

Hellenic Complex Systems Laboratory

# Intelligent Quality: A Software Tool for Exploring the Application of Convolutional Neural Networks to Quality Control Samples of Very Small Size

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# Intelligent Quality: A Software Tool for Exploring the Application of Convolutional Neural Networks to Quality Control Samples of Very Small Size

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**Search Terms:** QC, convolutional neural networks, probability for rejection, probability for false rejection, QC measurement

## Introduction

Although there is extensive literature on the application of artificial neural networks (NN) in quality control (QC), to monitor the conformity of a process to quality specifications, at least five QC measurements are required, increasing the related cost. To explore the application of NN to samples of QC measurements of very small size, four one-dimensional (1-D) convolutional NN (CNN) were designed, trained, and tested with datasets of  $n$ -tuples of simulated standardized normally distributed QC measurements, for  $1 \leq n \leq 4$ .

The designed CNN were compared to statistical QC functions with equal probabilities for false rejection ( $pfr$ ), applied to samples of the same size. When the  $n$ -tuples included at least two QC measurements distributed as  $\mathcal{N}(\mu, \sigma^2)$ , where  $0.2 < |\mu| \leq 6.0$ , and  $1.0 < \sigma \leq 7.0$ , the designed CNN outperformed the respective statistical QC functions. Therefore, 1-D CNN applied to samples of 2-4 quality control measurements can be used to increase the probability of detection of the nonconformity of a process to the quality specifications, with lower cost.

The program *Intelligent Quality* was developed to explore the application of the trained CNN to QC samples of very small size and compare them to statistical QC functions.

## The program

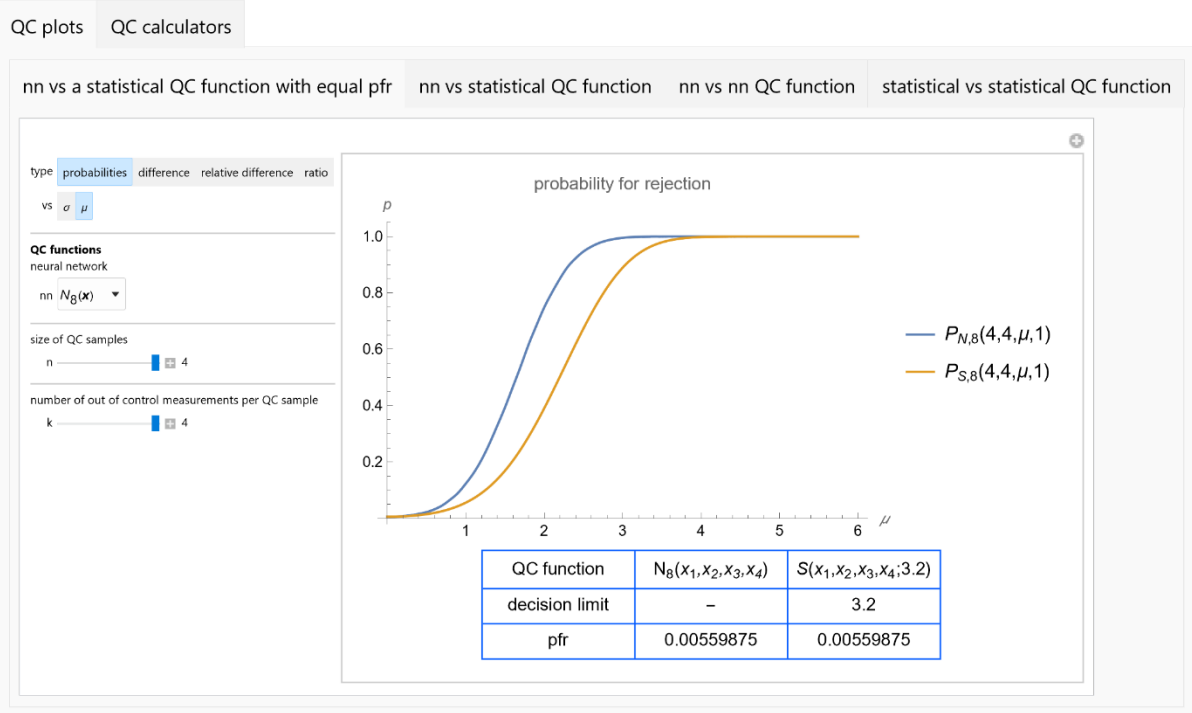
The program provides two modules:

