Hellenic Complex Systems Laboratory

Intelligent Quality: A Software Tool for Exploring the Application of Convolutional Neural Networks to Quality Control Samples of Very Small Size

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Rallou A. Chatzimichail and Aristides T. Hatjimihail

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Intelligent Quality: A Software Tool for Exploring the Application of Convolutional Neural Networks to Quality Control Samples of Very Small Size

Rallou A. Chatzimichail, PhD [a], Aristides T. Hatjimihail, MD, PhD [a]

[a] Hellenic Complex Systems Laboratory

Search Terms: QC, convolutional neural networks, probability for rejection, probability for false rejection, QC measurement

Introduction

Although there is extensive literature on the application of artificial neural networks in quality control (QC), to monitor the conformity of a process to quality specifications, at least five QC measurements are required, increasing the related cost. To explore the application of neural networks to samples of QC measurements of very small size, four one-dimensional (1-D) convolutional neural networks (CNN) were designed, trained, and tested with datasets of *n* -tuples of simulated standardized normally distributed QC measurements, for $1 \leq n \leq 4$.

The designed neural networks were compared to statistical QC functions with equal probabilities for false rejection (*pfr*), applied to samples of the same size. When the *n* -tuples included at least two QC measurements distributed as $\mathcal{N}(\mu, \sigma^2)$, where $0.2 < |\mu| \leq 6.0$, and $1.0 < \sigma \leq 7.0$, the designed neural networks outperformed the respective statistical QC functions. Therefore, 1-D CNN applied to samples of 2-4 quality control measurements can be used to increase the probability of detection of the nonconformity of a process to the quality specifications, with lower cost.

The program *Intelligent Quality* was developed to explore the application of the trained neural networks to QC samples of very small size and compare them to statistical QC functions.

The program

The program provides two modules:

QC Plots

This module plots the probabilities for rejection of the application of neural networks and statistical QC functions to the respective datasets, their differences, relative differences, and ratios.

QC Calculators

This module calculates the probabilities for rejection of the application of neural networks and statistical QC functions for a user defined critical mean and standard deviation of the measurement process, and their probabilities for false rejection.

