Hellenic Complex Systems Laboratory

Intelligent Quality: A Software Tool for Exploring the Application of Convolutional Neural Networks to Quality Control Samples of Very Small Size

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Intelligent Quality: A Software Tool for Exploring the Application of Convolutional Neural Networks to Quality Control Samples of Very Small Size

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Search Terms: quality control, convolutional neural networks, probability for rejection, probability for false rejection, quality control measurement

Introduction

Although there is extensive literature on the application of artificial neural networks (NNs) in quality control (QC) to monitor the conformity of a process to quality specifications, at least five QC measurements are required, increasing the related cost. To explore the application of NNs to samples of QC measurements of very small size, four one-dimensional (1-D) convolutional NNs (CNNs) were designed, trained, and tested with datasets of n -tuples of simulated standardized normally distributed QC measurements for $1 \leq n \leq 4$.

The designed CNNs were compared to statistical single-value (SSV) QC functions with equal probabilities for false rejection (*pfr*), applied to samples of the same size. When the n -tuples included at least two QC measurements distributed as $\mathcal{N}(\mu, \sigma^2)$, where $0.2 < |\mu| \leq 6.0$, and $1.0 < \sigma \leq 7.0$, the designed CNNs outperformed the respective QC functions. Therefore, 1-D CNNs applied to samples of 2-4 quality control measurements can replace SSV QC functions to increase the probability of detection of the nonconformity of a process to the quality specifications with lower cost.

The program *Intelligent Quality* was developed to explore the application of the trained CNNs to QC samples of very small size and compare them to SSV QC functions.

The program

The program provides two modules:

QC Plots

This module plots the probabilities for rejection of the application of NN and SSV QC functions to the respective datasets, their differences, relative differences, and ratios (see Figures 1 and 2).

QC Calculators

This module calculates the probabilities for rejection of the application of NN and SSV QC functions for a user defined critical mean and standard deviation of the measurement process and their probabilities for false rejection (see Figures 3 and 4).