### QC Plots

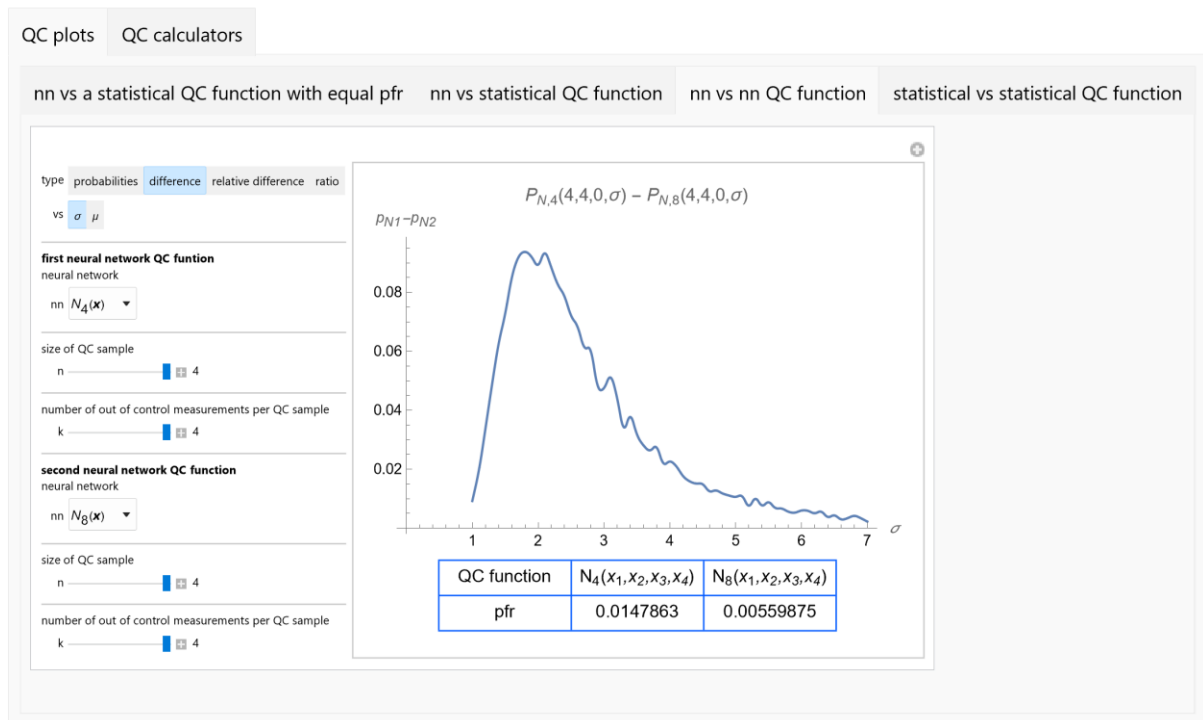
This module plots the probabilities for rejection of the application of NN and statistical QC functions to the respective datasets, their differences, relative differences, and ratios (see Figure 1 and 2).

