Hellenic Complex Systems Laboratory

# Uncertainty of Measurement and Areas Over and Under the ROC Curves

Technical Report VI

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WOLFRAM Demonstrations Project

# Uncertainty of Measurement and Areas Over and Under the ROC Curves

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Search Terms: ROC curves, uncertainty of measurement, sensitivity, specificity, diagnostic test, clinical accuracy, diagnostic accuracy, diagnostic inaccuracy, permissible uncertainty, normal distribution, binormal distribution, area under ROC curves, area over ROC curves, AUC, AOC, relative change, rate of change, relative rate of change

## Short Description of the Demonstration

This Demonstration plots the change, the relative change, the rate of change, and the relative rate of change of the area under (blue plot) and the area over (orange plot) the receiver operating characteristic (ROC) curve of a diagnostic test, as the uncertainty of measurement increases from 0 to a user defined upper bound. The test measures a measurand on normally distributed nondiseased and diseased populations, for various values of the mean and standard deviation of the populations. A normal distribution of the uncertainty is assumed. The type of plot is selected using the "plot" menu. The five parameters that can be varied using the sliders are measured in arbitrary units.

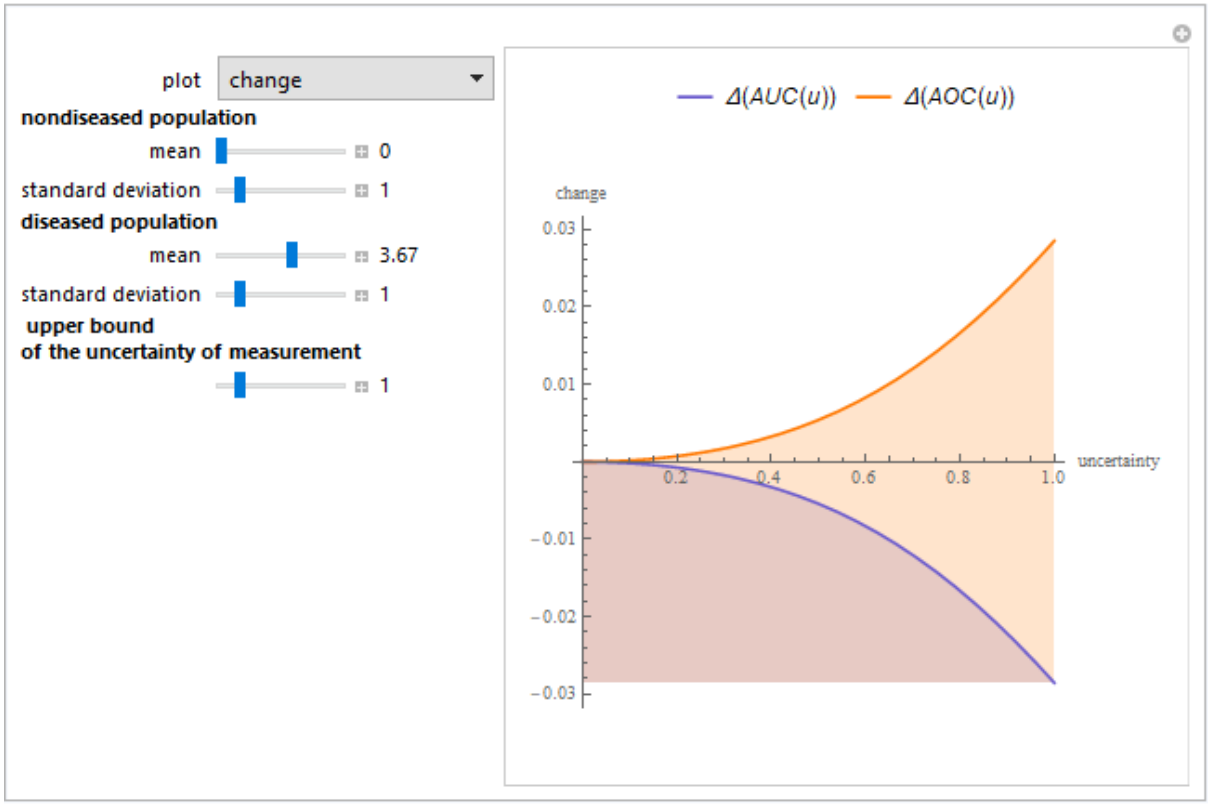


Figure 1: *Plot of the change of the areas under (blue plot) and over (orange plot) the receiver operating characteristic (ROC) curve of a diagnostic test, with the settings shown at the right.*

