N-Body Results from a Barnes-Hut Implementation

Darian Lagman 7917268

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Introduction

Simulating the dynamics of gravitational systems is a central problem in astrophysics. Accurate simulations of systems with many interacting bodies such as star clusters or galaxies can provide insights into their long-term evolution. However, the computational complexity of directly calculating gravitational interactions for every pair of bodies grows exponentially with the number of bodies, making such simulations unfeasible for large systems. To address this, we employ the Barnes-Hut algorithm, a method that significantly reduces computational costs while maintaining reasonable accuracy. This report outlines the results of this implementation on a n = 1000-star cluster including the energy profiles of the cluster and its density profile.

Approach

The most computationally taxing part of the algorithm is computing the forces that each body applies to each other. In the naïve approach we would simply brute force the calculation for each body; this would result in $\frac{n(n-1)}{2}$ pairs and $O(n^2)$ runtime. However, this simulation will utilize a modified Barnes-Hut algorithm to reduce this to $O(nlog(n))$. This is crucial for analysing results from a cluster of stars as it allows for a large number of bodies to be simulated in a reasonable amount of time (Ruhr-University Bochum, n.d.).

Barnes-Hut Algorithm

The Barnes-Hut algorithm is a hierarchical method for approximating gravitational forces in an N-body simulation. Instead of calculating the pairwise force between every body, it approximates the force of distant bodies by grouping them into clusters and treating each cluster as a single point mass. This approximation relies on the concept of spatial decomposition and is implemented using a tree structure (Arbor.js Team, n.d.). The description for the Barnes-Hut algorithm is originally for a system in 2D space which uses a quadtree in its construction. With some modifications to the data structure – using an octree instead of a quadtree – we are able to model the movements of the bodies in a 3D space. I will explain the three main steps of the algorithm, however, I will use supporting pictures from the creation of a 2D implementation as it is simpler to follow.

