Hellenic Complex Systems Laboratory

# Quality: A Software Tool for Statistical Quality Control Design

Technical Report XX

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# *Quality*: A Software Tool for Statistical Quality Control Design

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Search Terms: quality control, quality control design, measurement, total allowable analytical error, fraction nonconforming, critical systematic error, critical random error, decision limits, probability for rejection, probability for false rejection

## Introduction

Alternative quality control (QC) rules and procedures can be applied to a process to test statistically the null hypothesis, that the process conforms to the quality specifications and consequently is in control, against the alternative, that the process is out of control. When a true null hypothesis is rejected, a statistical type I error is committed. We have then a false rejection of a run of the process. The probability of a type I error is called probability of false rejection. When a false null hypothesis is accepted, a statistical type II error is committed. We fail then to detect a significant change in the probability density function of a quality characteristic of the process. The probability of rejection of a false null hypothesis equals the probability of detection of the nonconformity of the process to the quality specifications.

The program *Quality* was developed for designing and exploring statistical QC rules to meet defined quality specifications.

### The program modules

The program provides two modules:

#### 1. *Design*

This module estimates various parameters of a measurement process and designs the QC rule to be applied and plots the probability density function (pdf) and the cumulative density function (cdf) of the control measurements, and the probability for rejection for the random error and systematic error of the statistical QC rule.

#### 2. *Compare*

This module explores and compares alternative statistical QC rules $S(\boldsymbol{x}; m, s, l_i)$ (see Notation) applied on tuples $\boldsymbol{x}$ of $n_i$ control measurements, $k_i$ of them distributed as $\mathcal{N}(m, s^2)$, and $n_i$-$k_i$ as $\mathcal{N}(0,1)$, plotting the probabilities for rejection $P(n_i, k_i, m, s; 0,1, l_i)$ of the two statistical QC rules, their difference, relative difference and ratio.

### The program interface

#### 1. Design

##### 1.1. *Input.*

###### 1.1.1. Parameters of the QC measurements (in arbitrary measurement units)

1.1.1.1. The assigned mean $m_0$ $(0.01 \leq m_0 \leq 1000.0)$, 1.1.1.2. The observed mean $m$ when the process is in control ($0.01 \leq m \leq 1000.0$), and

1.1.1.3. The standard deviation $s$ when the process is in control ($0.01 \leq s \leq 1000.0$).

### 1.1.2. Quality specifications of the measurement process

1.1.2.1. The total allowable error $t$ (as a percentage of the assigned mean) ($0.01 \leq t \leq 200.0$) ,

1.1.2.2. The maximum acceptable fraction $f_{max}$ of measurements nonconforming to the specifications ($0.01 \leq f_{max} \leq 1.0$),

1.1.2.3. The minimum acceptable probability $p_{s_c}$ for detection of the critical random error ($0.01 \leq p_{s_c} \leq 0.99$).

1.1.2.4. The minimum acceptable probability $p_{m_c}$ for detection of the critical systematic error ($0.01 \leq p_{m_c} \leq 0.99$).

1.1.2.5. The number $n$ of control measurements ($1 \leq n \leq 6$).

## *1.2. Output*

### 1.2.1. Table of QC parameters

1.2.1.1. The fraction nonconforming $f$,

1.2.1.2. The critical random error $s_c$ ,

1.2.1.3. The critical systematic error $m_c$ ,

1.2.1.4. The QC decision limits $m \pm l\,s$ ,

1.2.1.5. The probability $p_{s_c}$ for critical random error detection,

1.2.1.6. The probability $p_{m_c}$ for critical systematic error detection

1.2.1.7. The probability $p_{fr}$ for false rejection

### 1.2.2. Plots

1.2.2.1. The probability density function (pdf) plot of the QC measurements,

1.2.2.2. The cumulative density function (cdf) plot of the QC measurements,

1.2.2.3. the probability for rejection plot for the random error ($p_r$ vs $s$), and

1.2.2.4. the probability for rejection plot for the systematic error ($p_r$ vs $m$).

