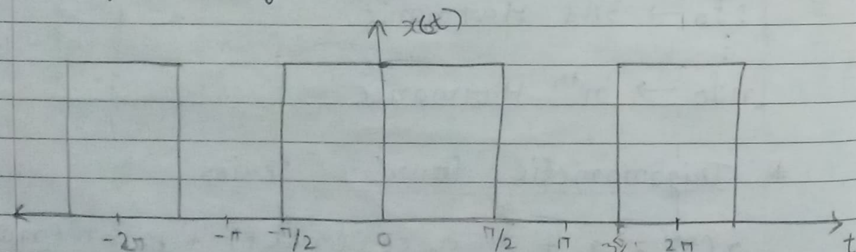
$\rightarrow a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos(n\omega_0 t)\, dt$

$\rightarrow b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin(n\omega_0 t)\, dt$

→ cosine signal is called even function of time. Because $f(t) = f(-t)$ (even signal)

→ sinusoidal signal is called odd function of time (odd signal). Because $f(t) = -f(-t)$

Q. Find A trigonometric fourier series expansion of given signal.

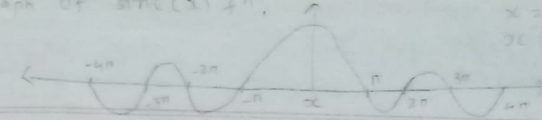


$\rightarrow x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin(n\omega_0 t)$

$\rightarrow T_0 = \pi - (-\pi) = 2\pi$

$\omega_0 = \frac{2\pi}{T_0} = 1$

→ Graph of sinc(x) fn.



$x = 0$, sinc = 1

$x = \pm n\pi$, sinc = 0

$\rightarrow a_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt$

$\therefore a_0 = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} 1\, dt$ બાકી અંતરાલમાં $x(t) = 0$ છે એટલે ન લીધું.

$$\boxed{\therefore a_0 = \frac{1}{2}}$$

$\rightarrow a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos(n\omega_0 t)\, dt$

$\therefore a_n = \frac{2}{2\pi} \int_{-\pi/2}^{\pi/2} 1 \cdot \cos(nt)\, dt$

$\therefore a_n = \frac{1}{\pi} \left[\frac{\sin(nt)}{n} \right]_{-\pi/2}^{\pi/2}$

$\therefore a_n = \frac{1}{n\pi} \left[\sin\frac{n\pi}{2} + \sin\frac{n\pi}{2} \right]$

$$\boxed{\therefore a_n = \frac{2 \sin n\pi/2}{n\pi}} = \frac{\sin(n\pi/2)}{n\pi/2} = \frac{\sin x}{x} = \text{sinc}(x) = \text{sinc}\left(\frac{n\pi}{2}\right)$$

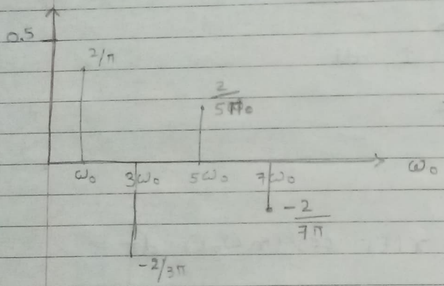
$\rightarrow b_n = 0$ $\{\cos n\pi/2 = 0\}$

$\rightarrow x(t) = 0.5 + \sum_{n=1}^{\infty} \left(\frac{2}{n\pi} \sin\frac{n\pi}{2} \right) \cos nt$

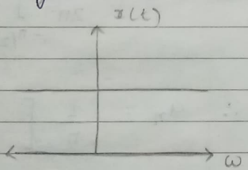
$= 0.5 + \frac{2}{\pi}\cos t - \frac{2}{3\pi}\cos 3t + \frac{2}{5\pi}\cos 5t + \ldots$

→ Triangle wave માટે $x(t) = 1$ મોટું ના લખી શકાય
square wave માટે લખી શકાય.

Amplitude spectrum (If say)



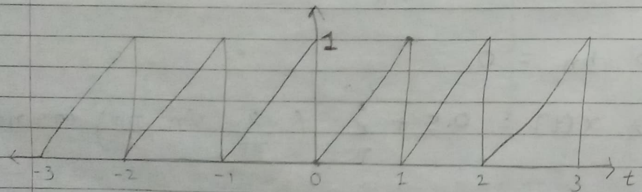
Q. એ Horizontal symetrical signal હોય :



→ તો ખાલી a_0 નું સૂત્ર
પછી $a_0 = 0$, ખાલી
લખવાનું given signal
is Horizontal symetrical

Q.2 If signal is even → તો $b_n = 0$:

Q.3 If signal is odd → તો $a_n = 0$



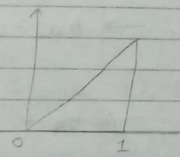
Q Find the trignomatric furiour seriose, it's a time domain signal

→ $T_0 = 1$ sec T_0 → time for complete 1 cycle

→ $\omega_0 = \dfrac{2\pi}{T_0} = \dfrac{2\pi}{1} = 2\pi$

→ $a_0 = \dfrac{1}{T_0} \displaystyle\int_{T_0} x(t)\, dt$

find $x(t)$ for this



$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-0}{1-0} = 1$

→ $y = mx + c$

$y = x + c$,

when $x = 0$, $y = 0$ → $c = 0$

$\therefore y = x$

$\therefore \boxed{x(t) = t}$ for $0 < t < 1$

$\therefore a_0 = \dfrac{1}{1} \displaystyle\int_0^1 t \, dt$

$\therefore a_0 = \left[\dfrac{t^2}{2}\right]_0^1 \boxed{= \dfrac{1}{2}}$

→ $a_n = \dfrac{2}{T_0} \displaystyle\int_{T_0} x(t) \cos(n\omega_0 t)\, dt$

$\therefore a_n = \dfrac{2}{1} \displaystyle\int_0^1 t \cos(n \cdot 2\pi \cdot t)\, dt$ $u = t$, $v = \cos 2\pi n t$