1. Constructing the octree:

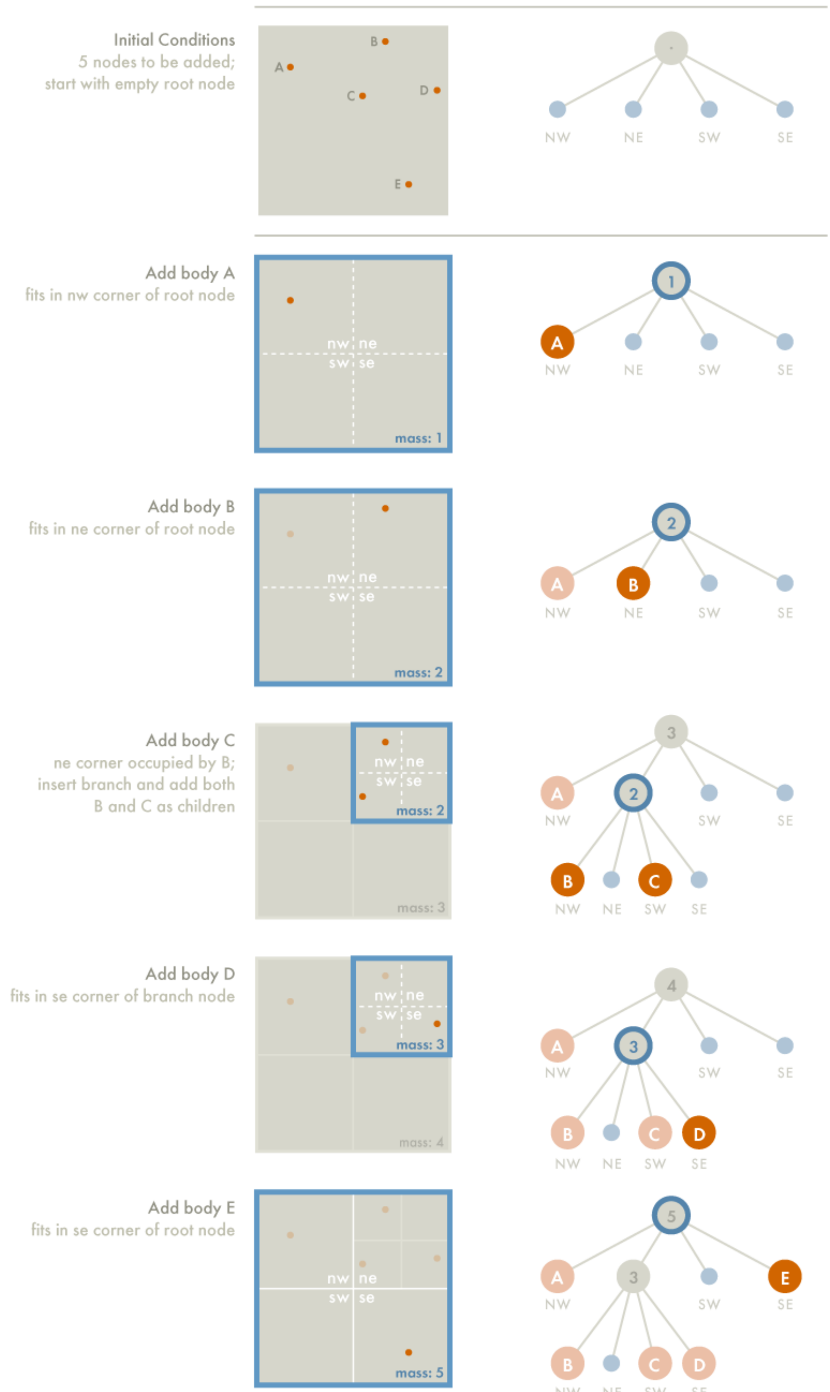


Figure 1. Creation of Quadtree for Barnes-Hut Algorithm

- The simulation space is initially divided into eight smaller regions using an octree. Each node in the octree represents a region of space and contains information about the bodies within it. Bodies will then begin to be placed within the space (Ruhr-University Bochum, n.d.).

- If a region contains more than one body, it is subdivided into eight smaller regions (octants). This process continues, adding one body at a time and subdividing if a sector contains more than one body.

2. Calculating centre of mass:

Figure 2 Visualization of Barnes-Hut algorithm for 8 bodies in 2D

- For each node in the octree, the algorithm calculates the total mass and center of mass of all bodies within the region it represents. This information is stored at the node and is used to approximate the gravitational influence of distant regions.

3. Estimating forces:

- To compute the gravitational force on a given body, the algorithm traverses the octree. For each node, it checks whether the node is sufficiently far away from the body. This is done by computing $\frac{s}{d}$ where s is the width of the region of the internal node. With d being the physical distance between the body being computed and the centre of mass for the node (Princeton University, n.d.). If this ratio falls beneath some threshold θ, then the node is treated just as a single point mass and the force is calculated directly (Princeton University, n.d.). The effects of different values for these variables s, d, and θ can be seen as:
    - A smaller s means the region containing the particles is compact. If s is small relative to d, the particles in this region are close together, making it more reasonable to approximate their combined gravitational effect as a single point mass. This leads to fewer calculations, improving the efficiency of the algorithm.
    - A larger d means the particle is far from the region. When d is large relative to s, the distance diminishes the effect of the individual positions of particles.
    - A smaller θ value means stricter criteria for approximation. The algorithm will perform more direct calculations, increasing accuracy but also computational cost.

- A larger θ value allows more approximations, reducing the number of calculations and speeding up the simulation, but at the cost of accuracy.

Results

With the Barnes-Hut algorithm implemented, we can now analyze the results of the simulation. The following sections present the outcomes of simulating a star cluster, focusing on energies and radial density profiles, which provide insights into the dynamical behavior and structure of the system over time. Specifically, we will look at a cluster of n=1000 bodies, with a timespan of 10^6 seconds. Moreover, for the system, we define a radius for our cluster to exist in order to allow stars to “leave” our system. We will assume that once a star passes this radius, it has a sufficiently large velocity to escape the system.

