

Hellenic Complex Systems Laboratory

Uncertainty of Measurement and Areas Over and Under the ROC Curves

Technical Report VI

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Uncertainty of Measurement and Areas Over and Under the ROC Curves

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Search Terms: ROC curves, uncertainty of measurement, sensitivity, specificity, diagnostic test, clinical accuracy, diagnostic accuracy, diagnostic inaccuracy, permissible uncertainty, normal distribution, binormal distribution, area under ROC curves, area over ROC curves, AUC, AOC, relative change, rate of change, relative rate of change

Abstract

This Demonstration plots the change, the relative change, the rate of change, and the relative rate of change of the area under (blue plot) and the area over (orange plot) the receiver operating characteristic (ROC) curve of a diagnostic test, as the uncertainty of measurement increases from 0 to a user defined upper bound. The test measures a measurand on normally distributed nondiseased and diseased populations, for various values of the mean and standard deviation of the populations. A normal distribution of the uncertainty is assumed. The type of plot is selected using the menu. The five parameters that can be varied using the sliders are measured in arbitrary units.

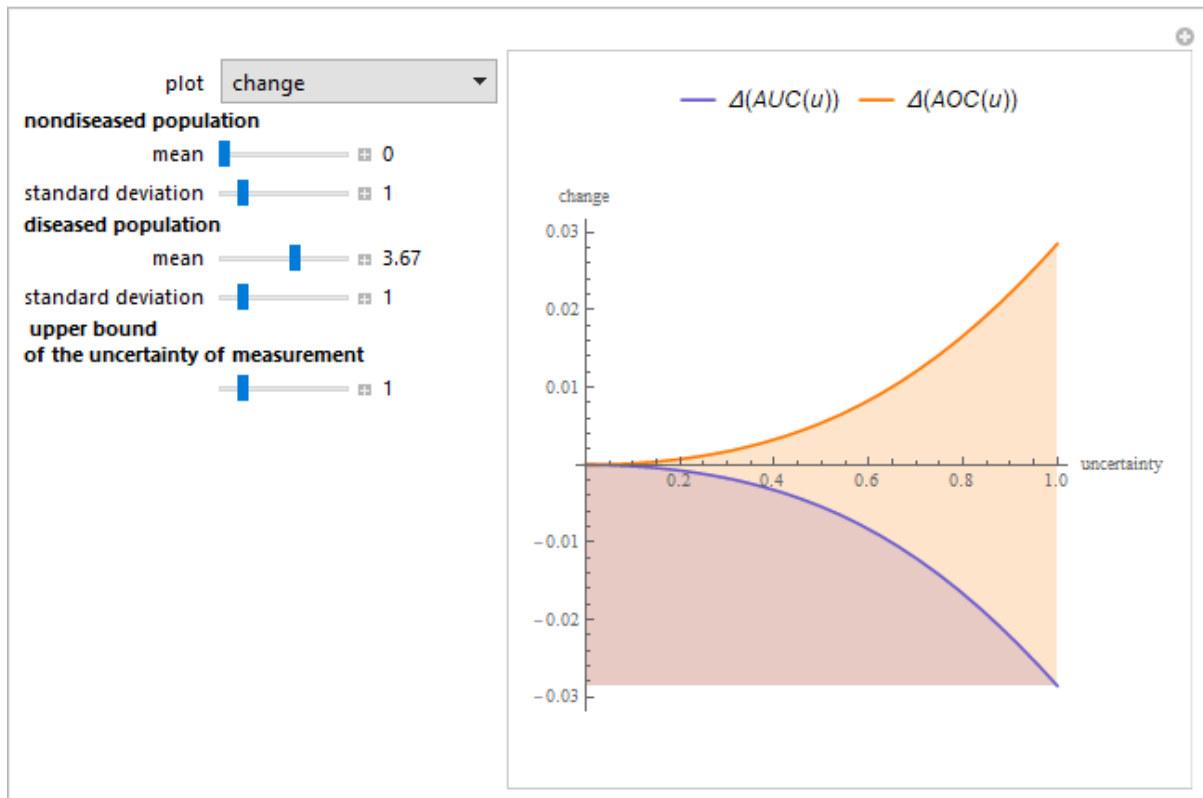


Figure 1: Plot of the change of the areas under (blue plot) and over (orange plot) the receiver operating characteristic (ROC) curve of a diagnostic test, when the measurement uncertainty u varies from 0.00 to 1.00 units. Mean and standard deviation of the measurand on the nondiseased population: 0.00 and 1.00 units respectively, mean and standard deviation of the measurand on the diseased population: 3.67 and 1.0 units respectively.