The parameters are estimated and the functions are plotted if $10^{-100} \leq f \leq f_{max}$

# 2. Compare

## *2.1. Input.*

### 2.1.1. Plot options

#### *2.1.1.1. Type of error:*

2.1.1.1.1. Random error

2.1.1.1.2. Systematic error

#### 2.1.1.2. *Type of plot*:

2.1.1.2.1. The probabilities for rejection of both QC rules

2.1.1.2.2. The difference between the probabilities for rejection

2.1.1.2.3. The relative difference between the probabilities for rejection

2.1.1.2.4. The ratio of the probabilities for rejection

#### *2.1.1.3. Plot range*

2.1.1.3.1. The range $r$ of the $y$-axis (1.0 – 10.0)

### 2.1.2. QC parameters

The following parameters are defined for two alternative QC procedures:

2.1.2.1. The number $n$ of the control measurements ($1 \leq n \leq 6$).

2.1.2.2. The number $k$ of the control measurements distributed as $\mathcal{N}(m, s^2)$

2.1.2.3. Either:

2.1.2.3.1. The probability $p_{fr}$ for false rejection ($0.000001 \leq p_{fr} \leq 0.1$) or

2.1.2.3.2. The QC decision limit $l$ ($1.0 \leq l \leq 4.0$)

## *2.2. Output*

### 2.2.1. Table of QC parameters

2.2.1.1. The QC decision limit $l\,s$,

2.2.1.2. The probability $p_0$ for false rejection

### 2.2.2. Plots

2.2.2.1. The probabilities for rejection of both QC rules

2.2.2.2. The difference between the probabilities for rejection

2.2.2.3. The relative difference between the probabilities for rejection

2.2.2.4. The ratio of the probabilities for rejection

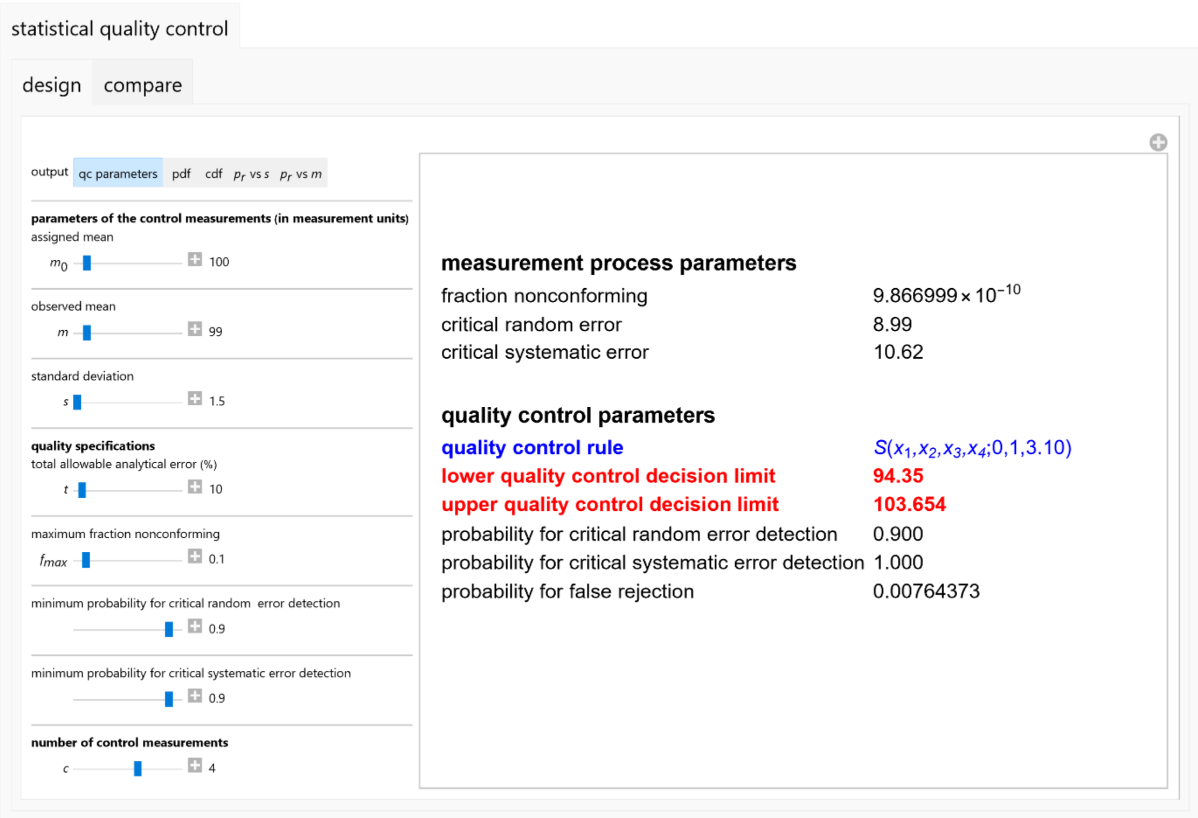


Figure 1: *The measurement process parameters and the QC parameters of a measurement process, with the settings shown at the left.*

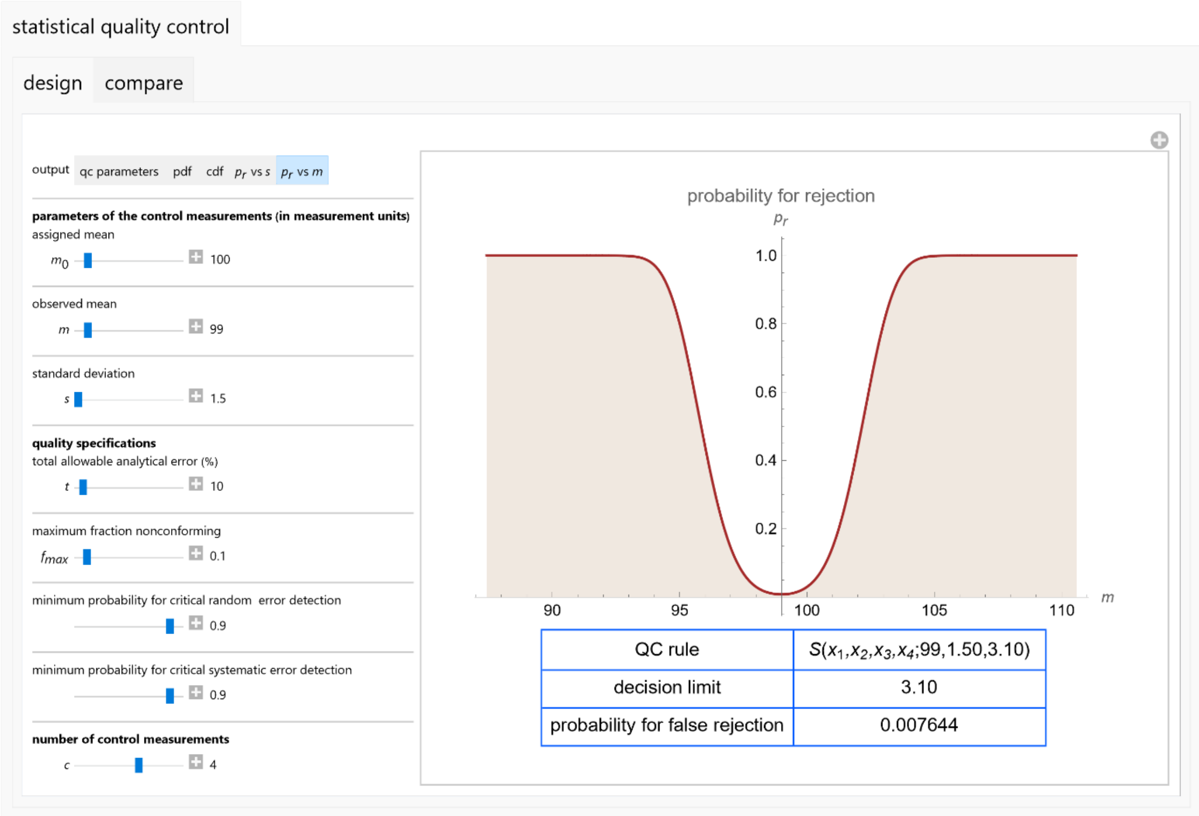


Figure 2: *The probability for rejection of a QC rule vs* the mean $m$ *of the control measurements, with the settings shown at the left.*