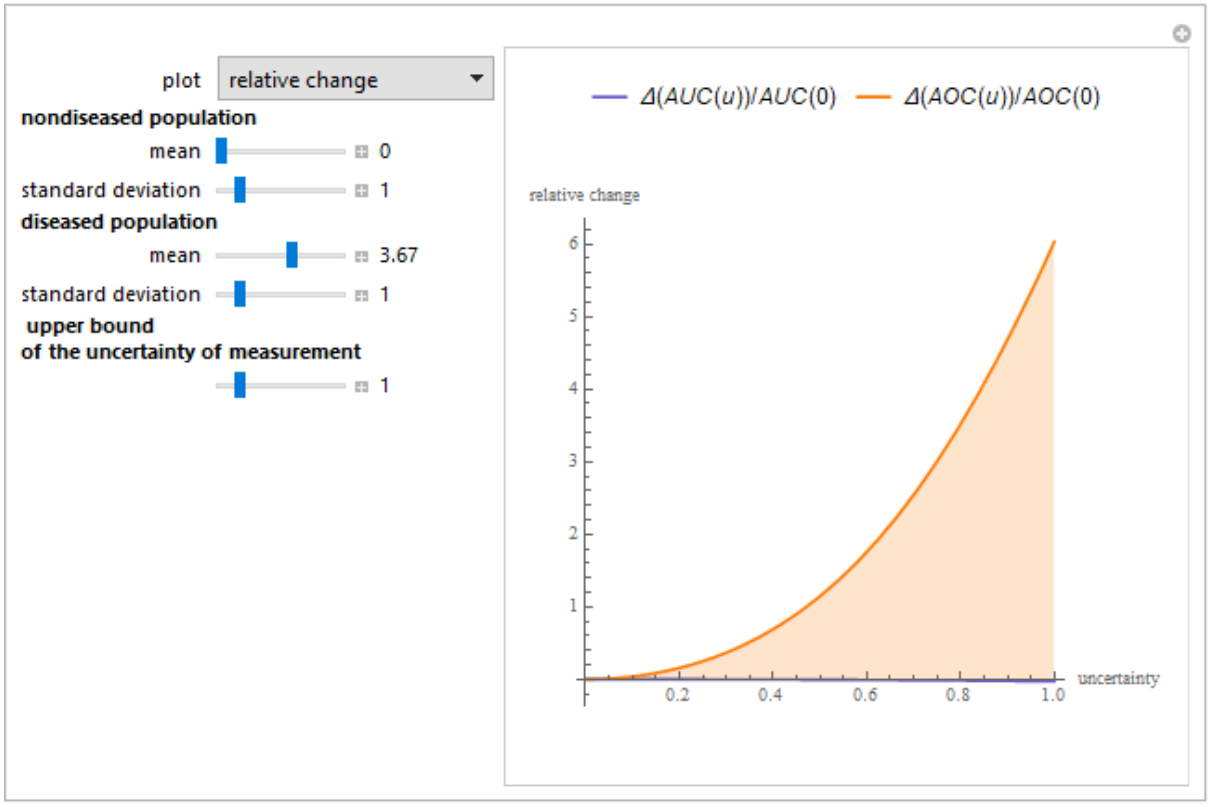


Figure 2: *Plot of the relative change of the areas under (blue plot) and over (orange plot) the receiver operating characteristic (ROC) curve of a diagnostic test, with the settings shown at the right.*