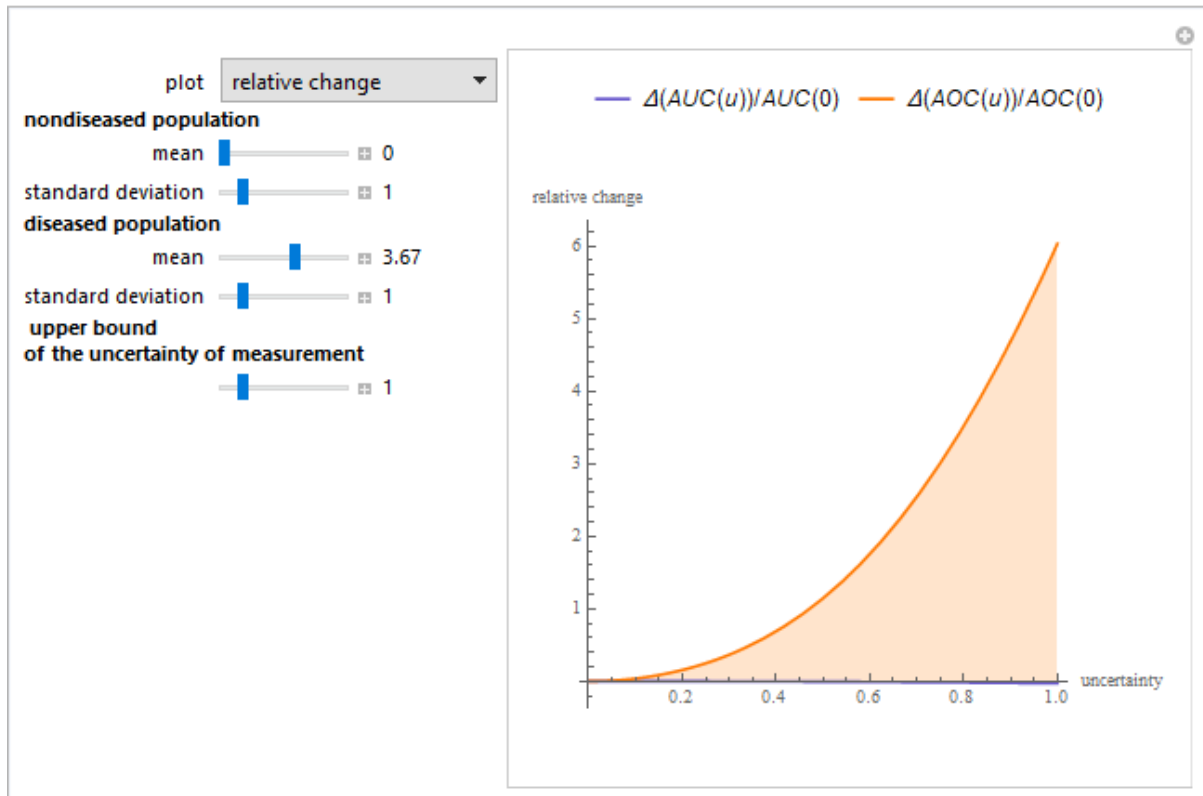


Figure 2: Plot of the relative change of the areas under (blue plot) and over (orange plot) the receiver operating characteristic (ROC) curve of a diagnostic test, when the measurement uncertainty u varies from 0.00 to 1.00 units. Mean and standard deviation of the measurand on the nondiseased population: 0.00 and 1.00 units respectively, mean and standard deviation of the measurand on the diseased population: 3.67 and 1.0 units respectively.