$\therefore a_n = 2 \left[t \left(\dfrac{\sin(2\pi n t)}{2\pi n} \right) - \displaystyle\int \dfrac{\sin(2\pi n t)}{2\pi n} dt \right]$

$\therefore a_n = 2 \left[\dfrac{t \sin(2\pi n t)}{2\pi n} + \dfrac{\cos(2\pi n t)}{4\pi^2 n^2} \right]_0^1$

$$\therefore a_n = 2\left[0 + \frac{1}{(2\pi n)^2} - \left(0 + \frac{1}{(2\pi n)^2}\right)\right]$$

$$\therefore a_n = 0$$

$$\rightarrow b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin(2\pi n t)\, dt$$

$$\therefore b_n = 2\int_0^1 t\sin(2\pi n t)\, dt$$

$$\therefore b_n = 2\left[t\left(-\frac{\cos(2\pi n t)}{2\pi n}\right) + \int \frac{\cos(2\pi n t)}{2\pi n}\, dt\right]$$

$$\therefore b_n = 2\left[-\frac{t}{2\pi n}\cos(2\pi n t) + \frac{\sin(2\pi n t)}{(2\pi n)^2}\right]_0^1$$

$$\therefore b_n = 2\left[\frac{-1}{2\pi n} + 0 - (0 - 0)\right]$$

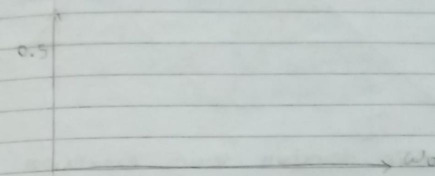
$$\boxed{\therefore b_n = \frac{-1}{n\pi}}$$

$$\rightarrow x(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)$$

$$\therefore x(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{-1}{n\pi}\sin(n\omega_0 t)$$

$$\boxed{\therefore x(t) = \frac{1}{2} - \frac{1}{\pi}\sin(\omega_0 t) - \frac{1}{2\pi}\sin(2\omega_0 t) - \frac{1}{3\pi}\sin(3\omega_0 t) + \dots}$$

- Spectrum



* compact trigonomatric fourier series (M-2)

Q. → square wave example.

$$\rightarrow a\cos x + b\sin x = c\cos(x + \theta)$$

$$c = \sqrt{a_n^2 + b_n^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\therefore x(t) = C_0 + \sum_{n=1}^{\infty} C_n\cos(n\omega_0 t + \theta)$$

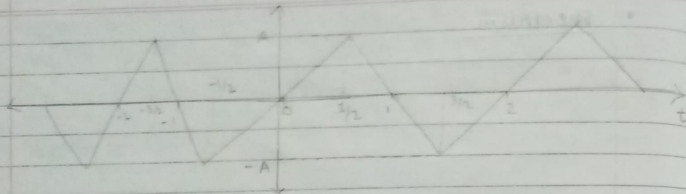
$$C_n = \sqrt{a_n^2 + b_n^2}, \quad \theta = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$$

$$C_n = \sqrt{\left(\frac{2\sin(n\pi/2)}{n\pi}\right)^2 + 0^2} = \frac{2\sin(n\pi/2)}{n\pi}$$

$$\theta = \tan^{-1}\left(\frac{0}{a_n}\right) = 0 \qquad C_0 = a_0 = 1/2$$

$$\therefore x(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2\sin(n\pi/2)}{n\pi}\cos(n\omega_0 t)$$

Q.



For 6 Marks

find trignomatric fourier series expansion for given function.

$\rightarrow T_0 = 2$

$\rightarrow \omega_0 = \frac{2\pi}{2} = \pi$

* $x(t)$ for $-1/2 < x < 1/2$

$\rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{A + A}{1/2 + 1/2} = 2A$

$\rightarrow y = mx + c$

$\therefore y = (2A)x + c$

at $x = 0$, $y = 0$

$\therefore c = 0$

$y = mx \longrightarrow y = x(t)$, $x = t$

$\boxed{\therefore x(t) = 2At}$

* $x(t)$ for $1/2 < x < 3/2$

$\rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-A - A}{3/2 - 1/2} = -2A$

$\rightarrow y = mx + c$

$\therefore y = (-2A)x + c$

at $x = 1$, $y = 0$

$\therefore c = 2A$

$\rightarrow y = (-2A)x + 2A$

$\boxed{\therefore x(t) = 2A - 2At}$

$\rightarrow a_0 = \frac{1}{T_0}\int_{-1/2}^{3/2} x(t)\,dt$

$\therefore a_0 = \frac{1}{T_0}\int_{-1/2}^{1/2} x(t)\,dt + \frac{1}{T_0}\int_{1/2}^{3/2} x(t)\,dt$

$\therefore a_0 = \frac{1}{2}\int_{-1/2}^{1/2} 2At\,dt + \frac{1}{2}\int_{1/2}^{3/2} 2A - 2At\,dt$

$\therefore a_0 = \frac{A}{2}[t^2]_{-1/2}^{1/2} + A[t]_{1/2}^{3/2} - \frac{A}{2}[t^2]_{1/2}^{3/2}$

$\therefore a_0 = \frac{A}{2}\left[\frac{1}{4} - \frac{1}{4}\right] + A\left[\frac{3}{2} - \frac{1}{2}\right] - \frac{A}{2}\left[\frac{9}{4} - \frac{1}{4}\right]$

$\therefore a_0 = 0 + A - A \boxed{= 0}$

$\rightarrow a_n = \frac{2}{T_0}\int_{-1/2}^{3/2} x(t)\cos(n\pi t)\,dt$

$\therefore a_n = \frac{2}{2}\int_{-1/2}^{1/2} 2At\cos(n\pi t)\,dt + \frac{2}{2}\int_{1/2}^{3/2} (2A - 2At)\cos(n\pi t)\,dt$

$\therefore a_n = 2A\left[t\sin\left(\frac{n\pi t}{n\pi}\right) - \int (1)\sin\left(\frac{n\pi t}{n\pi}\right)\right]_{-1/2}^{1/2}$