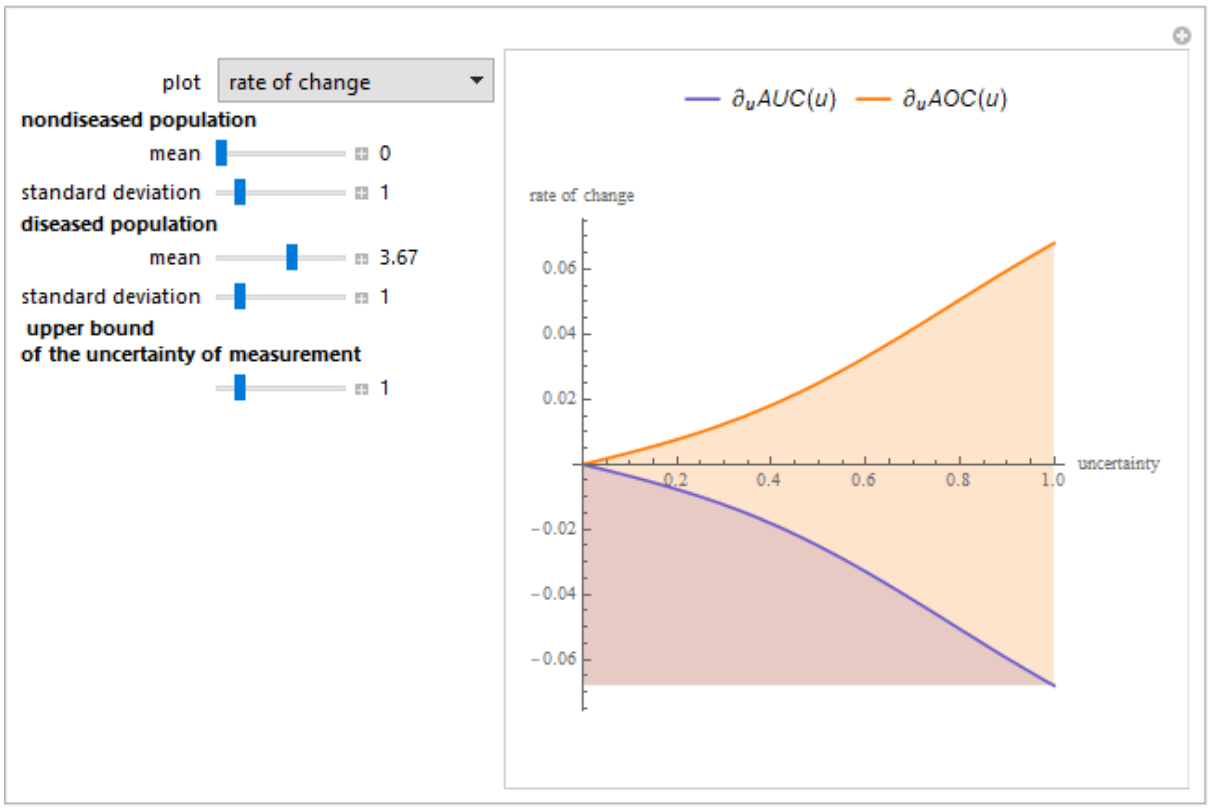


Figure 3: *Plot of the rate of change of the areas under (blue plot) and over (orange plot) the receiver operating characteristic (ROC) curve of a diagnostic test, with the settings shown at the right.*