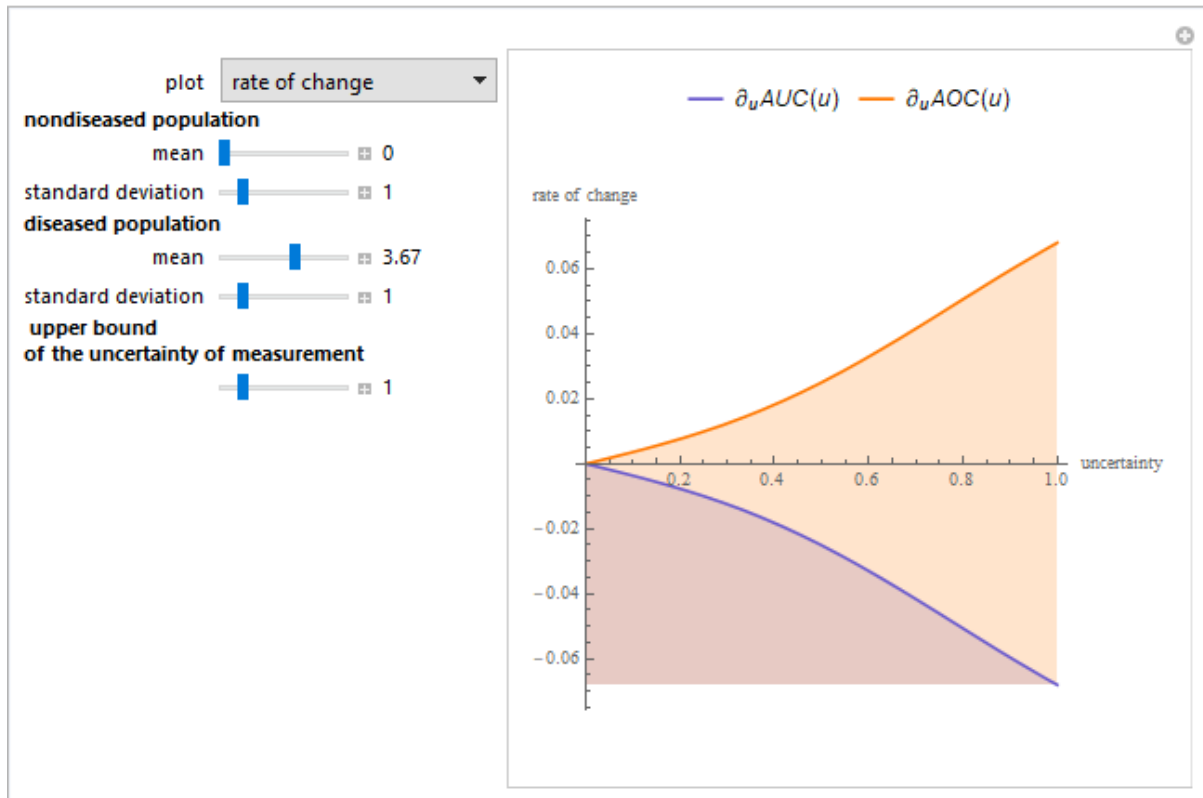


Figure 3: Plot of the rate of change of the areas under (blue plot) and over (orange plot) the receiver operating characteristic (ROC) curve of a diagnostic test, when the measurement uncertainty u varies from 0.00 to 1.00 units. Mean and standard deviation of the measurand on the nondiseased population: 0.00 and 1.00 units respectively, mean and standard deviation of the measurand on the diseased population: 3.67 and 1.0 units respectively.