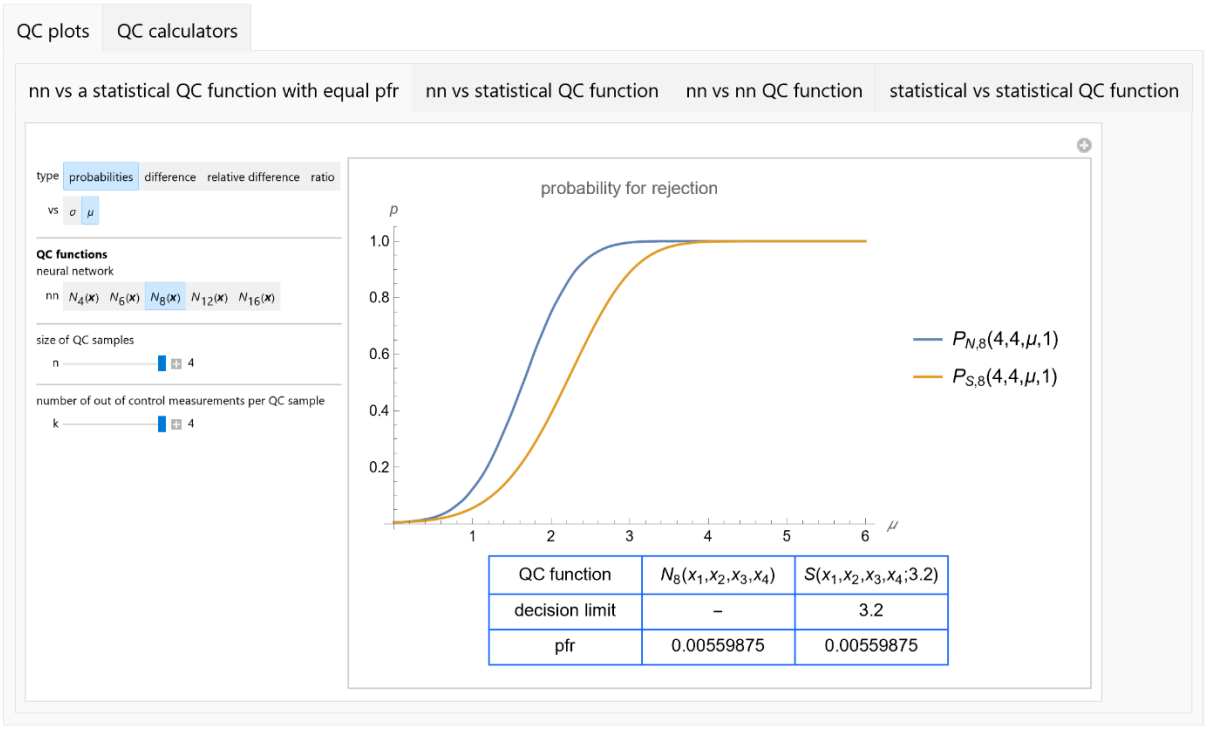


Figure 1: The plots of the probabilities for rejection $P_{N,8}(4,4,\mu,1)$ and $P_{S,8}(4,4,\mu,1)$ of the $N_8(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4)$ and $\mathrm{S}_8(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4) = \mathrm{S}(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4;3.2)$ QC functions, with the settings shown at the left.