### QC Calculators

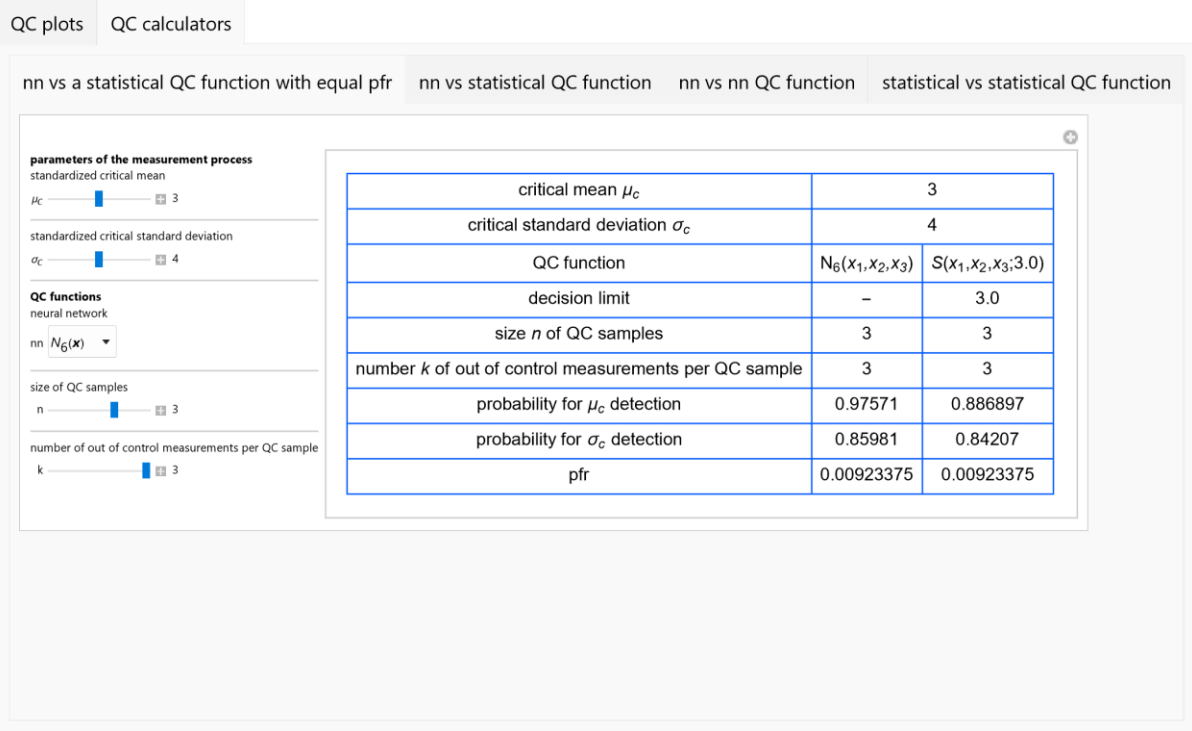
This module calculates the probabilities for rejection of the application of NN and statistical QC functions for a user defined critical mean and standard deviation of the measurement process, and their probabilities for false rejection (see Figure 3 and 4).



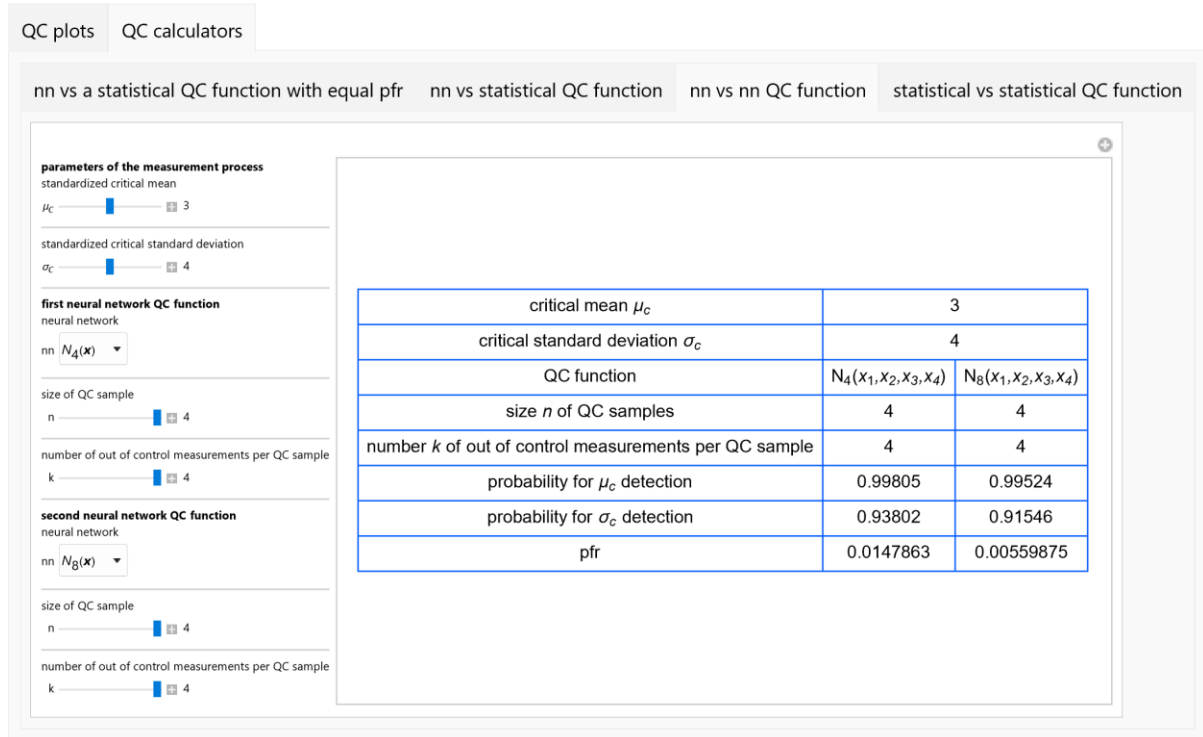
**Figure 1:** The plots of the probabilities for rejection  $P_{N,8}(4,4, \mu, 1)$  and  $P_{S,8}(4,4, \mu, 1)$  of the  $N_8(x_1, x_2, x_3, x_4)$  and  $S_8(x_1, x_2, x_3, x_4) = S(x_1, x_2, x_3, x_4; 3.2)$  QC functions, with the settings shown at the left.