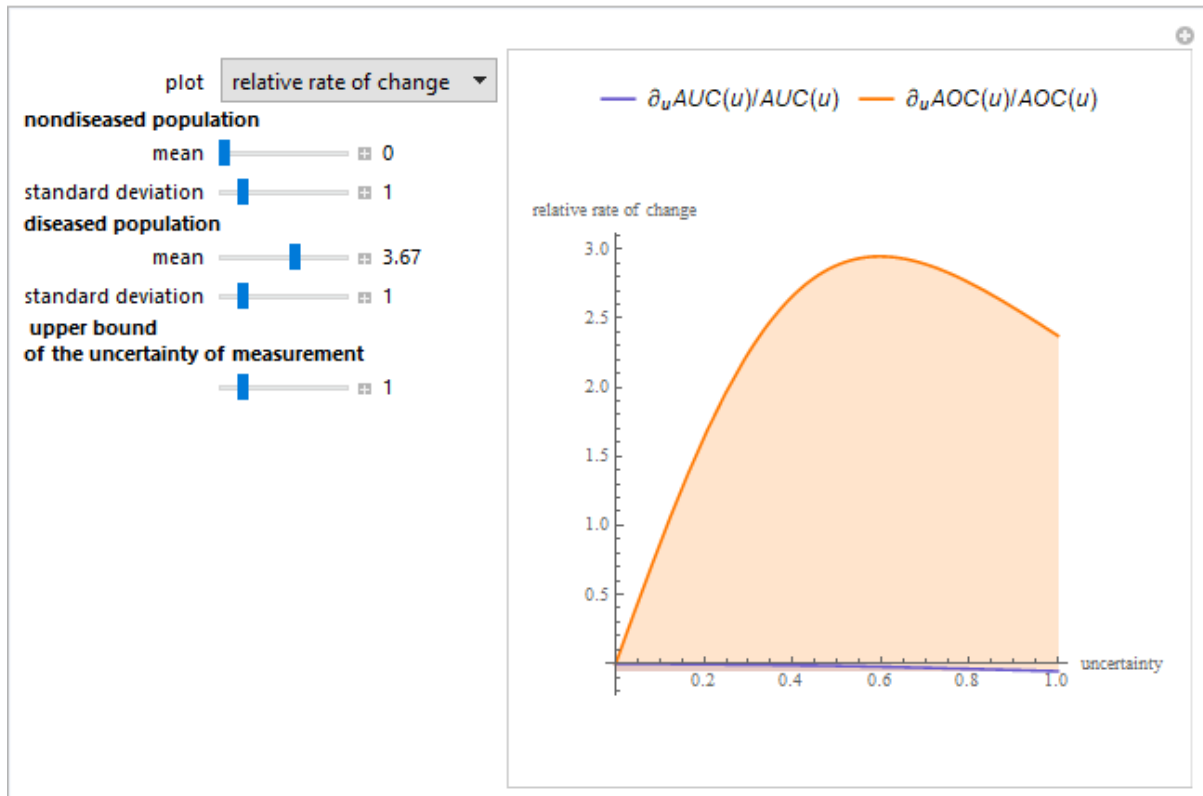


Figure 4: Plot of the relative rate of change of the areas under (blue plot) and over (orange plot) the receiver operating characteristic (ROC) curve of a diagnostic test, when the measurement uncertainty u varies from 0.00 to 1.00 units. Mean and standard deviation of the measurand on the nondiseased population: 0.00 and 1.00 units respectively, mean and standard deviation of the measurand on the diseased population: 3.67 and 1.0 units respectively.

Details

The ROC curves are used in the evaluation of the clinical accuracy of a diagnostic test applied to a diseased and a nondiseased population. They display the sensitivity of the test versus 1-specificity. Sensitivity is the fraction of the diseased population with a positive test, while specificity is the fraction of the nondiseased population with a negative test. Therefore, the ROC curves display the true positive fraction versus the false positive fraction. The uncertainty of measurement of the diagnostic test effects the ROC curve [1].

Let us denote as $AUC(u)$ the area under a ROC curve and $AOC(u)$ the area over the ROC curve for an uncertainty of measurement u . We define the changes $\Delta(AUC(u))$ and $\Delta(AOC(u))$ as $AUC(u) - AUC(0)$ and $AOC(u) - AOC(0)$, the relative changes as $\Delta(AUC(u))/AUC(0)$ and $\Delta(AOC(u))/AOC(0)$, the rates of change as $\frac{\partial AUC(u)}{\partial u}$ and $\frac{\partial AOC(u)}{\partial u}$, and the relative rates of change as $\frac{\frac{\partial AUC(u)}{\partial u}}{AUC(u)}$ and as $\frac{\frac{\partial AOC(u)}{\partial u}}{AOC(u)}$ respectively.

The area under a ROC curve is used as an index of the diagnostic accuracy of the respective test. As $AOC(u) = 1 - AUC(u)$, we can consider the area over the ROC curve as an index of its diagnostic inaccuracy. Actually the plots show that the relative change, the rate of change, and the relative rate of change of the area over a ROC curve versus the uncertainty of measurement are greater than the respective measures of the area under the ROC curve, for the same populations.

This Demonstration could be useful in evaluating the maximum medically permissible uncertainty of measurement of a diagnostic test using the area over the ROC curve as an inaccuracy index. For example, in the thumbnail and the snapshots the populations data describe a bimodal distribution of serum glucose measurements with a non diabetic and a diabetic population [1].

References:

1. Hatjimihail AT. Receiver Operating Characteristic Curves and Uncertainty of Measurement. Wolfram Demonstrations Project, Champaign: Wolfram Research, Inc., 2009.
 2. T. O. Lim, R. Bakri, Z. Morad, and M. A. Hamid, "Bimodality in Blood Glucose Distribution—Is It Universal?," *Diabetes Care*, **25**(12), 2002 pp. 2212-2217. Reference
- [1] T. O. Lim, R. Bakri, Z. Morad, and M. A. Hamid. Bimodality in Blood Glucose Distribution: Is It Universal? *Diabetes Care*, **25**(12), 2002 pp. 2212–2217.

Source Code

The updated Wolfram Mathematica® source code is available at:

<https://www.hcsl.com/Tools/UncertaintyOfMeasurementAndAreasOverAndUnderTheROCCurves-author.nb>

Permanent Citation of the Demonstration:

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