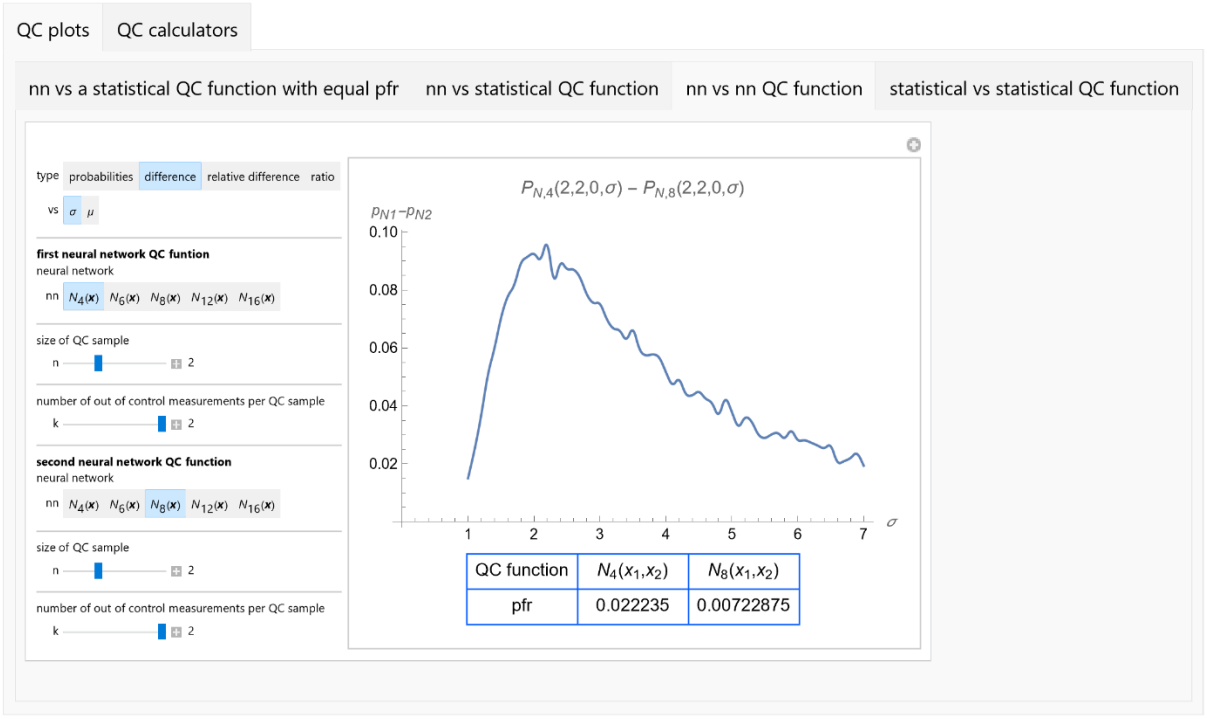
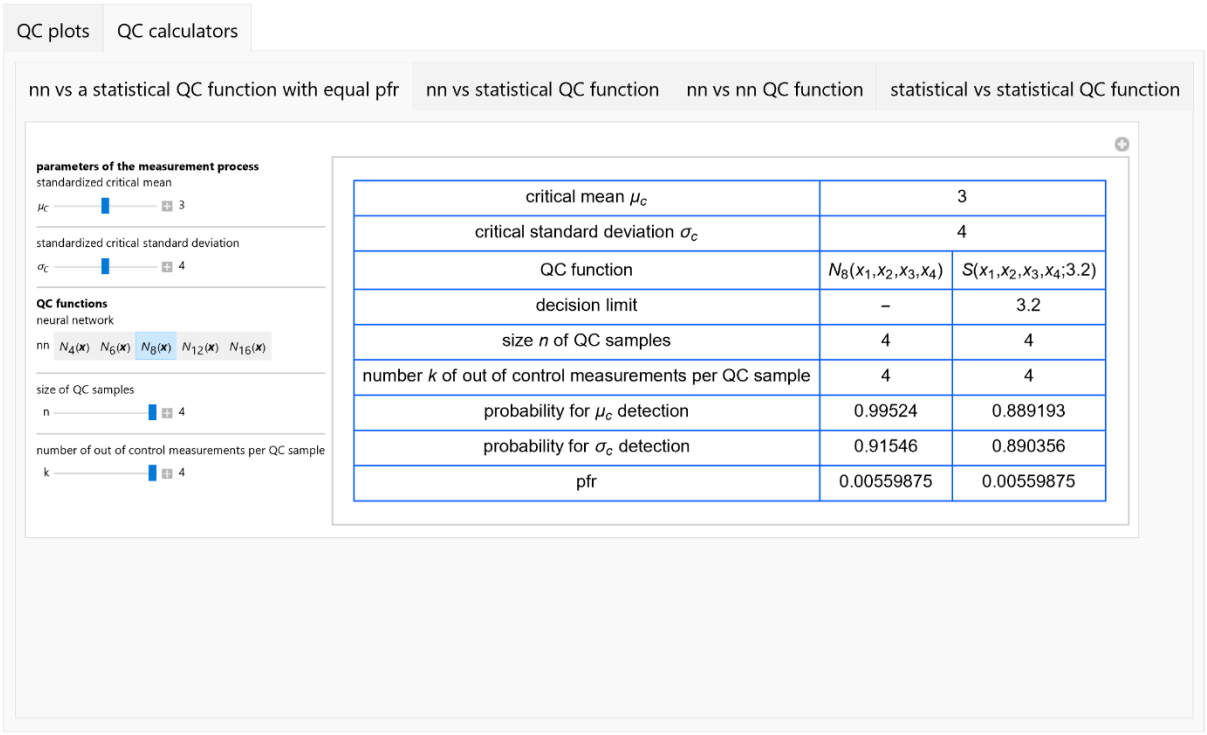
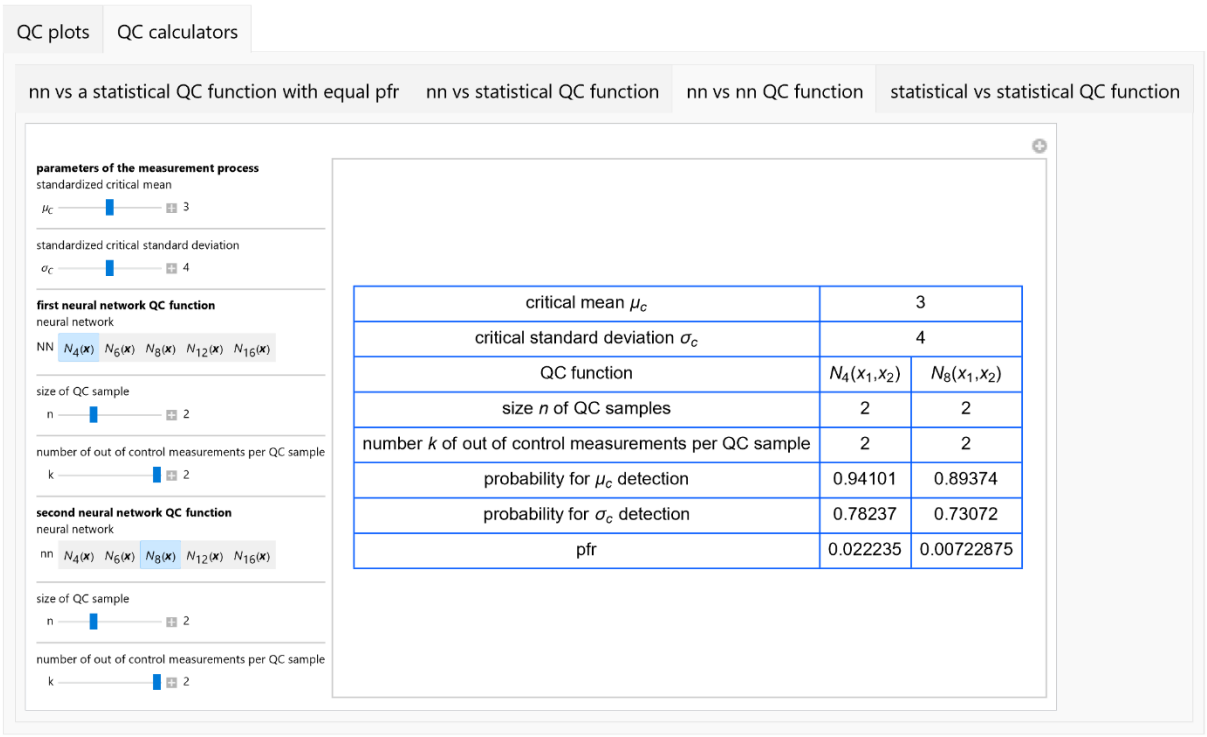