**Figure 2:** The plot of the difference of the probabilities for rejection  $P_{N,4}(2,2,0, \sigma)$  and  $P_{N,8}(2,2,0, \sigma)$  of the  $N_4(x_1, x_2)$  and  $N_8(x_1, x_2)$  QC functions, with the settings shown at the left.



**Figure 3:** The table presenting the probabilities for rejection of the critical standard deviation and mean of the  $N_8(x_1, x_2, x_3, x_4)$  and  $S_8(x_1, x_2, x_3, x_4) = S(x_1, x_2, x_3, x_4; 3.2)$  QC functions.



**Figure 4:** The table presenting the probabilities for rejection of the critical standard deviation and mean of the  $N_4(x_1, x_2)$  and  $N_8(x_1, x_2)$  QC functions, with the settings shown at the left.

## Details

Four 1-D CNN were designed, trained, and tested with datasets of  $n$ -tuples of simulated standardized normally distributed QC measurements, for  $1 \leq n \leq 4$ , as described in [1].

For  $a \in \{1,2,3,4,6,8,12,16\}$ ,  $n = \#\mathbf{x}$  (that is  $n$  equal to the cardinality of  $\mathbf{x}$ ) and  $1 \leq n \leq 4$ , each 1-D CNN  $N_a(\mathbf{x})$  consists of:

1. The input layer, with  $n$  nodes.
2. Ten parallel arrays of five convolutional layers each. Every convolutional layer has  $n$  output channels, and kernels of size  $1 \times 1$ .
3. A concatenating layer, which concatenates the ten parallel arrays.
4. A pooling layer, performing 1-D pooling with kernels of size  $2 \times 1$ .
5. Ten convolutional net layers having  $n$  output channels, and kernels of size  $1 \times 1$ .
6. A pooling net layer, performing 1-D pooling with kernels of size  $2 \times 1$ .
7. Four linear net layers with output vectors of decreasing size.
8. A softmax net layer, normalizing the exponential of the output vector of the fourth linear layer.
9. The output layer, which is a decoder with two nodes.

Let  $C$  be the probability for rejection of the statistical QC function  $S(\mathbf{x}; l)$  applied to a  $n$ -tuple  $\mathbf{x}$  of QC measurements, for  $n = \#\mathbf{x}$ ,  $1 \leq n \leq 4$ ,  $1 \leq k \leq n$ , where  $k$  of them are distributed as  $\mathcal{N}(\mu, \sigma^2)$  and  $(n - k)$  as  $\mathcal{N}(0,1)$ . Then:

$$P_S(n, k, \mu, \sigma; l) = 1 - \frac{1}{2^k} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right)^{n-k} \left( \operatorname{erf}\left(\frac{\mu+l}{\sigma\sqrt{2}}\right) - \operatorname{erf}\left(\frac{\mu-l}{\sigma\sqrt{2}}\right) \right)^k \text{ for } 0 > k \leq n$$

Given that  $P_S(n, k, 0, 1; l) = p$ , to define  $l$  we solve the following equation:

$$P_S(n, k, 0, 1; l) = p \Rightarrow l = \sqrt{2} \operatorname{erf}^{-1}\left((1 - p)^{\frac{1}{n}}\right)$$

Therefore:

$$P_{S,a}(n, k, \mu, \sigma) = P_S\left(n, k, \mu, \sigma; \sqrt{2} \operatorname{erf}^{-1}\left((1 - P_{N,a}(n, n, 0, 1))^{\frac{1}{n}}\right)\right)$$

The program *Intelligent Quality* compares each designed CNN  $N_a(\mathbf{x})$ :

1. To statistical QC functions  $S_a(\mathbf{x})$  with equal pfr, applied to samples of the same size.
2. To statistical QC functions  $S(\mathbf{x}; l)$  with user defined pfr.
3. To other designed CNN  $N_a(\mathbf{x})$ .

In addition, statistical QC functions  $S(\mathbf{x}; l)$  with user defined pfr can be compared to each other.