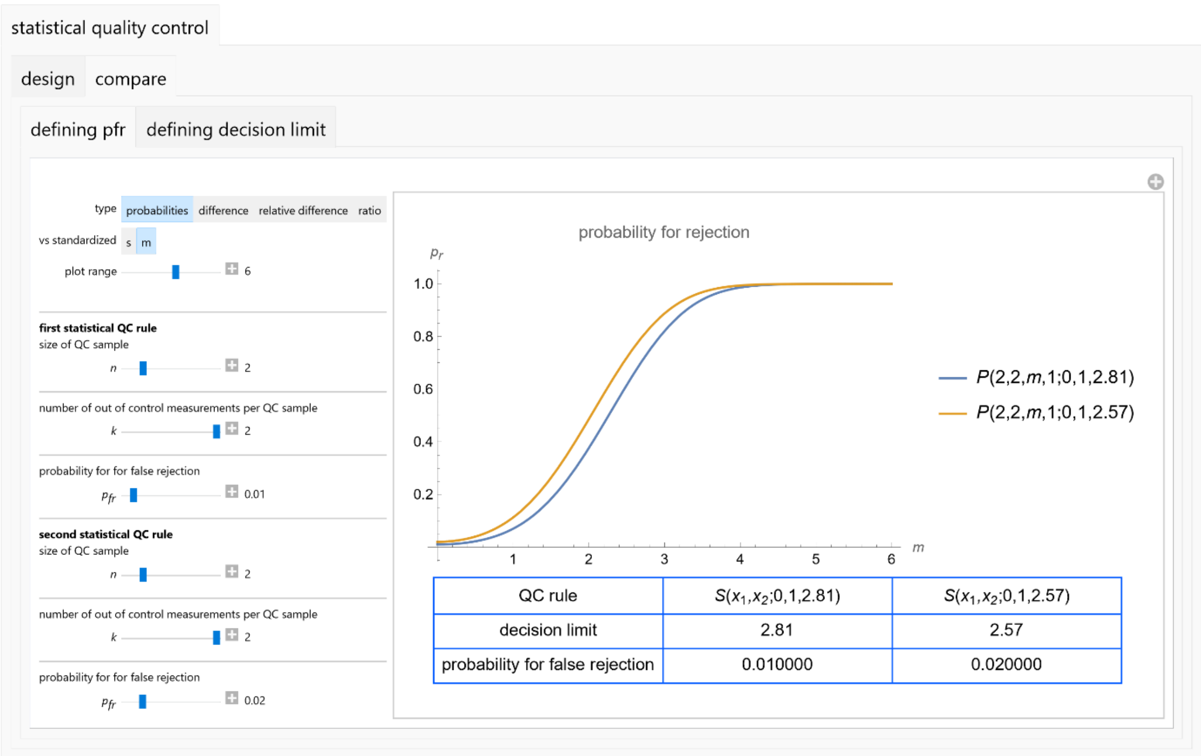


Figure 3: *The probabilities for rejection of two alternative QC rules vs* the standardized observed mean $m$ *of the control measurements, with the settings shown at the left.*