Figure 2: The plot of the difference of the probabilities for rejection $P_{N,4}(2,2,0,\sigma)$ and $P_{N,4}(2,2,0,\sigma)$ of the $N_4(\mathrm{x}_1,\mathrm{x}_2)$ and $\mathrm{N}_8(\mathrm{x}_1,\mathrm{x}_2)$ QC functions, with the settings shown at the left.



critical mean μ_c	3	
critical standard deviation σ_c	4	
QC function	$N_8(x_1,x_2,x_3,x_4)$	$S(x_1,x_2,x_3,x_4;3.2)$
decision limit	–	3.2
size n of QC samples	4	4
number k of out of control measurements per QC sample	4	4
probability for μ_c detection	0.99524	0.889193
probability for σ_c detection	0.91546	0.890356
pfr	0.00559875	0.00559875

Figure 3: The table presenting the probabilities for rejection of the critical standard deviation and mean of the $N_8(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4)$ and $\mathrm{S}_8(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4) = \mathrm{S}(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4;3.2)$ QC functions.



critical mean μ_c	3	
critical standard deviation σ_c	4	
QC function	$N_4(x_1,x_2)$	$N_8(x_1,x_2)$
size n of QC samples	2	2
number k of out of control measurements per QC sample	2	2
probability for μ_c detection	0.94101	0.89374
probability for σ_c detection	0.78237	0.73072
pfr	0.022235	0.00722875

Figure 4: The table presenting the probabilities for rejection of the critical standard deviation and mean of the $N_4(\mathrm{x}_1,\mathrm{x}_2)$ and $\mathrm{N}_8(\mathrm{x}_1,\mathrm{x}_2)$ QC functions, with the settings shown at the left.

Details

Four 1-D CNN were designed, trained, and tested with datasets of *n* -tuples of simulated standardized normally distributed QC measurements, for 1 ≤ *n* ≤ 4 [1].

The program *Intelligent Quality* compares each designed neural network $N_a(\boldsymbol{x})$, for $a \in \{4,6,8,12,16\}$.

1. To statistical QC functions $\mathrm{S}_a(\boldsymbol{x})$ with equal pfr, applied to samples of the same size.
2. To statistical QC functions $S(\boldsymbol{x}; l)$ with user defined *pfr.*
3. To other designed neural networks QC functions $N_a(\boldsymbol{x})$.

In addition, statistical QC functions $\mathrm{S}(\boldsymbol{x}; l)$ with user defined *pfr* can be compared to each other.