## Conclusion

The program *Intelligent Quality* can be used as an educational or laboratory tool for the evaluation and comparison of the designed CNN and statistical QC functions.

## References

1. Chatzimichail RA, Hatjimihail AT. Quality Control Using Convolutional Neural Networks Applied to Samples of Very Small Size. Technical Report XXI. Drama: Hellenic Complex Systems Laboratory, 2022. Available at: <https://www.hcsl.com/Documents/hcsltr21.pdf>

## Notation

$\mathbf{x}$  :  $n$ -tuple  $(x_1, x_2, \dots, x_n)$  of QC measurements

$\#\mathbf{x}$  : the cardinality of  $\mathbf{x}$

$s_c$  : the critical random error

$m_c$  : the critical systematic error

$\mathcal{N}(\mu, \sigma^2)$ : normal distribution with mean  $\mu$  and standard deviation  $\sigma$ .

$N_a(\mathbf{x})$ : NN QC function applied to a tuple  $\mathbf{x}$  of QC measurements, for  $a \in \{4,6,8,12,16\}$ . The index  $a$  is related to the ratio of the simulated QC measurements of the training set distributed as  $\mathcal{N}(0,1)$  and

$\mathcal{N}(\mu, \sigma^2)$ , for  $0 < |\mu| \leq 6$  or  $1 < |\sigma| \leq 7$

$S_a(\mathbf{x})$ : statistical QC function applied to a tuple  $\mathbf{x}$  of QC measurements, for  $a \in \{4,6,8,12,16\}$ ;  $N_a(\mathbf{x})$  and  $S_a(\mathbf{x})$  have equal probabilities for false rejection.

$S(\mathbf{x}; m, s, l)$ : statistical QC function with decision limits  $m \pm l s$ , applied to a tuple  $\mathbf{x}$  of QC measurements.

The function rejects an analytical run if:  $\exists x_i \in \mathbf{x}: x_i < m - l s \vee x_i > m + l s$

$S(\mathbf{x}; l) = S(\mathbf{x}; 0, 1, l)$

$P_{N,a}(n, k, \mu, \sigma)$ : probability for rejection of the NN QC function  $N_a(\mathbf{x})$  applied to a  $n$ -tuple  $\mathbf{x}$  of control measurements,  $k$  of them distributed as  $\mathcal{N}(\mu, \sigma^2)$  and  $n - k$  as  $\mathcal{N}(0,1)$ , for  $a \in \{4,6,8,12,16\}$ , and  $0 < |\mu| \leq 6$  or  $1 < |\sigma| \leq 7$  and

$P_{S,a}(n, k, \mu, \sigma)$ : probability for rejection of the statistical QC function  $S_a(\mathbf{x})$  applied to a  $n$ -tuple  $\mathbf{x}$  of control measurements,  $k$  of them distributed as  $\mathcal{N}(0,1)$  and  $n - k$  as  $\mathcal{N}(\mu, \sigma^2)$ , for  $a \in \{4,6,8,12,16\}$ , and  $0 < |\mu| \leq 6$  or  $1 < |\sigma| \leq 7$ .

$P_S(n, k, \mu, \sigma; l)$  : probability for rejection of the statistical QC function  $S(\mathbf{x}; l)$  applied on a  $n$ -tuple  $\mathbf{x}$  of control measurements,  $k$  of them distributed as  $\mathcal{N}(0,1)$  and  $n - k$  as  $\mathcal{N}(\mu, \sigma^2)$ , for  $a \in \{4,6,8,12,16\}$ , and  $0 < |\mu| \leq 6$  or  $1 < |\sigma| \leq 7$ .

## The program interface

### QC Plots

#### 1. NN vs a statistical QC function with equal *pfr*

##### 1.1. Input.

##### 1.1.1. Plot options

##### 1.1.1.1. *x-axis*

1.1.1.1.1. The standardized standard deviation ( $\sigma$ ).

1.1.1.1.2. The standardized mean ( $\mu$ ).

##### 1.1.1.2. *The type of the plot:*

1.1.1.2.1. The probabilities for rejection of the QC functions

1.1.1.2.2. Their difference.

1.1.1.2.3. Their relative difference.

1.1.1.2.4. Their ratio.

### 1.1.2. QC parameters

The following are defined:

- 1.1.2.1. The type of neural network  $N_a(\mathbf{x})$ ,  $a \in \{1,2,3,4,6,8,12,16\}$ .
- 1.1.2.2. The size  $n$  of the QC samples ( $1 \leq n \leq 4$ ).
- 1.1.2.3. The number  $k$  of the out of control QC measurements per sample ( $1 \leq k \leq n$ ).