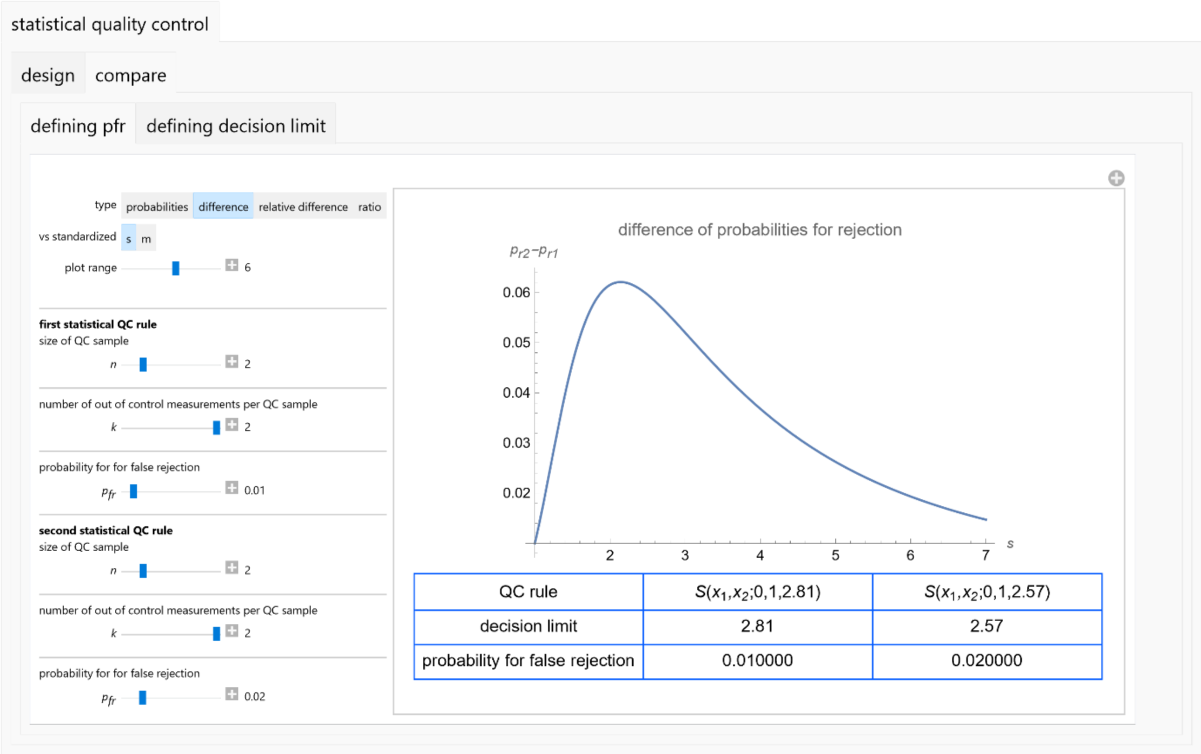


Figure 4: *The difference probabilities for rejection of two alternative QC rules vs* the standardized standard deviation $s$ *of the control measurements, with the settings shown at the left.*

## Details

Let $m_0$, $m$, and $s$ be the assigned mean, the observed mean, and the standard deviation of the control measurements. Let $t$ be the total allowable analytical error (expressed as percentage of the assigned mean). Then the following equations are used to estimate the respective parameters [1, 2, 3]:

(a) the fraction nonconforming $f$:

$$f = 1 - \int_{m_0\left(1-\frac{t}{100}\right)}^{m_0\left(1+\frac{t}{100}\right)} \left( \frac{e^{\frac{(-m+z)^2}{2s^2}}}{s\sqrt{2\pi}} \right) dz$$

(b) the critical random error $s_c$:

$$1 - \int_{m_0\left(1-\frac{t}{100}\right)}^{m_0\left(1+\frac{t}{100}\right)} \left( \frac{e^{\frac{(-m+z)^2}{2s_c^2}}}{s_c\sqrt{2\pi}} \right) dz = f_{max}$$

(c) the critical systematic error $m_c$:

$$\int_{m_0\left(1-\frac{t}{100}\right)}^{m_0\left(1+\frac{t}{100}\right)} \left( \frac{e^{\frac{(\epsilon\, m_c-m+z)^2}{2s^2}}}{s\sqrt{2\pi}} \right) dz = f_{max}$$

where $\epsilon=1$ if $m_o < m_a$ and $\epsilon = -1$ otherwise,

(d) the factor $l$ of the decision limits $m \pm l\,s$ of the QC rule $S(\boldsymbol{x}; m, s, l\,)$ is the minimum solution of both of the following two equations for the variable $l$:

$$\left(\int_{m-l\,s}^{m+l\,s}\left(\frac{e^{\frac{(-m+z)^2}{2s_c^2}}}{s_c\sqrt{2\pi}}\right)dz\right)^n = p_{s_c}$$