$\therefore a_n = 2A\left[\frac{t}{n\pi}\sin(n\pi t) + \frac{\cos(n\pi t)}{(n\pi)^2}\right]_{-1/2}^{1/2}$

~~$\therefore a_n = 2A\left[0 + A\frac{\cos(n\pi/2)}{(n\pi)^2} + \right.$~~

$$\therefore\ a_n = 2A\left[\frac{1/2}{n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{\cos\left(\frac{n\pi}{2}\right)}{(n\pi)^2} - \left(\frac{-1/2}{n\pi}\sin\left(\frac{-n\pi}{2}\right) + \frac{\cos(-n\pi/2)}{(n\pi)^2}\right)\right]$$

$\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$

$$\therefore\ a_n = 2A[0] = 0 \quad \text{—— (i)}$$

$$\rightarrow\ a_n = \int_{1/2}^{3/2} (2A - 2At)\cos(n\pi t)\,dt$$

$$\therefore\ a_n = 2A\left[\frac{\sin(n\pi t)}{(n\pi)}\right]_{1/2}^{3/2} - 2A\left[\frac{t}{n\pi}\sin(n\pi t) + \frac{\cos(n\pi t)}{(n\pi)^2}\right]_{1/2}^{3/2}$$

$$\therefore\ a_n = \frac{2A}{n\pi}\left[\sin\left(\frac{3n\pi}{2}\right) - \sin\left(\frac{n\pi}{2}\right)\right] - 2A\left[\frac{3/2}{n\pi}\sin\left(\frac{3n\pi}{2}\right) + \frac{\cos(3n\pi/2)}{(n\pi)^2} - \left(\frac{1/2}{n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{\cos(n\pi/2)}{(n\pi)^2}\right)\right]$$

$$\therefore\ a_n = \frac{2A}{n\pi}\sin\left(\frac{3n\pi}{2}\right) - \frac{2A}{n\pi}\sin\left(\frac{n\pi}{2}\right) - \frac{3A}{n\pi}\sin\left(\frac{3n\pi}{2}\right) - \frac{2A}{(n\pi)^2}\cos\left(\frac{3n\pi}{2}\right) + \frac{A}{n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{2A}{(n\pi)^2}\cos\left(\frac{n\pi}{2}\right)$$

$$\therefore\ a_n = \frac{-A}{n\pi}\sin\left(\frac{3n\pi}{2}\right) - \frac{A}{n\pi}\sin\left(\frac{n\pi}{2}\right) + \frac{2A}{(n\pi)^2}\cos\left(\frac{n\pi}{2}\right) - \frac{2A}{(n\pi)^2}\cos\left(\frac{3n\pi}{2}\right)$$

$\sin\left(\frac{3n\pi}{2}\right) = \sin\left(n\pi + \frac{n\pi}{2}\right) = -\sin\left(\frac{n\pi}{2}\right)$

$\cos\left(\frac{3n\pi}{2}\right) = \cos\left(n\pi + \frac{n\pi}{2}\right) = -\cos\left(\frac{n\pi}{2}\right)$

$$\boxed{\therefore\ a_n = 0}$$

$$\rightarrow\ b_n = \underbrace{\frac{2}{2}\int_{-1/2}^{1/2} 2At\sin(n\pi t)\,dt}_{I_1} + \underbrace{\frac{2}{2}\int_{1/2}^{3/2} (2A - 2At)\sin(n\pi t)\,dt}_{I_2}$$

$$\therefore\ I_1 = 2A\left[t\left(-\frac{\cos(n\pi t)}{n\pi}\right) + \int (1)\frac{\cos(n\pi t)}{n\pi}\right]_{-1/2}^{1/2}$$

$$\therefore\ I_1 = 2A\left[\frac{-t}{n\pi}\cos(n\pi t) + \frac{\sin(n\pi t)}{(n\pi)^2}\right]_{-1/2}^{1/2}$$

$$\therefore\ I_1 = 2A\left[\frac{-1/2}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{\sin\left(\frac{n\pi}{2}\right)}{(n\pi)^2} - \left(\frac{1/2}{n\pi}\cos\left(\frac{-n\pi}{2}\right) + \frac{\sin\left(\frac{-n\pi}{2}\right)}{(n\pi)^2}\right)\right]$$

$$\therefore\ I_1 = \frac{-A}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{2A}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right) - \frac{A}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{2A}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right).$$

$$\rightarrow\ I_2 = -2A\left[\frac{\cos(n\pi t)}{n\pi}\right]_{1/2}^{3/2} - 2A\left[\frac{-t}{n\pi}\cos(n\pi t) + \frac{\sin(n\pi t)}{(n\pi)^2}\right]_{1/2}^{3/2}$$

$$\therefore I_2 = -2A\left[\frac{1}{n\pi}\cos\left(\frac{3n\pi}{2}\right) - \frac{1}{n\pi}\cos\left(\frac{n\pi}{2}\right)\right]$$

$$-2A\left[\frac{-3/2}{n\pi}\cos\left(\frac{3n\pi}{2}\right) + \frac{\sin}{(n\pi)^2}\left(\frac{3\pi n}{2}\right) - \left(\frac{-1/2}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{\sin(n\pi/2)}{(n\pi)^2}\right)\right]$$

$$\therefore I_2 = \frac{2A}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{2A}{n\pi}\cos\left(\frac{n\pi}{2}\right) - \frac{A}{n\pi}\cos\left(\frac{n\pi}{2}\right)$$

$$+ \cancel{\sin}\ \frac{2A}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right) - \frac{A}{n\pi}\cos\left(\frac{n\pi}{2}\right)$$

$$+ \frac{2A}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)$$

$$\therefore I_2 = \frac{2A}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{4A}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)$$

$$\rightarrow b_n = I_1 + I_2$$

$$\therefore b_n = \frac{-2A}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{4A}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right) + \frac{2A}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{4A}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)$$

$$\therefore b_n = \frac{8A}{(n\pi)^2}\sin\left(\frac{n\pi}{2}\right)$$

$$\rightarrow x(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos(n\pi t) + b_n\sin(n\pi t)$$

$$a_0 = a_n = 0$$

$$\therefore x(t) = \frac{8A}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\left(\frac{n\pi}{2}\right)\sin(n\pi t)$$

$$\therefore \cancel{x(t) = \frac{8A}{\pi^2}\left[0 + 0 A\right.}$$

$$\therefore x(t) = \frac{8A}{\pi^2}\left[\sin(\pi t) - \frac{1}{9}\sin(3\pi t) + \frac{1}{25}\sin(5\pi t) + \dots\right]$$