## 1.2. Output

### 1.2.1. Table of QC parameters

- 1.2.1.1. The decision limit  $l$  of the statistical QC function  $S(\mathbf{x}; l) = S_a(\mathbf{x})$ .
- 1.2.1.2. The probability  $pfr$  for false rejection of the  $N_a(\mathbf{x})$  and  $S_a(\mathbf{x})$ .

### 1.2.2. Plots of:

- 1.2.2.1. The probabilities for rejection  $P_{N,a}(n, k, \mu, \sigma)$  and  $P_{S,a}(n, k, \mu, \sigma)$  of the  $N_a(\mathbf{x})$  and  $S_a(\mathbf{x})$ .
- 1.2.2.2. Their difference.
- 1.2.2.3. Their relative difference.
- 1.2.2.4. Their ratio.

## 2. NN vs statistical QC function

### 2.1. Input

#### 2.1.1. Plot options

##### 2.1.1.1. $x$ -axis

- 2.1.1.1.1. The standardized standard deviation ( $\sigma$ ).
- 2.1.1.1.2. The standardized mean ( $\mu$ ).

##### 2.1.1.2. The type of the plot:

- 2.1.1.2.1. The probabilities for rejection.
- 2.1.1.2.2. Their difference.
- 2.1.1.2.3. Their relative difference.
- 2.1.1.2.4. Their ratio.

### 2.1.2. QC parameters

- 2.1.2.1. The neural network  $N_a(\mathbf{x})$ ,  $a \in \{1,2,3,4,6,8,12,16\}$
- 2.1.2.2. The size  $n$  of the QC samples ( $1 \leq n \leq 4$ ), for the  $N_a(\mathbf{x})$ .
- 2.1.2.3. The number  $k$  of the out of control QC measurements per sample ( $1 \leq k \leq n$ ), for the  $N_a(\mathbf{x})$ .
- 2.1.2.4. The size  $n$  of the QC samples ( $1 \leq n \leq 4$ ), for the statistical QC function  $S(\mathbf{x}; l)$ .
- 2.1.2.5. The number  $k$  of the out of control QC measurements per sample ( $1 \leq k \leq n$ ), for the  $S(\mathbf{x}; l)$ .
- 2.1.2.6. The probability  $pfr$  for false rejection of the  $S(\mathbf{x}; l)$ .

## 2.2. Output

### 2.2.1. Table of QC parameters

- 2.2.1.1. The decision limit  $l$  of  $S(\mathbf{x}; l)$ .
- 2.2.1.2. The probabilities  $pfr$  for false rejection of the  $N_a(\mathbf{x})$  and  $S_a(\mathbf{x})$ .

### 2.2.2. Plots of:

2.2.2.1. The probabilities for rejection  $P_{N,a}(n, k, \mu, \sigma)$  and  $P_S(n, k, \mu, \sigma; l)$  of the  $N_a(\mathbf{x})$  and  $S(\mathbf{x}; l)$ .

2.2.2.2. Their difference.

2.2.2.3. Their relative difference.

2.2.2.4. Their ratio.

## 3. NN vs NN QC function

### 3.1. Input.

#### 3.1.1. Plot options

##### 3.1.1.1. *x-axis*

3.1.1.1.1. The standardized standard deviation ( $\sigma$ ).

3.1.1.1.2. The standardized mean ( $\mu$ ).

##### 3.1.1.2. *The type of the plot:*

3.1.1.2.1. The probabilities for rejection.

3.1.1.2.2. Their difference.

3.1.1.2.3. Their relative difference.

3.1.1.2.4. Their ratio.

#### 3.1.2. QC parameters

For each of the two NN QC functions  $N_a(\mathbf{x})$ :

3.1.2.1. The type of neural network  $N_a(\mathbf{x})$ ,  $a \in \{1, 2, 3, 4, 6, 8, 12, 16\}$ .

3.1.2.2. The size  $n$  of the QC samples ( $1 \leq n \leq 4$ ), for the  $N_a(\mathbf{x})$ .

3.1.2.3. The number  $k$  of the out of control QC measurements per sample ( $1 \leq k \leq n$ ), for the  $N_a(\mathbf{x})$ .