$$\left(\int_{m-l\,s}^{m+l\,s}\left(\frac{e^{\frac{(m_c-m+z)^2}{2s^2}}}{s\sqrt{2\pi}}\right)dz\right)^n = p_{m_c}$$

with $\epsilon$ as before in (c),

(e) the probability for false rejection of the QC rule $S\,(\boldsymbol{x}; m, s, l\,)$:

$$P(n, n, m, s; m, s, l\,) = 1 - \left(\int_{m-l\,s}^{m+l\,s}\left(\frac{e^{\frac{(-m+z)^2}{2s^2}}}{s\sqrt{2\pi}}\right)dz\right)^n$$

(f) the probability for rejection for the random error $s_e$ and the systematic error $m_e$ of the QC rule $S\,(\boldsymbol{x}; m, s, l\,)$:

$$P(n, n, m_e, s_e, m, s, l\,) = 1 - \left(\int_{m-l\,s}^{m+l\,s}\left(\frac{e^{\frac{(m_e-m+z)^2}{2s_e{}^2}}}{s_e\sqrt{2\pi}}\right)dz\right)^n$$

$$P(n, k, m_e, s_e; m, s, l) = P(k, k, m_e, s_e; m, s, l\,) + P(n-k, n-k, m, s; m, s, l\,) -$$

$$P(k, k, m_e, s_e; m, s, l\,)P(n-k, n-k, m, s; m, s, l\,)$$

## Conclusion

The program *Quality* can be used as an educational or laboratory tool for the design and evaluation of alternative QC rules for a measurement process.

## References

[1] A. T. Hatjimihail, A Tool for the Design and Evaluation of Alternative Quality Control Procedures, Clinical Chemistry 38, 1992 pp. 204–210.

[2] A.T. Hatjimihail. Tool for Quality Control Design and Evaluation, Wolfram Demonstrations Project, Champaign: Wolfram Research, Inc., 2010.

[3] A. T. Hatjimihail, Estimation of the Optimal Statistical Quality Control Sampling Time Intervals Using a Residual Risk Measure, PLoS ONE 4(6), 2009 p. e5770.

## Notation

$\boldsymbol{x}$ : $n$-tuple $(x_1, x_2, \dots, x_n)$ of control measurements

$m_0$: the assigned mean of the control measurements

$m$ : the observed mean of the control measurements

$s$ : the observed standard deviation of the control measurements

$m_e$ : the observed mean of the control measurements when the measurement process is out of control

$s_e$ : the observed standard deviation of the control measurements when the measurement process is out of control

$s_c$ : the critical random error

$m_c$ : the critical systematic error

$p_r$ : probability for rejection

$p_{fr}$ : probability for false rejection

$p_{s_c}$ : the probability for critical random error detection

$p_{m_c}$ : the probability for critical systematic error detection

$S(\boldsymbol{x}; m, s, l)$: statistical QC rule with decision limits $m \pm l\, s$, applied on $\boldsymbol{x}$. The rule rejects an analytical run if: $\exists\, x_i \in \boldsymbol{x}: x_i < m - ls \lor x_i > m + l$

$\mathcal{N}(m, s^2)$ : normal distribution with mean $m$ and standard deviation $s$.

$P_s(n, k, m_e, s_e; m, s, l)$ : probability for rejection of the QC rule $S(\boldsymbol{x}; m, s, l)$ applied on a $n$-tuple $\boldsymbol{x}$ of control measurements, $k$ of them distributed as $\mathcal{N}(m_e, s_e^2)$ and $n$-$k$ as $\mathcal{N}(m, s^2)$

## Source Code

Programming language: Wolfram Language

Availability: The updated source code is available at: https://www.hcsl.com/Tools/Quality/Quality.nb

## Software Requirements

Operating systems: Microsoft Windows, Linux, Apple iOS

Other software requirements: Wolfram Player®, freely available at: https://www.wolfram.com/player/ or Wolfram Mathematica®.

## System Requirements

*Processor*: Intel Core i7® or equivalent CPU

*System memory (RAM)*: 8 GB+ recommended.

## Permanent Citation:

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