Conclusion

The program *Intelligent Quality* can be used as an educational or laboratory tool for the evaluation and comparison of the designed neural networks and statistical QC functions.

References

1. Chatzimichail RA, Hatjimihail AT. Quality Control Using Convolutional Neural Networks Applied to Samples of Very Small Size. Technical Report XXI. Drama: Hellenic Complex Systems Laboratory, 2022. Available at: https://www.hcsl.com/Documents/hcsltr21.pdf

Notation

$\boldsymbol{x}$: *n*-tuple $(x_1, x_2, \ldots, x_n)$ of QC measurements

s_c : the critical random error

m_c : the critical systematic error

$\mathcal{N}(\mu, \sigma^2)$: normal distribution with mean μ and standard deviation σ.

$N_a(\boldsymbol{x})$: neural networks QC function applied to a tuple $\boldsymbol{x}$ of QC measurements, for $a \in \{4,6,8,12,16\}$. The index a is related to the ratio of the simulated QC measurements of the training set distributed as $\mathcal{N}(0,1)$ and $\mathcal{N}(\mu, \sigma^2)$, for $0<|\mu|\leq 6$ or $1<|\sigma|\leq 7$

$\mathrm{S}_a(\boldsymbol{x})$: statistical QC function applied to a tuple $\boldsymbol{x}$ of QC measurements, for $a \in \{4,6,8,12,16\}$; $N_a(\boldsymbol{x})$ and $\mathrm{S}_a(\boldsymbol{x})$ have equal probabilities for false rejection.

$S(\boldsymbol{x}; m, s, l)$: statistical QC function with decision limits $m \pm l\, s$, applied to a tuple $\boldsymbol{x}$ of QC measurements. The function rejects an analytical run if: $\exists\, x_i \in \boldsymbol{x}: x_i < m - l\, s \vee x_i > m + l\, s$

$S(\boldsymbol{x}; l) = S(\boldsymbol{x}; 0, 1, l)$

$P_{N,a}(n, k, \mu, \sigma)$: probability for rejection of the neural networks QC function $N_a(\boldsymbol{x})$ applied to a *n* -tuple $\boldsymbol{x}$ of control measurements, *k* of them distributed as $\mathcal{N}(\mu, \sigma^2)$ and $n - k$ as $\mathcal{N}(0,1)$, for $a \in \{4,6,8,12,16\}$, and $0<|\mu|\leq 6$ or $1<|\sigma|\leq 7$ and

$P_{S,a}(n, k, \mu, \sigma)$: probability for rejection of the statistical QC function $\mathrm{S}_a(\boldsymbol{x})$ applied to a *n* -tuple $\boldsymbol{x}$ of control measurements, *k* of them distributed as $\mathcal{N}(0,1)$ and $n - k$ as $\mathcal{N}(\mu, \sigma^2)$, for $a \in \{4,6,8,12,16\}$, and $0<|\mu|\leq 6$ or $1<|\sigma|\leq 7$.

$P_S(n, k, \mu, \sigma; l)$: probability for rejection of the QC function $S(\boldsymbol{x}; l)$ applied on a *n* -tuple $\boldsymbol{x}$ of control measurements, *k* of them distributed as $\mathcal{N}(0,1)$ and $n - k$ as $\mathcal{N}(\mu, \sigma^2)$, for $a \in \{4,6,8,12,16\}$, and $0<|\mu|\leq 6$ or $1<|\sigma|\leq 7$.

The program interface

QC Plots

1. neural networks vs a statistical QC function with equal *pfr*

1.1. Input.

1.1.1. Plot options

1.1.1.1. x-axis

1.1.1.1.1. The standardized standard deviation (σ).

1.1.1.1.2. The standardized mean (μ).

1.1.1.2. The type of the plot:

1.1.1.2.1. The probabilities for rejection of the QC functions

1.1.1.2.2. Their difference.

1.1.1.2.3. Their relative difference.

1.1.1.2.4. Their ratio.

1.1.2. QC parameters

The following are defined:

1.1.2.1. The type of neural network $N_a(\boldsymbol{x})$, a $\in \{4,6,8,12,16\}$.

1.1.2.2. The size *n* of the QC samples ($1 \le n \le 4$).

1.1.2.3. The number *k* of the out of control QC measurements per sample ($1 \le k \le n$).

1.2. Output

1.2.1. Table of QC parameters

1.2.1.1. The decision limit *l* of the statistical QC function $\mathrm{S}(\boldsymbol{x}; l) = \mathrm{S}_a(\boldsymbol{x})$.