### 3.2. Output

#### 3.2.1. Table of QC parameters

3.2.1.1. The probability *pfr* for false rejection of each of the  $N_a(\mathbf{x})$ .

#### 3.2.2. Plots of:

3.2.2.1. The probabilities for rejection  $P_{N,a}(n, k, \mu, \sigma)$  of each of the  $N_a(\mathbf{x})$ .

3.2.2.2. Their difference.

3.2.2.3. Their relative difference.

3.2.2.4. Their ratio.

## 4. Statistical vs statistical QC function

### 4.1. Input.

#### 4.1.1. Plot options

##### 4.1.1.1. *x-axis*

4.1.1.1.1. The standardized standard deviation ( $\sigma$ ).

4.1.1.1.2. The standardized mean ( $\mu$ ).

##### 4.1.1.2. *The type of the plot:*

4.1.1.2.1. The probabilities for rejection of the QC functions.

4.1.1.2.2. Their difference.



4.1.1.2.3. Their relative difference.

4.1.1.2.4. Their ratio.

#### 4.1.2. QC parameters

For each of the two statistical QC functions  $S(\mathbf{x}; l)$ :

4.1.2.1. The size  $n$  of the QC samples ( $1 \leq n \leq 4$ ).

4.1.2.2. The number  $k$  of the out of control QC measurements per sample ( $1 \leq k \leq n$ ).

4.1.2.3. The probability  $pfr$  for false rejection.

#### 4.2. Output

##### 4.2.1. Table of QC parameters

4.2.1.1. The decision limits  $l$  of  $S(\mathbf{x}; l)$ .

4.2.1.2. The probabilities  $pfr$  for false rejection of each of the  $S(\mathbf{x}; l)$ .

##### 4.2.2. Plots of:

4.2.2.1. The probabilities for rejection  $P_S(n, k, \mu, \sigma; l)$  of each of the  $S(\mathbf{x}; l)$ .

4.2.2.2. Their difference.

4.2.2.3. Their relative difference.

4.2.2.4. Their ratio.

### QC Calculators

#### 5. NN vs a statistical QC function with equal $pfr$

##### 5.1. Input

##### 5.1.1. Parameters of of the measurement process

5.1.1.1. The standardized critical mean ( $m_c$ ).

5.1.1.2. The standardized critical standard deviation ( $s_c$ ).

##### 5.1.2. QC parameters

The following are defined:

5.1.2.1. The type of the neural network  $N_a(\mathbf{x})$ ,  $a \in \{1, 2, 3, 4, 6, 8, 12, 16\}$ .

5.1.2.2. The size  $n$  of the QC samples ( $1 \leq n \leq 4$ ).

5.1.2.3. The number  $k$  of the out of control QC measurements ( $1 \leq k \leq n$ ).

##### 5.2. Output

##### 5.2.1. Table of QC parameters

5.2.1.1. The decision limit  $l$  of the statistical QC function  $S(\mathbf{x}; l) = S_a(\mathbf{x})$ .

5.2.1.2. The size  $n$  of the QC samples.

5.2.1.3. The number  $k$  of the out of control QC measurements per sample.

5.2.1.4. The probabilities for critical standard deviation  $s_c$  detection of the  $N_a(\mathbf{x})$  and  $S_a(\mathbf{x})$ .

5.2.1.5. The probabilities for critical mean  $m_c$  detection of the  $N_a(\mathbf{x})$  and  $S_a(\mathbf{x})$ .

5.2.1.6. The probability  $pfr$  for false rejection of of the  $N_a(\mathbf{x})$  and  $S_a(\mathbf{x})$ .

#### 6. NN vs statistical QC function

##### 6.1. Input

##### 6.1.1. Parameters of of the measurement process

6.1.1.1. The standardized critical standard deviation ( $s_c$ )

6.1.1.2. The standardized critical mean ( $m_c$ )

### 6.1.2. QC parameters

6.1.2.1. The type of the neural network  $N_a(\mathbf{x})$ ,  $a \in \{1,2,3,4,6,8,12,16\}$

6.1.2.2. The size  $n$  of the QC sample ( $1 \leq n \leq 4$ ), for the  $N_a(\mathbf{x})$ .

6.1.2.3. The number  $k$  of the out of control QC measurements per sample ( $1 \leq k \leq n$ ), for the  $N_a(\mathbf{x})$ .

6.1.2.4. The size  $n$  of the QC samples ( $1 \leq n \leq 4$ ), for the statistical QC function  $S(\mathbf{x}; l)$ .

6.1.2.5. The number  $k$  of the out of control QC measurements per sample ( $1 \leq k \leq n$ ), for the  $S(\mathbf{x}; l)$ .