* Exponential Fouriour series

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\rightarrow x(t) = C_0 + \sum_{n=1}^{\infty} C_n\cos(n\omega_0 t + \theta_n)$$

$$\therefore x(t) = C_0 + C_n\sum_{n=1}^{\infty}\frac{e^{j(n\omega_0 t + \theta_n)} + e^{-j(n\omega_0 t + \theta_n)}}{2}$$

$$\therefore x(t) = C_0 + \frac{C_n}{2}\sum_{n=1}^{\infty} e^{jn\omega_0 t}\cdot e^{j\theta_n} + e^{-jn\omega_0 t}\cdot e^{-j\theta_n}$$

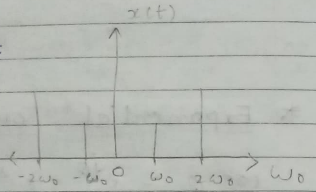
$$\therefore x(t) = C_0 + \sum_{n=1}^{\infty}\frac{C_n e^{j\theta_n}}{2}\cdot e^{jn\omega_0 t} + \sum_{n=1}^{\infty}\frac{C_{-n} e^{-j\theta_n}}{2}e^{-jn\omega_0 t}$$

$$\therefore x(t) = C_0 + \sum_{n=1}^{\infty} C_n e^{jn\omega_0 t} + \sum_{n=1}^{\infty} C_{-n} e^{-jn\omega_0 t}$$

$$\therefore x(t) = D_0 + \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$\rightarrow D_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$$

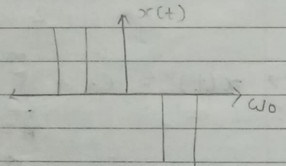
$$\rightarrow D_0 = \frac{1}{T_0}\int_{T_0} x(t)\, dt$$



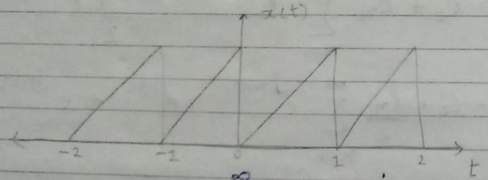
$$\rightarrow D_n = a + jb$$

$$\therefore |D_n| = \sqrt{a^2+b^2}$$

$$\rightarrow \theta_n = -\tan^{-1}\left(\frac{b}{a}\right)$$



Q.



$$\rightarrow x(t) = D_0 + \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$\rightarrow T_0 = 1 \text{ sec}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{1} \boxed{= 2\pi}$$

$$\rightarrow x(t) = t$$

$$D_n = \frac{-j}{2\pi n}$$

$$\rightarrow (-j2\pi n)^2 = -(j^2)(4)\pi^2 n^2 = 4\pi^2 n^2$$

$$\rightarrow D_0 = \frac{1}{T_0}\int_{T_0} x(t)\, dt$$

$$\therefore D_0 = \frac{1}{1}\int_0^1 t\, dt$$

$$\therefore D_0 = \left[\frac{t^2}{2}\right]_0^1 = \frac{1}{2}$$

$$\rightarrow D_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$\therefore D_n = \frac{1}{1}\int_0^1 t\, e^{-jn(2\pi)t}$$

$$\therefore D_n = \left[t\left(\frac{e^{-jn(2\pi)t}}{-jn2\pi}\right) - \int (1)\frac{e^{-jn(2\pi)t}}{-jn2\pi}\right]_0^1$$

$$\therefore D_n = \left[\frac{-t\,e^{-jn(2\pi)t}}{j2n\pi} - \frac{e^{-jn(2\pi)t}}{(jn2\pi)^2}\right]_0^1$$

$$\therefore D_n = \left[\left(\frac{-e^{-jn2\pi}}{j2n\pi} - \frac{e^{-jn(2\pi)}}{(jn2\pi)^2}\right) - \left(0 - \frac{1}{(jn2\pi)^2}\right)\right]$$

$$\therefore D_n = \left[\frac{1}{(jn2\pi)^2} - \frac{e^{-jn(2\pi)}}{(jn2\pi)^2}\right] - \frac{e^{-jn2\pi}}{jn2\pi}$$

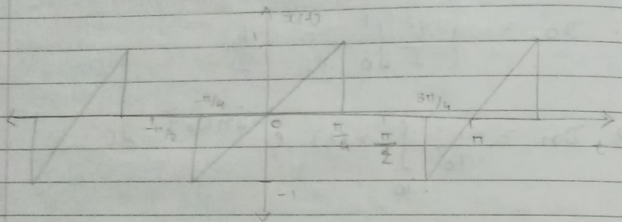
$$\left\{ \begin{array}{l} e^{-j2\pi n} = \cos(2\pi n) - j\sin(2\pi n) \\ \text{For each } n,\ e^{-j2\pi n} = 1 \end{array} \right\} \star$$

$$\therefore D_n = \frac{-1}{jn2\pi} = \frac{-j}{(j)^2(2n\pi)} \boxed{= \frac{j}{2\pi n}}$$

$\rightarrow \quad |D_n| = \sqrt{0 + \left(\frac{j}{2\pi n}\right)^2} = \frac{j}{2\pi n}$

$\rightarrow \quad \theta = -\tan^{-1}\left(\frac{j/2\pi n}{0}\right) = \infty$

Q.