1.2.1.2. The probability *pfr* for false rejection of the $N_a(\boldsymbol{x})$ and $\mathrm{S}_a(\boldsymbol{x})$.

1.2.2. Plots of:

1.2.2.1. The probabilities for rejection $P_{N,a}(n,k,\mu,\sigma)$ and $P_{S,a}(n,k,\mu,\sigma)$ of the $N_a(\boldsymbol{x})$ and $\mathrm{S}_a(\boldsymbol{x})$.

1.2.2.2. Their difference.

1.2.2.3. Their relative difference.

1.2.2.4. Their ratio.

2. neural networks vs a statistical QC function

2.1. Input.

2.1.1. Plot options

2.1.1.1. x-axis

2.1.1.1.1. The standardized standard deviation (σ).

2.1.1.1.2. The standardized mean (μ).

2.1.1.2. The type of the plot:

2.1.1.2.1. The probabilities for rejection.

2.1.1.2.2. Their difference.

2.1.1.2.3. Their relative difference.

2.1.1.2.4. Their ratio.

2.1.2. QC parameters

2.1.2.1. The neural network $N_a(\boldsymbol{x})$, a ∈ {4,6,8,12,16}

2.1.2.2. The size *n* of the QC samples (1 ≤ *n* ≤ 4), for the $N_a(\boldsymbol{x})$.

2.1.2.3. The number *k* of the out of control QC measurements per sample (1 ≤ *k* ≤ *n*), for the $N_a(\boldsymbol{x})$.

2.1.2.4. The size *n* of the QC samples (1 ≤ *n* ≤ 4), for the statistical QC function $\mathrm{S}(\boldsymbol{x}; l)$.

2.1.2.5. The number *k* of the out of control QC measurements per sample (1 ≤ *k* ≤ *n*), for the $\mathrm{S}(\boldsymbol{x}; l)$.

2.1.2.6. The probability *pfr* for false rejection of the $\mathrm{S}(\boldsymbol{x}; l)$.

2.2. Output

2.2.1. Table of QC parameters

2.2.1.1. The decision limit *l* of $\mathrm{S}(\boldsymbol{x}; l)$.

2.2.1.2. The probabilities *pfr* for false rejection of the of the $N_a(\boldsymbol{x})$ and $\mathrm{S}_a(\boldsymbol{x})$.

2.2.2. Plots of:

2.2.2.1. The probabilities for rejection $P_{N,a}(n, k, \mu, \sigma)$ and $P_S(n, k, \mu, \sigma; l)$ of the $N_a(\boldsymbol{x})$ and $\mathrm{S}(\boldsymbol{x}; l)$.

2.2.2.2. Their difference.

2.2.2.3. Their relative difference.

2.2.2.4. Their ratio.

3. neural networks vs a neural networks QC function

3.1. Input.

3.1.1. Plot options

3.1.1.1. x-axis

3.1.1.1.1. The standardized standard deviation (*σ*).

3.1.1.1.2. The standardized mean (*μ*).

3.1.1.2. The type of the plot:

3.1.1.2.1. The probabilities for rejection.

3.1.1.2.2. Their difference.

3.1.1.2.3. Their relative difference.

3.1.1.2.4. Their ratio.

3.1.2. QC parameters

For each of the two neural networks QC functions $N_a(\boldsymbol{x})$.:

3.1.2.1. The type of neural network $N_a(\boldsymbol{x})$, $a \in \{4,6,8,12,16\}$.

3.1.2.2. The size *n* of the QC samples (1 ≤ *n* ≤ 4), for the $N_a(\boldsymbol{x})$.

3.1.2.3. The number *k* of the out of control QC measurements per sample (1 ≤ *k* ≤ *n*), for the $N_a(\boldsymbol{x})$.

3.2. Output

3.2.1. Table of QC parameters

3.2.1.1. The probability *pfr* for false rejection of each of the $N_a(\boldsymbol{x})$.

3.2.2. Plots of:

3.2.2.1. The probabilities for rejection $P_{N,a}(n,k,\mu,\sigma)$ of each of the $N_a(\boldsymbol{x})$.

3.2.2.2. Their difference.

3.2.2.3. Their relative difference.

3.2.2.4. Their ratio.

4. statistical vs a statistical QC function

4.1. Input.

4.1.1. Plot options

4.1.1.1. x-axis

4.1.1.1.1. The standardized standard deviation (σ).

4.1.1.1.2. The standardized mean (μ).

4.1.1.2. The type of the plot:

4.1.1.2.1. The probabilities for rejection of the QC functions.

4.1.1.2.2. Their difference.

4.1.1.2.3. Their relative difference.

4.1.1.2.4. Their ratio.

4.1.2. QC parameters

For each of the two statistical QC functions $\mathrm{S}(\boldsymbol{x};l)$:

4.1.2.1. The size *n* of the QC samples ($1 \le n \le 4$).

4.1.2.2. The number *k* of the out of control QC measurements per sample ($1 \le k \le n$).

4.1.2.3. The probability *pfr* for false rejection.

4.2. Output

4.2.1. Table of QC parameters

4.2.1.1. The decision limits *l* of $\mathrm{S}(\boldsymbol{x};l)$.

4.2.1.2. The probabilities *pfr* for false rejection of each of the $\mathrm{S}(\boldsymbol{x};l)$.

4.2.2. Plots of:

4.2.2.1. The probabilities for rejection $P_S(n,k,\mu,\sigma;l)$ of each of the $\mathrm{S}(\boldsymbol{x};l)$.

4.2.2.2. Their difference.

4.2.2.3. Their relative difference.

4.2.2.4. Their ratio.

QC Calculators

5. neural networks vs a statistical QC function with equal *pfr*

5.1. Input.

5.1.1. Parameters of of the measurement process

5.1.1.1. The standardized critical mean (m_c).

5.1.1.2. The standardized critical standard deviation (s_c).

5.1.2. QC parameters

The following are defined:

5.1.2.1. The type of the neural network $N_a(\boldsymbol{x})$, a $\in \{4,6,8,12,16\}$.

5.1.2.2. The size *n* of the QC samples (1 ≤ *n* ≤ 4).

5.1.2.3. The number *k* of the out of control QC measurements (1 ≤ *k* ≤ *n*).

5.2. Output

5.2.1. Table of QC parameters

5.2.1.1. The decision limit *l* of the statistical QC function $\mathrm{S}(\boldsymbol{x}; l) = \mathrm{S}_a(\boldsymbol{x})$.

5.2.1.2. The size *n* of the QC samples.

5.2.1.3. The number *k* of the out of control QC measurements per sample.

5.2.1.4. The probabilities for critical standard deviation s_c detection of the $N_a(\boldsymbol{x})$ and $\mathrm{S}_a(\boldsymbol{x})$.

5.2.1.5. The probabilities for critical mean m_c detection of the $N_a(\boldsymbol{x})$ and $\mathrm{S}_a(\boldsymbol{x})$.

5.2.1.6. The probability *pfr* for false rejection of of the $N_a(\boldsymbol{x})$ and $\mathrm{S}_a(\boldsymbol{x})$.

6. neural networks vs a statistical QC function

6.1. Input.

6.1.1. Parameters of of the measurement process

6.1.1.1. The standardized critical standard deviation (s_c)

6.1.1.2. The standardized critical mean (m_c)

6.1.2. QC parameters

6.1.2.1. The type of the neural network $N_a(\boldsymbol{x})$, a $\in \{4,6,8,12,16\}$

6.1.2.2. The size *n* of the QC sample (1 ≤ *n* ≤ 4), for the $N_a(\boldsymbol{x})$.

6.1.2.3. The number *k* of the out of control QC measurements per sample (1 ≤ *k* ≤ *n*), for the $N_a(\boldsymbol{x})$.

6.1.2.4. The size *n* of the QC samples (1 ≤ *n* ≤ 4), for the statistical QC function $\mathrm{S}(\boldsymbol{x}; l)$.

6.1.2.5. The number *k* of the out of control QC measurements per sample (1 ≤ *k* ≤ *n*), for the $\mathrm{S}(\boldsymbol{x}; l)$.