6.1.2.6. The probability  $pfr$  for false rejection of the statistical QC function  $S(\mathbf{x}; l)$ .

## 6.2. Output

### 6.2.1. Table of QC parameters

6.2.1.1. The decision limit  $l$  of  $S(\mathbf{x}; l)$ .

6.2.1.2. The size  $n$  of the QC samples.

6.2.1.3. The number  $k$  of the out of control QC measurements per sample.

6.2.1.4. The probabilities for critical standard deviation  $s_c$  detection of both QC functions.

6.2.1.5. The probabilities for critical mean  $m_c$  detection of both QC functions.

6.2.1.6. The probabilities  $pfr$  for false rejection of the  $N_a(\mathbf{x})$  and  $S(\mathbf{x}; l)$ .

### 6.2.2. Plots of:

6.2.2.1. The probabilities for rejection  $P_{N,a}(n, k, \mu, \sigma)$  and  $P_S(n, k, \mu, \sigma; l)$  of the  $N_a(\mathbf{x})$  and  $S(\mathbf{x}; l)$ .

6.2.2.2. Their difference.

6.2.2.3. Their relative difference.

6.2.2.4. Their ratio.

## 7. NN vs NN QC function

### 7.1. Input

#### 7.1.1. Plot options

##### 7.1.1.1. x-axis

7.1.1.1.1. The standardized mean ( $\mu$ )

7.1.1.1.2. The standardized standard deviation ( $\sigma$ )

#### 7.1.2. QC parameters

For each NN QC function:

7.1.2.1. The type of the neural network  $N_a(\mathbf{x})$ ,  $a \in \{1,2,3,4,6,8,12,16\}$

7.1.2.2. The size  $n$  of the QC samples ( $1 \leq n \leq 4$ ), for the  $N_a(\mathbf{x})$ .

7.1.2.3. The number  $k$  of the out of control QC measurements per sample ( $1 \leq k \leq n$ ), for the  $N_a(\mathbf{x})$ .

### 7.2. Output

#### 7.2.1. Table of QC parameters

7.2.1.1. The size  $n$  of the QC samples.

7.2.1.2. The number  $k$  of the out of control QC measurements per sample.

- 7.2.1.3. The probabilities for critical standard deviation  $s_c$  detection of each of the  $N_a(\mathbf{x})$ .
- 7.2.1.4. The probabilities for critical mean  $m_c$  detection of each of the  $N_a(\mathbf{x})$ .
- 7.2.1.5. The probabilities  $pfr$  for false rejection of each of the  $N_a(\mathbf{x})$ .

## 8. Statistical vs statistical QC function

### 8.1. Input

#### 8.1.1. QC parameters

For each statistical QC function  $S(\mathbf{x}; l)$ :

- 8.1.1.1. The size  $n$  of the QC samples ( $1 \leq n \leq 4$ ).
- 8.1.1.2. The number  $k$  of the out of control QC measurements per sample ( $1 \leq k \leq n$ ).
- 8.1.1.3. The probability  $pfr$  for false rejection.

### 8.2. Output

#### 8.2.1. Table of QC parameters

- 8.2.1.1. The decision limits  $l$  of each of the  $S(\mathbf{x}; l)$ .
- 8.2.1.2. The size  $n$  of the QC samples.
- 8.2.1.3. The number  $k$  of the out of control QC measurements per sample.
- 8.2.1.4. The probabilities for critical standard deviation  $s_c$  detection of each of the  $S(\mathbf{x}; l)$ .
- 8.2.1.5. The probabilities for critical mean  $m_c$  detection of each of the  $S(\mathbf{x}; l)$ .
- 8.2.1.6. The probabilities  $pfr$  for false rejection of each of the  $S(\mathbf{x}; l)$ .

## Source Code

*Programming language:* Wolfram Language

*Availability:* The updated source code is available at: <https://www.hcsl.com/Tools/IntelligentQuality/IntelligentQuality.nb>

## Supplement

The trained CNN models, are available as Wolfram Language Objects at:

<https://www.hcsl.com/Supplements/hcsltr22s.zip>

## Software Requirements

Operating systems: Microsoft Windows, Linux, Apple iOS

Other software requirements: Wolfram Player®, freely available at: <https://www.wolfram.com/player/> or Wolfram Mathematica®.

## System Requirements

*Processor:* Intel Core i5® or equivalent CPU

*System memory (RAM):* 4 GB+ recommended.

## Permanent Citation:

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