$\rightarrow \quad T_0 = \frac{3\pi}{4} - \left(\frac{-\pi}{4}\right) = \frac{4\pi}{4} = \pi$

$\rightarrow \quad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{\pi} = 2$

$\rightarrow \quad \underline{-\pi/4 < x(t) < \pi/4}$ $\qquad m = \frac{1}{\pi/4} = \frac{4}{\pi}$

$x(t) = \frac{4}{\pi} t$ $\qquad \rightarrow y = \left(\frac{4}{\pi}\right) x + c$

$\qquad x(t) = \frac{4}{\pi} t$

$\rightarrow \quad \underline{\pi/4 < t < 3\pi/4}$

$x(t) = 0$

$\rightarrow \quad D_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt$

$\therefore \quad D_0 = \frac{1}{\pi} \int_{-\pi/4}^{\pi/4} \frac{4}{\pi} t \, dt$

$D_n = \frac{-j}{n\pi}\left[\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) - \cos\left(\frac{n\pi}{2}\right)\right]_{n \neq 0}$

$\therefore \quad D_0 = \frac{4}{\pi^2}\left[\frac{t^2}{2}\right]_{-\pi/4}^{\pi/4}$

$\therefore \quad D_0 = \frac{2}{\pi^2}\left[\frac{\pi^2}{16} - \frac{\pi^2}{16}\right] \boxed{= 0}$

$\rightarrow \quad D_n = \frac{1}{\pi} \int_{-\pi/4}^{\pi/4} \frac{4}{\pi} t \, e^{-jn2t} \, dt$

$\therefore \quad D_n = \frac{4}{\pi^2}\left[t\left(\frac{e^{-j2nt}}{-j2n}\right) + \int (1) \frac{e^{-j2nt}}{j2n} dt\right]$

$\therefore \quad D_n = \frac{4}{\pi^2}\left[\frac{-t}{j2n} e^{-j2nt} - \frac{e^{-j2nt}}{(j2n)^2}\right]_{-\pi/4}^{\pi/4}$

$\therefore \quad D_n = \frac{4}{\pi^2}\left[\frac{-\pi/4}{2nj} e^{-j2n(\pi/4)} - \frac{e^{-j2n(\pi/4)}}{(j2n)^2} - \left(\frac{\pi/4}{2nj} e^{j2n(\pi/4)} - \frac{e^{j2n(\pi/4)}}{(j2n)^2}\right)\right]$

$\therefore \quad D_n = \frac{4}{\pi^2}\left[\frac{-\pi}{8nj} e^{-\frac{jn\pi}{2}} - \frac{e^{-\frac{jn\pi}{2}}}{(j2n)^2} - \frac{\pi}{8nj} e^{\frac{jn\pi}{2}} + \frac{e^{\frac{jn\pi}{2}}}{(j2n)^2}\right]$

$\therefore \quad D_n = \frac{4}{\pi^2}\left[\frac{e^{\frac{jn\pi}{2}} - e^{\frac{-jn\pi}{2}}}{2(jn)^2\, 2}\right] - \frac{4}{\pi^2}\left[\frac{\pi}{8nj}\left(e^{\frac{jn\pi}{2}} + e^{\frac{-jn\pi}{2}}\right)\right]$

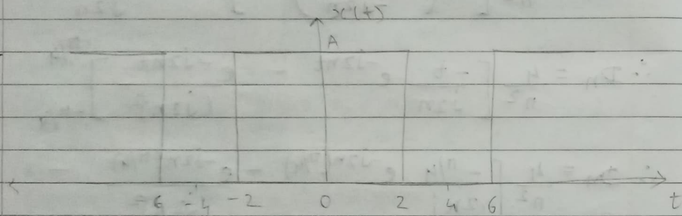
$\therefore \quad D_n = \frac{2}{\pi^2 n^2 j}\left[\frac{e^{\frac{jn\pi}{2}} - e^{\frac{-jn\pi}{2}}}{2j}\right] - \frac{1}{\pi n j}\left[\frac{e^{\frac{jn\pi}{2}} + e^{\frac{-jn\pi}{2}}}{2}\right]$

$$\rightarrow D_n = \frac{2}{\pi^2 n^2 j}\sin\left(\frac{n\pi}{2}\right) - \frac{1}{\pi n j}\cos\left(\frac{n\pi}{2}\right)$$

$$\therefore D_n = \frac{1(j)}{\pi n j(j)}\left[\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) - \cos\left(\frac{n\pi}{2}\right)\right]$$

$$\boxed{\therefore D_n = \frac{-j}{n\pi}\left[\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) - \cos\left(\frac{n\pi}{2}\right)\right]}$$

Q.



$$\rightarrow T_0 = 6 - (-2) = 8$$

$$\rightarrow \omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$\rightarrow D_0 = \frac{1}{T_0}\int_{T_0} x(t)\, dt$$

$$\therefore D_0 = \frac{1}{8}\int_{-2}^{2} A\, dt$$

$$\therefore D_0 = \frac{A}{8}\left[t\right]_{-2}^{2} = \frac{A}{8}\left[2 - (-2)\right] = \frac{A}{8}(4)$$

$$\boxed{= \frac{A}{2}}$$

$$\rightarrow D_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$\therefore D_n = \frac{1}{8}\int_{-2}^{2} A\, e^{-jn\omega_0 t}\, dt$$

$$\therefore D_n = \frac{A}{8}\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{-2}^{2}$$

$$\therefore D_n = \frac{A\times 4}{8}\left[\frac{e^{-jn(\frac{\pi}{4})t}}{-jn\pi}\right]_{-2}^{2}$$

$$\therefore D_n = \frac{A}{2}\left[\frac{e^{-jn(\pi/2)}}{-jn\pi} - \left(\frac{e^{jn(\pi/2)}}{-jn\pi}\right)\right]$$

$$\therefore D_n = \frac{A}{-jn\pi}\left[\frac{e^{-jn(\frac{\pi}{2})} - e^{jn(\frac{\pi}{2})}}{2}\right]$$

$$\therefore D_n = \frac{A}{n\pi}\left[\frac{e^{jn(\frac{\pi}{2})} - e^{-jn(\frac{\pi}{2})}}{2j}\right]$$

$$\boxed{\therefore D_n = \frac{A}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right)\right]}$$

Q A continuous time $x(t)$ has fundamental time period 8 sec. The non zero Fourier coefficients for $x(t)$ are:

$a_1 = a_{-1} = 2$

$a_3 = a_{-3}^* = j4$

Express the $x(t) = \sum_{n=0}^{\infty} C_n \cos(n\omega_0 t + \theta)$

Ans.