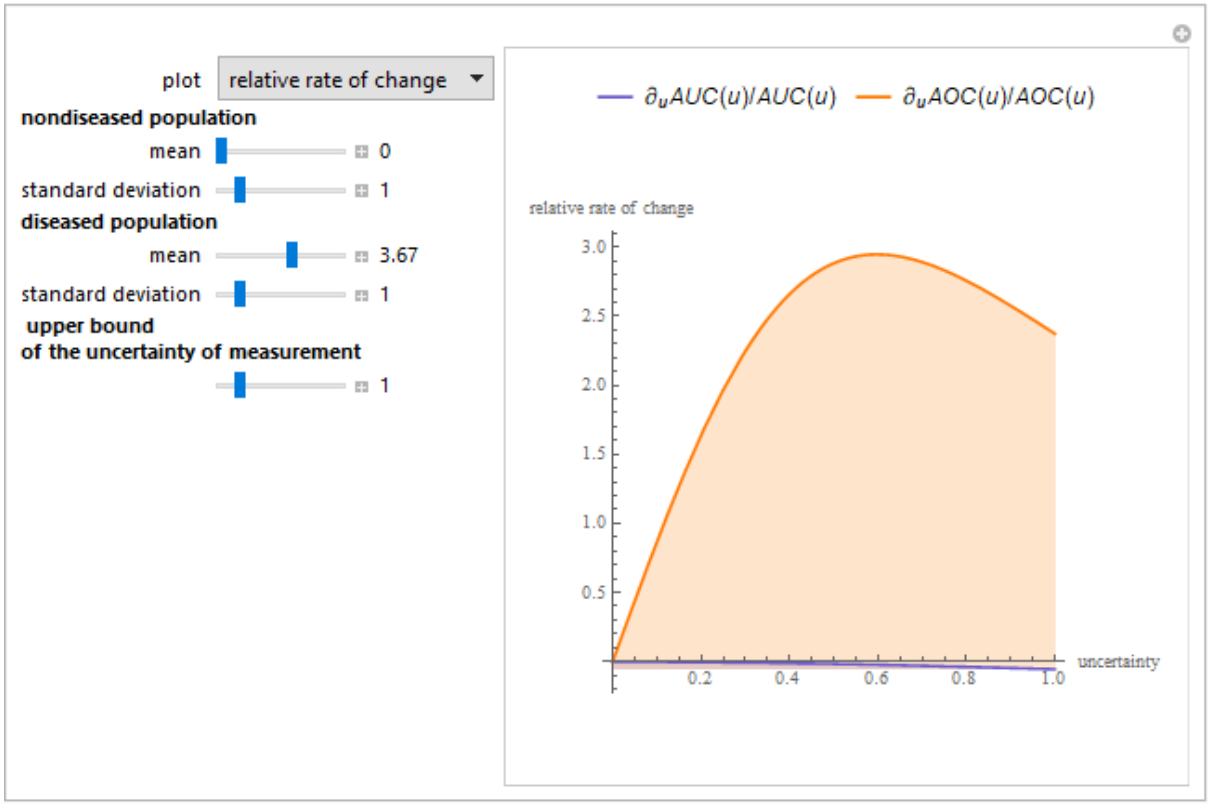


Figure 4: *Plot of the relative rate of change of the areas under (blue plot) and over (orange plot) the receiver operating characteristic (ROC) curve of a diagnostic test, with the settings shown at the right.*

## Details

The ROC curves are used in the evaluation of the clinical accuracy of a diagnostic test applied to a diseased and a nondiseased population. The ROC curves display the sensitivity of the test against 1-specificity. The term sensitivity is used to describe the fraction of the diseased population with a positive test result, while specificity describes the fraction of the nondiseased population with a negative test result. Therefore, the ROC curves display the true positive fraction against the false positive fraction. The area under a ROC curve is used as an index of the diagnostic accuracy of the respective test. The area under the curve decreases as the uncertainty of measurement of the diagnostic test increases [1].

Assuming $AUC(u)$ is the area under a ROC curve and $AOC(u)$ is the area over the ROC curve for an uncertainty of measurement $u$, the changes $\Delta\big(AUC(u)\big)$ and $\Delta\big(AOC(u)\big)$ are defined as $AUC(u) - AUC(0)$ and $AOC(u) - AOC(0)$, the relative changes as $\Delta(AUC(u))/AUC(0)$ and $\Delta(AOC(u))/AOC(0)$, the rates of change as $\frac{\partial AUC(u)}{\partial u}$ and $\frac{\partial AOC(u)}{\partial u}$ and the relative rates of change as $\frac{\frac{\partial AUC(u)}{\partial u}}{AUC(u)}$ and $\frac{\frac{\partial AOC(u)}{\partial u}}{AOC(u)}$ respectively.

As $AOC(u) = 1 - AUC(u)$, it can be considered that the area over the ROC curve is an index of diagnostic inaccuracy. In fact, as the plots show, the relative change, the rate of change, and the relative rate of change of the area over a ROC curve against the uncertainty of measurement are greater than the respective measures of the area under the ROC curve, for the same populations.

To the best of my knowledge, measures of the area over the ROC curve against the uncertainty of measurement have not been discussed in the literature.

The area over the ROC curve could be used in the evaluation of a diagnostic test, as a diagnostic inaccuracy index. For example, in the thumbnail and the snapshots, the population data describe a bimodal distribution of serum glucose on a non-diabetic and a diabetic population [2].

## References:

1. Hatjimihail AT. Receiver Operating Characteristic Curves and Uncertainty of Measurement. Wolfram Demonstrations Project, Champaign: Wolfram Research, Inc., 2009.

2. T. O. Lim, R. Bakri, Z. Morad, and M. A. Hamid, "Bimodality in Blood Glucose Distribution—Is It Universal?" *Diabetes Care,* **25**(12), 2002 pp. 2212-2217.

## Source Code

The updated Wolfram Mathematica© source code is available at:
https://www.hcsl.com/Tools/UncertaintyOfMeasurementAndAreasOverAndUnderTheROCCurves-author.nb

### Permanent Citation of the Demonstration:

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http://demonstrations.wolfram.com/UncertaintyOfMeasurementAndAreasOverAndUnderTheROCCurves/

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