6.1.2.6. The probability *pfr* for false rejection of the $\mathrm{S}(\boldsymbol{x}; l)$.

6.2. Output

6.2.1. Table of QC parameters

6.2.1.1. The decision limit *l* of $\mathrm{S}(\boldsymbol{x}; l)$.

6.2.1.2. The size *n* of the QC samples.

6.2.1.3. The number *k* of the out of control QC measurements per sample.

6.2.1.4. The probabilities for critical standard deviation s_c detection of both QC functions.

6.2.1.5. The probabilities for critical mean m_c detection of both QC functions.

6.2.1.6. The probabilities *pfr* for false rejection of the $N_a(\boldsymbol{x})$ and $\mathrm{S}(\boldsymbol{x}; l)$.

6.2.2. Plots of:

6.2.2.1. The probabilities for rejection $P_{N,a}(n, k, \mu, \sigma)$ and $P_S(n, k, \mu, \sigma; l)$ of the $N_a(\boldsymbol{x})$ and $\mathrm{S}(\boldsymbol{x}; l)$.

6.2.2.2. Their difference.

6.2.2.3. Their relative difference.

6.2.2.4. Their ratio.

7. neural networks vs a neural networks QC function

7.1. Input.

7.1.1. Plot options

7.1.1.1. x-axis

7.1.1.1.1. The standardized mean (μ)

7.1.1.1.2. The standardized standard deviation (σ)

7.1.2. QC parameters

For each neural networks QC function:

7.1.2.1. The type of the neural network $N_a(\boldsymbol{x}), a \in \{4,6,8,12,16\}$

7.1.2.2. The size *n* of the QC samples ($1 \le n \le 4$), for the $N_a(\boldsymbol{x})$.

7.1.2.3. The number *k* of the out of control QC measurements per sample ($1 \le k \le n$), for the $N_a(\boldsymbol{x})$.

7.2. Output

7.2.1. Table of QC parameters

7.2.1.1. The size *n* of the QC samples.

7.2.1.2. The number *k* of the out of control QC measurements per sample.

7.2.1.3. The probabilities for critical standard deviation s_c detection of each of the $N_a(\boldsymbol{x})$.

7.2.1.4. The probabilities for critical mean m_c detection of each of the $N_a(\boldsymbol{x})$.

7.2.1.5. The probabilities *pfr* for false rejection of each of the $N_a(\boldsymbol{x})$.

8. statistical vs a statistical QC function

8.1. Input.

8.1.1. QC parameters

For each statistical QC function $\mathrm{S}(\boldsymbol{x}; l)$:

8.1.1.1. The size *n* of the QC samples ($1 \le n \le 4$).

8.1.1.2. The number *k* of the out of control QC measurements per sample ($1 \le k \le n$).

8.1.1.3. The probability *pfr* for false rejection.

8.2. Output

8.2.1. Table of QC parameters

8.2.1.1. The decision limits *l* of each of the $\mathrm{S}(\boldsymbol{x}; l)$.

8.2.1.2. The size *n* of the QC samples.

8.2.1.3. The number *k* of the out of control QC measurements per sample.

8.2.1.4. The probabilities for critical standard deviation s_c detection of each of the $\mathrm{S}(\boldsymbol{x}; l)$.

8.2.1.5. The probabilities for critical mean m_c detection of each of the $\mathrm{S}(\boldsymbol{x}; l)$.

8.2.1.6. The probabilities *pfr* for false rejection of each of the $\mathrm{S}(\boldsymbol{x}; l)$.

Source Code

Programming language: Wolfram Language

Availability: The updated source code is available at: https://www.hcsl.com/Tools/IntelligentQuality/IntelligentQuality.nb

Neural Networks

The designed neural networks models, are available in various formats at:
https://www.hcsl.com/Supplements/hcsltr21s1.zip

Software Requirements

Operating systems: Microsoft Windows, Linux, Apple iOS
Other software requirements: Wolfram Player®, freely available at: https://www.wolfram.com/player/ or Wolfram Mathematica®.

System Requirements

Processor: Intel Core i7® or equivalent CPU
System memory (RAM): 8 GB+ recommended.

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