→ $T_0 = 8$ sec

$\omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t}$$

$$= a_{-3} e^{-j3\omega_0 t} + a_1 e^{-j\omega_0 t} + a_1 e^{j\omega_0 t} + a_3 e^{j3\omega_0 t}$$

$$= -j4 e^{-j3\omega_0 t} + 2 e^{-j\omega_0 t} + 2 e^{j\omega_0 t} + j4 e^{j3\omega_0 t}$$

$$= 4 e^{-j(3\omega_0 t + \frac{\pi}{2})} + 2(e^{j\omega_0 t} + e^{-j\omega_0 t}) + 4 e^{j(3\omega_0 t + \pi/2)}$$

$$= 4 \cos(\omega_0 t) + 8 \cos\left(3\omega_0 t + \frac{\pi}{2}\right)$$

→ $x(t) = 4 \cos\left(\frac{\pi t}{4}\right) + 8 \cos\left(\frac{3\pi t}{4} + \frac{\pi}{2}\right)$

* Power signal

$0 < P_g < \infty$

$E_g \rightarrow 0$

* Energy signal

→ $0 < E_g < \infty$ ← Finite Energy

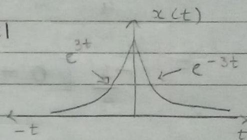
$P_g = 0$ ← Power is zero

→ magnitude → 0 when $t \to \infty$ (Energy → 0)

time bounded energy signal called non periodic signal.

$$\boxed{E_x = \int_{-\infty}^{\infty} x^2(t)\, dt}$$ (OR) $$\boxed{E_x = \int_{-\infty}^{\infty} |x^2(t)|\, dt}$$

Q. Get energy of $x(t) = e^{-3|t|}$



→ $E_g = \int_{-\infty}^{\infty} e^{-6|t|}\, dt$

$\therefore E_g = \int_{-\infty}^{0} e^{6t}\, dt + \int_{0}^{\infty} e^{-6t}\, dt$

$\therefore E_g = \left[\frac{e^{6t}}{6}\right]_{-\infty}^{0} + \left[\frac{e^{-6t}}{-6}\right]_{0}^{\infty}$

$\therefore E_g = \frac{1}{6} - 0 - \frac{1}{6}[0 - 1]$

$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \boxed{\frac{1}{3}}$

Q. $x(t) = e^{-4t}\, u(t)\, dt$

u(t) → unit step signal amplitude remain constant

$\rightarrow \int_{-\infty}^{\infty} e^{-4t}\, dt$

$\therefore \int_{-\infty}^{0} 0\, dt + \int_{0}^{\infty} e^{-4t}\, dt$

$= 0 + \left[\frac{e^{-4t}}{-4}\right]_0^{\infty}$

$u(t) = 1 \quad t \geq 0$

$u(t) = 0 \quad t < 0$

for $-\infty$ to 0

$= \frac{-1}{4}\left[e^{-4t}\right]_0^{\infty}$

$= \frac{-1}{4}[0 - 1] \quad \boxed{= \frac{1}{4}}$

→ If we want to frequency domain of ~~power~~ Energy signal by fourier transform.

* Fourier Transform

$\rightarrow X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

* Inverse Fourier Transform

→ Frequency domain to time domain.

$\rightarrow x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$

* convolution

$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau)\, d\tau$

→ Instead of convolution theorm

$\rightarrow Y(\omega) = X_1(\omega) \times X_2(\omega)$

than use inverse fouriour transform to get $y(t)$.

Q. Find the fouriour transform of signal $x(t) = e^{-at} \cdot u(t)$

Ans.

$\rightarrow X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt$

$\therefore X(\omega) = \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, (1)\, dt$

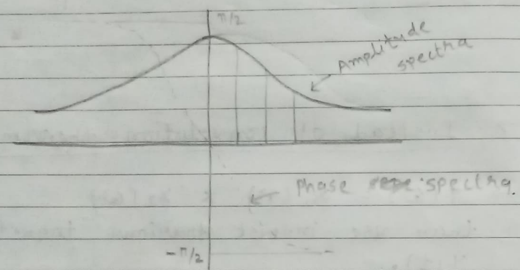
$\therefore X(\omega) = \int_{0}^{\infty} e^{-(a+j\omega)t} \cdot dt$

$\therefore X(\omega) = \frac{-1}{a+j\omega}\left[e^{-(a+j\omega)t}\right]_0^{\infty} \quad = \frac{1}{a+j\omega}$

- <u>amplitude</u>

$$|X(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{\omega}{a}\right)$$



Q. Fouriour transform of : $x(t) = e^{-a|t|}$

Ans. :- $X(\omega) = \dfrac{2a}{\omega^2+a^2}$

* <u>Gate Function</u> (Rectangle f^n / rect f^n)

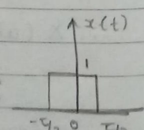
→ If we have signal : $x(t) = rect\left(\frac{t}{\tau}\right)$

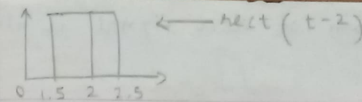
→ by this we get rectangle pulses.