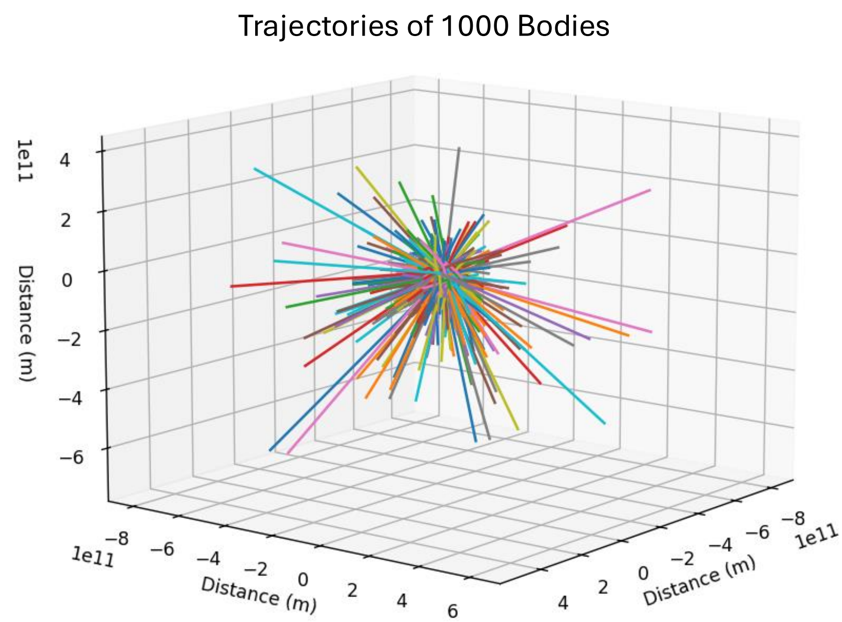
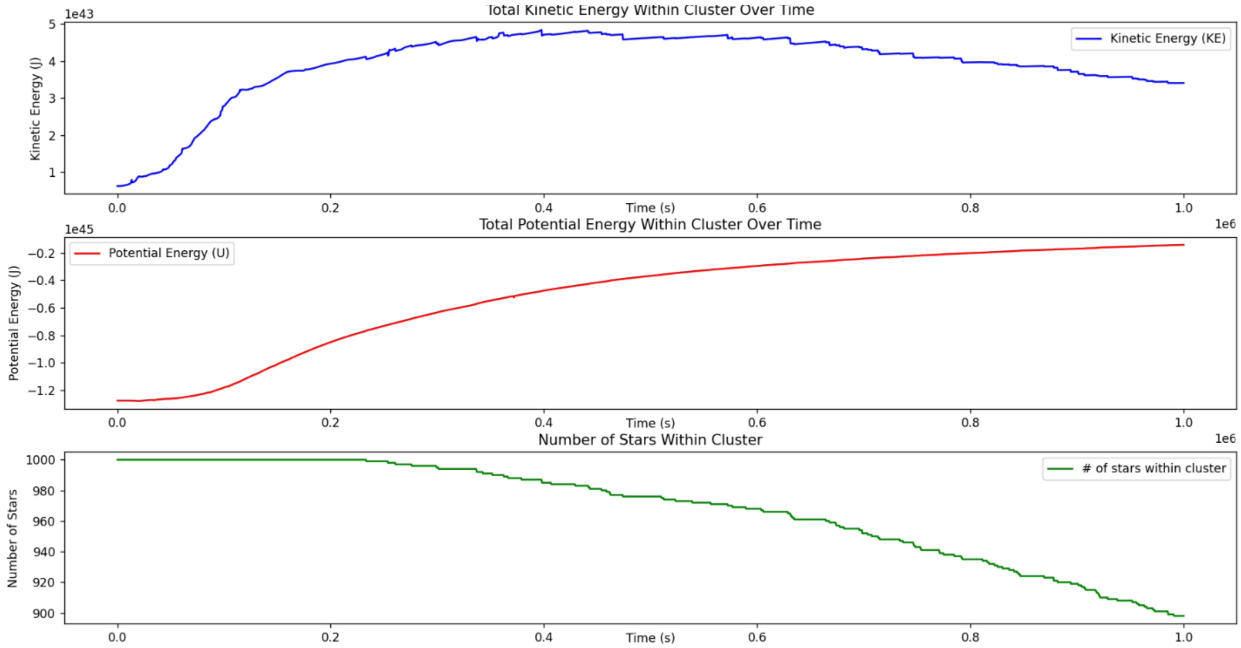


Figure 3 Traced movements of 1000 bodies

This 3D graphic traces the movements of the 1000 stars which have a central point of origin with most of them initially distributed around that center. Stars move outward from the center with some diverging significantly. This is indicating that these stars are being ejected from the cluster due to their high velocities showing a visual representation of these stars “evaporating” from the system. Furthermore, the violent interactions occurring – especially those that happened near the start of the simulation – are also visible with the linear radial trajectories. If left to evolve further, I expect the overall topology of the cluster to become more spherical or cloud-like.