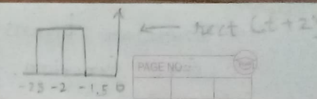
→ τ is width

→ $x(t) = 2\, rect\left(\frac{t}{\tau}\right)$ — only t હોય તો

Amplitude





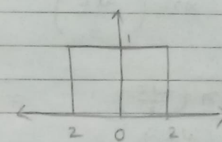


→ $rect\left(\frac{t}{\tau}\right) = 1 \rightarrow -\tau/2 < t < \tau/2$

$= 0 \rightarrow t \geq |\tau/2|$

* <u>Example</u>

$rect\left(\frac{t}{4}\right)$



Q. Fouriour transform of signal $rect(t/\tau)$

Ans.

→ $x(t) = 1$

$$\rightarrow X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt$$

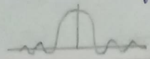
$$= \int_{-\tau/2}^{\tau/2} 1 \cdot e^{-j\omega t}\, dt$$

$$= \frac{-1}{j\omega}\left[e^{-j\omega t}\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{1}{j\omega}\left[e^{-j\omega t}\right]_{\tau/2}^{-\tau/2}$$

$$= \frac{1}{j\omega}\left[e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right]$$

→ Fouriour transform of rect signal is always sinc function.

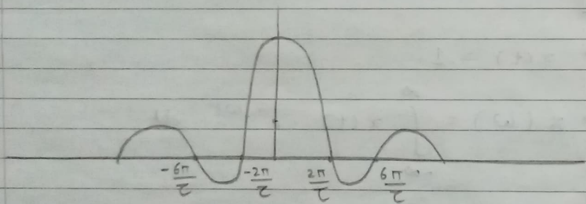


$$= \frac{2}{\omega}\left[\frac{e^{\frac{j\omega\tau}{2}} - e^{-\frac{j\omega\tau}{2}}}{2j}\right]$$

$$= \frac{2}{\omega} \sin\left(\frac{\omega\tau}{2}\right)$$

$$= \tau \cdot \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)} \quad \boxed{= \tau \cdot \text{sinc}\left(\frac{\omega\tau}{2}\right)}$$

• Spectrum representation of rectangular Pulse

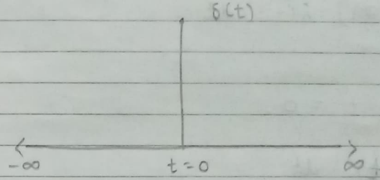


$$\frac{\omega\tau}{2} = \pm\pi$$

$$\boxed{\therefore \quad \omega = \frac{\pm 2\pi}{\tau}}$$

* Impulse signal (Direct function)

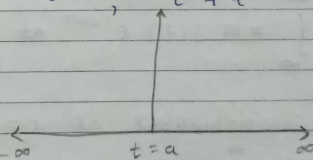
→ $\delta(t) = 1 \Rightarrow t = 0$



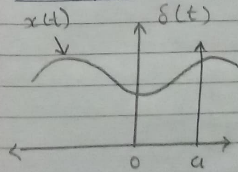
$$\boxed{\int_{-\infty}^{\infty} \delta(t)\, dt = 1} \leftarrow \text{avg is 1.}$$

* Example

→ $\delta(t-a) = 1, \; t = a$
$\qquad\qquad = 0, \; t \neq a$



* Example



→ $x(t) \cdot \delta(t) = x(0) \cdot \delta(t)$

→ $x(t) \cdot \delta(t) = x(a) \cdot \delta(t)$

Q. Find the integral of signal :

$$\int_{-\infty}^{\infty} [t^2+1]\,\delta(t)\,dt \qquad \int_{-\infty}^{\infty} [t^2+2]\,\delta(t)\,dt$$

$\boxed{= 2}$

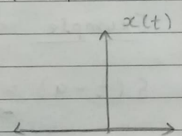
When $t = 0$,

$$= \int_{-\infty}^{\infty} \delta(t)\,dt$$

$\boxed{= 1}$

Q. Find the Fouriour transform of $x(t) = \delta(t)$

Ans. $$\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$



$$\therefore X(\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt$$

impulse is available at $t = 0$.

$$\therefore X(\omega) = e^{0} \int_{-\infty}^{\infty} \delta(t)\, dt \qquad \boxed{= 1}$$

→ Impulse in time change transforme is constant signal

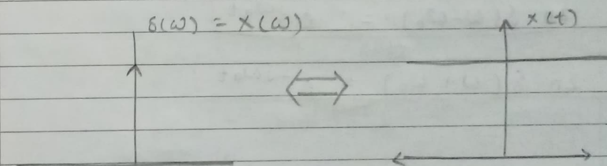
Q. $X(\omega) = \delta(\omega)$ find fouriour transform

$$\rightarrow x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

at $t = 0$

$$\therefore x(t) = \frac{1}{2\pi}\, e^{0} \int_{-\infty}^{\infty} X(\omega)\, d\omega \quad \leftarrow \text{ans} = 1$$

$$\therefore x(t) = \frac{1}{2\pi}$$



Q. Find Fouriour transform of : $X(\omega) = \delta(\omega - \omega_0)$

~~Assume that~~

$\rightarrow \omega - \omega_0 = x$

$\therefore d\omega = dx$

$\therefore \omega = x + \omega_0$

$$\rightarrow x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$\rightarrow$ 1 to 11 , 13 ~~to~~, 16 to 19

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(x)\, e^{j(x+\omega_0)t}\, dx$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(x)\, e^{jxt} \cdot \underbrace{e^{j\omega_0 t}}_{\text{constant}}\, dx$$

$$= \frac{e^{j\omega_0 t}}{2\pi}\int_{-\infty}^{\infty} \delta(x)\, e^{jxt}\, dx$$

$$\therefore\ x(t) = \frac{e^{j\omega_0 t}}{2\pi}$$

$$\boxed{\therefore\ 2\pi\, \delta(\omega-\omega_0) = e^{j\omega_0 t}}$$

(OR)

$$\boxed{\therefore\ 2\pi\, \delta(\omega+\omega_0) = e^{-j\omega_0 t}}$$

Q. $x(t) = \cos\omega_0 t$, Find Fouriour transform

$\rightarrow$ $\cos\omega_0 t = \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

$\therefore \cos\omega_0 t = \pi\,[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$

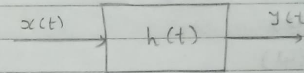
Q. $x(t) = \sin\omega_0 t$, Find Fouriour transform.

$\rightarrow$ $\sin\omega_0 t = \dfrac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$

$$= \frac{2\pi}{2j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$

$$= \frac{\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$

★ **Filtering of signal**



$h(t)$ = Impulse responance of system
$x(t)$ = Input
$y(t)$ = Output

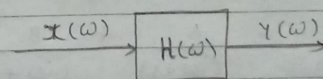
$\rightarrow$ $y(t) = h(t) * x(t)$

$$= \int_{-\infty}^{\infty} h(\tau)\cdot x(t-\tau)\, dt$$

(OR)

$$= \int_{-\infty}^{\infty} x(\tau)\cdot h(t-\tau)\, dt$$

$\rightarrow$ It is very complicated , so do by fourier transform.



$Y(\omega) = H(\omega) * X(\omega)$

↳ than apply Inverse Fourier Transform and get $y(t)$ which equals to $x(t) * h(t)$.

Q. Determine the input signal $x(t)$ that can produce an output of $y(t) = e^{-3t}u(t) - e^{-4t}u(t)$ with the frequency responce system is
$$H(\omega) = \frac{1}{(j\omega+2)}$$

Ans.

$\rightarrow$ $Y(\omega) = X(\omega) * H(\omega)$

$\therefore$ $X(\omega) = \dfrac{Y(\omega)}{H(\omega)}$

$\rightarrow$ $Y(\omega) = \int_{-\infty}^{\infty} e^{-3t}u(t) - e^{-4t}u(t)\, dt$

$\therefore$ $Y(\omega) = \dfrac{1}{j\omega+3} - \dfrac{1}{j\omega+4}$

$$\boxed{e^{-at}u(t) = \frac{1}{a+j\omega}}$$

$\therefore$ $Y(\omega) = \dfrac{1}{(j\omega+3)(j\omega+4)}$

$\rightarrow$ $X(\omega) = \dfrac{Y(\omega)}{H(\omega)} = \dfrac{(j\omega+2)}{(j\omega+3)(j\omega+4)}$

$\rightarrow$ $\dfrac{(j\omega+2)}{(j\omega+3)(j\omega+4)} = \dfrac{A}{(j\omega+3)} + \dfrac{B}{(j\omega+4)}$

Let $j\omega = t$

$*$ $= 1-13$

$\rightarrow$ $t+2 = A(t+4) + B(t+3)$

$*$ $t = -3$

$\therefore$ $-1 = A$

$*$ $t = -4$

$\therefore$ $-2 = -B$

$\therefore$ $B = 2$

$\rightarrow$ $X(\omega) = \dfrac{-1}{j\omega+3} + \dfrac{2}{j\omega+4}$

$$\boxed{\rightarrow x(t) = 2e^{-4t}u(t) - e^{-3t}u(t)}$$

Q. The impulse responce of the system is given by $h(t) = \dfrac{1}{RC} e^{-t/RC} \cdot u(t)$.

Find the frequency responce and ~~Amplitude~~ megnitude spectrum.

Q. For A system has impulse responce $h(t) = e^{-t}u(t) + e^{2t}u(t)$ & $x(t) = e^{-2t}u(t)$ Find $y(t)$.

Ans.

$\rightarrow$ $H(\omega) = \dfrac{1}{j\omega+1} + \dfrac{1}{2-j\omega}$

$$y(t) = e^{-t}u(t) - \frac{3}{4}e^{-2t}u(t) - \frac{1}{4}e^{2t}u(-t)$$

$\rightarrow X(\omega) = \dfrac{1}{j\omega + 2}$

$\rightarrow Y(\omega) = H(\omega) * X(\omega)$

$$= \left(\frac{1}{j\omega+1} + \frac{1}{2-j\omega}\right)\cdot\left(\frac{1}{j\omega+2}\right)$$

$$= \frac{1}{(j\omega+1)(j\omega+2)} + \frac{1}{(2-j\omega)(j\omega+2)}$$

$\rightarrow \dfrac{1}{(t+1)(t+2)} = \dfrac{A}{t+1} + \dfrac{B}{t+2}$

$\therefore$ $t = -1$

$\rightarrow A = 1$

$\therefore$ $t = -2$

$\rightarrow B = -1$

* $\dfrac{1}{(2-t)(t+2)} = \dfrac{A}{2-t} + \dfrac{B}{t+2}$

$\therefore$ $t = -2$

$\rightarrow B = \dfrac{1}{4}$

$\rightarrow A = \dfrac{1}{4}$

Justify Power signal has $E_x = \infty$
Energy " " $P_x = 0$

$$\therefore Y(\omega) = \frac{1}{(j\omega+1)} - \frac{1}{(j\omega+2)} + \frac{1/4}{2-j\omega} + \frac{1/4}{j\omega+2}$$

$\rightarrow y(t)$

* Power signal

$\rightarrow$ If signal is periodic signal strength measures in Power.

$\rightarrow$ $0 < P_x < \infty$

$E_x = \infty$

$$\boxed{\rightarrow P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt}$$

Q. Get power of $x(t) = K\cos\omega_0 t$

Ans.

$$\rightarrow P_x = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} K^2\cos^2\omega_0 t\,dt$$

$$= \lim_{T\to\infty}\frac{K^2}{2T}\int_{-T/2}^{T/2} 1 + \cos 2\omega_0 t\,dt$$

avg of cos = 0

$$= \lim_{T\to\infty}\frac{K^2}{2T}\left[t\right]_{-T/2}^{T/2} + \lim_{T\to\infty}\frac{K^2}{2T}\left[\frac{\sin 2\omega_0 t}{2\omega_0}\right]_{-T/2}^{T/2}$$

$$P_x = \frac{K^2}{2}$$

$$\therefore \quad P_x(rms) = \sqrt{\frac{K^2}{2}} \quad \boxed{= \frac{K}{\sqrt{2}}}$$