Energy

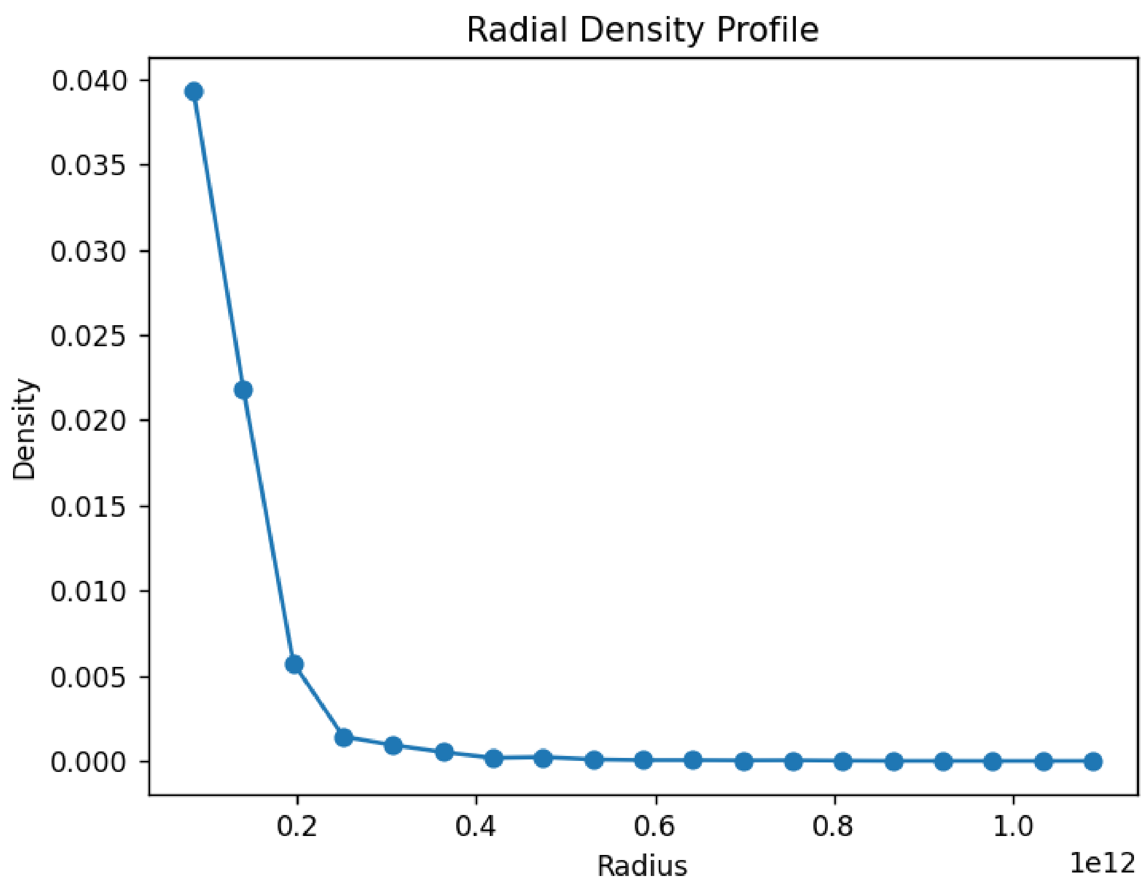


The total kinetic energy within the cluster increases over time then stays about the same. High-energy stars that acquire sufficient velocity to escape the cluster carry away a portion of the system's energy. Due to the negative specific heat capacity inherent in gravitationally bound systems, the removal of these stars leads to a contraction of the remaining cluster, accompanied by an increase in the mean kinetic energy of the bound stars. This behavior aligns with the energy exchange processes observed in globular cluster evolution and supports the theoretical framework of energy equipartition and evaporation dynamics.

The gravitational potential energy of the trend shows a steady increase (becoming less negative) over time. This reflects a diminishing of the cluster's overall gravitational binding. As stars are ejected, the remaining stars are less gravitationally bound to one another. The gravitational potential energy becomes less negative because the cluster's mass is decreasing with the escape of high-energy stars. The total mass within the system diminishes, reducing the overall gravitational attraction between the remaining stars.

The number of stars within the cluster shows a monotonically decreasing trend. As high-energy stars are ejected from the system, the total stellar population of the cluster decreases over time. This process of mass loss due to the ejection of stars directly affects the energy distribution in the system. Fewer stars lead to a reduction in the total potential energy, and a shift in the distribution of kinetic energy, as the mean velocity of the remaining stars increases. The stars that remain are typically those that were initially bound by the cluster's gravity but have had their velocities increased due to the interactions between stars. As the number of stars diminishes, the system's overall dynamical state continues to evolve, with fewer stars available to interact and contribute to the kinetic and potential energy exchanges.

Radial density profile



A radial density profile is constructed of the cluster with 20 shells. Each point represents the star density (stars/m^3). The density is the greatest at small radii, due to the concentration of stars near the cluster core. This is expected due to gravitational interactions causing stars to migrate toward the center. Furthermore, the density decreases sharply with increasing radius. There is a steep drop-off in the stellar population as you move away from the core. Beyond a certain radius, the density approaches a near-zero value. This represents the outskirts of the cluster where stars are sparsely distributed or have escaped the cluster due to evaporation.

References

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