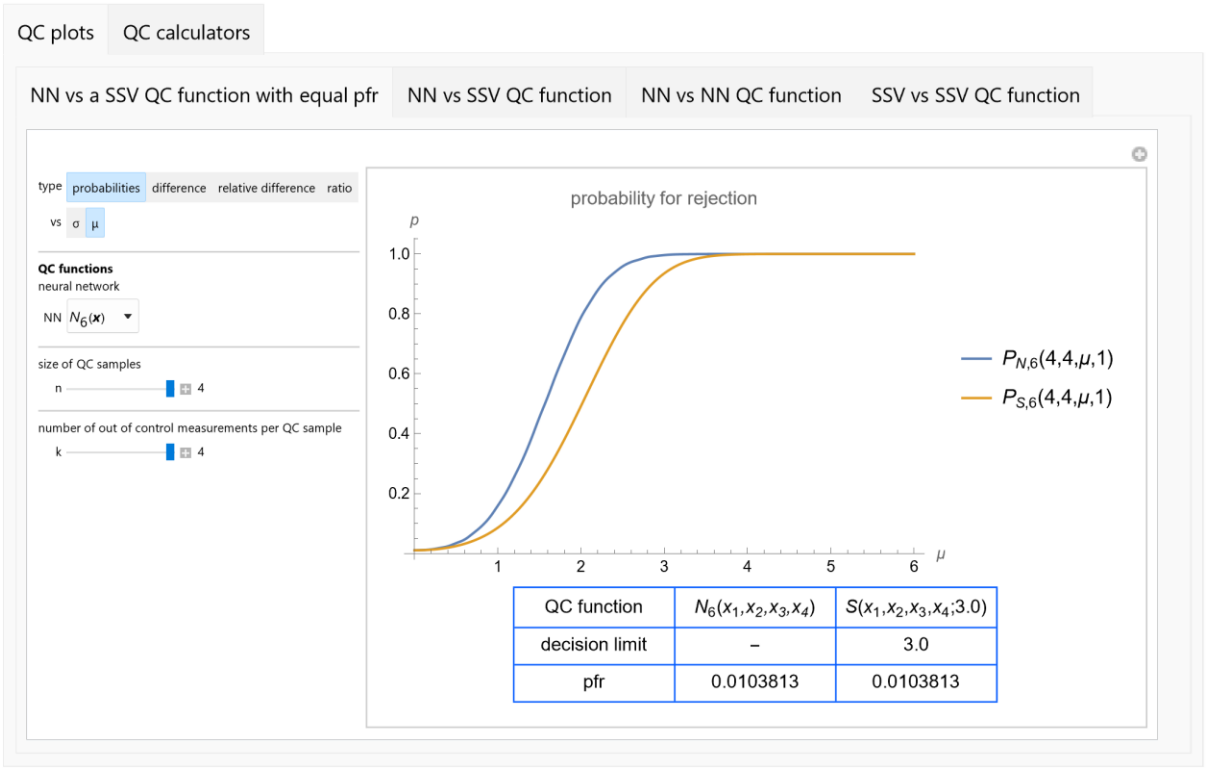


Figure 1: The plots of the probabilities for rejection $P_{N,6}(4,4,\mu,1)$ and $P_{S,6}(4,4,\mu,1)$ of the $N_8(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4)$ and $\mathrm{S}_6(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4) = \mathrm{S}(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4;3.0)$ QC functions, with the settings shown at the left.

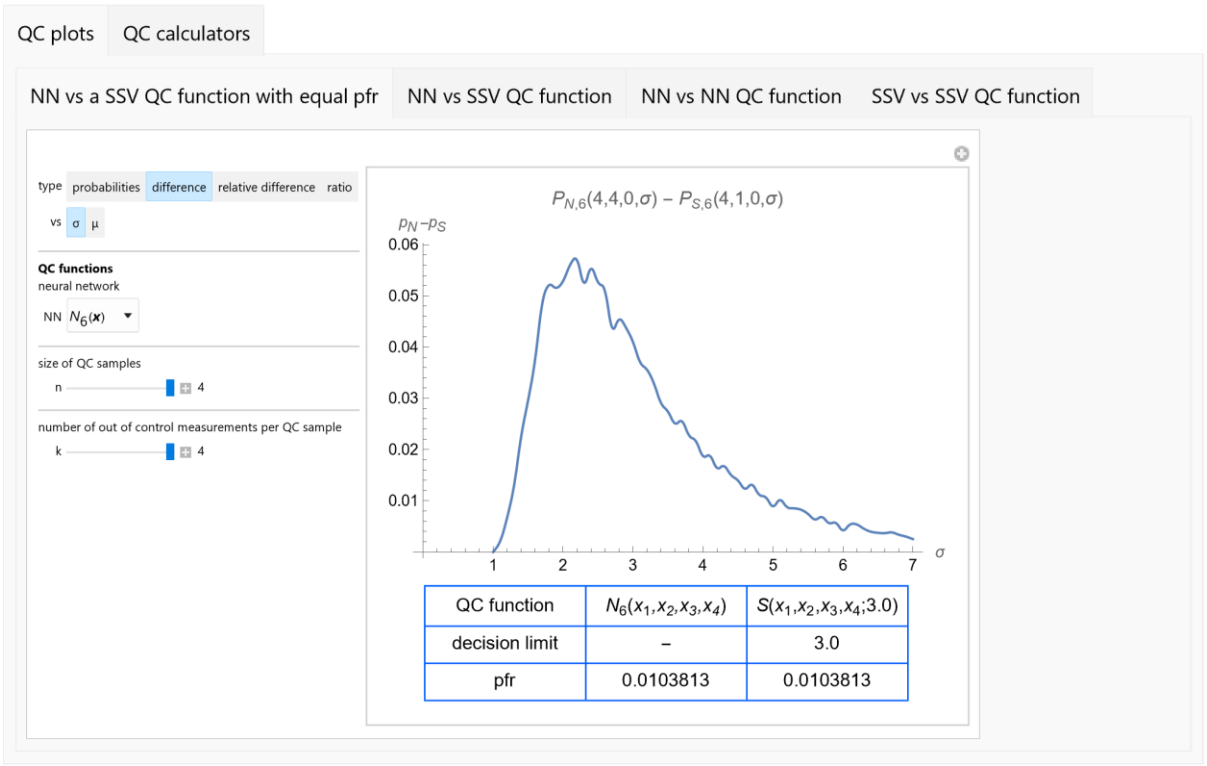
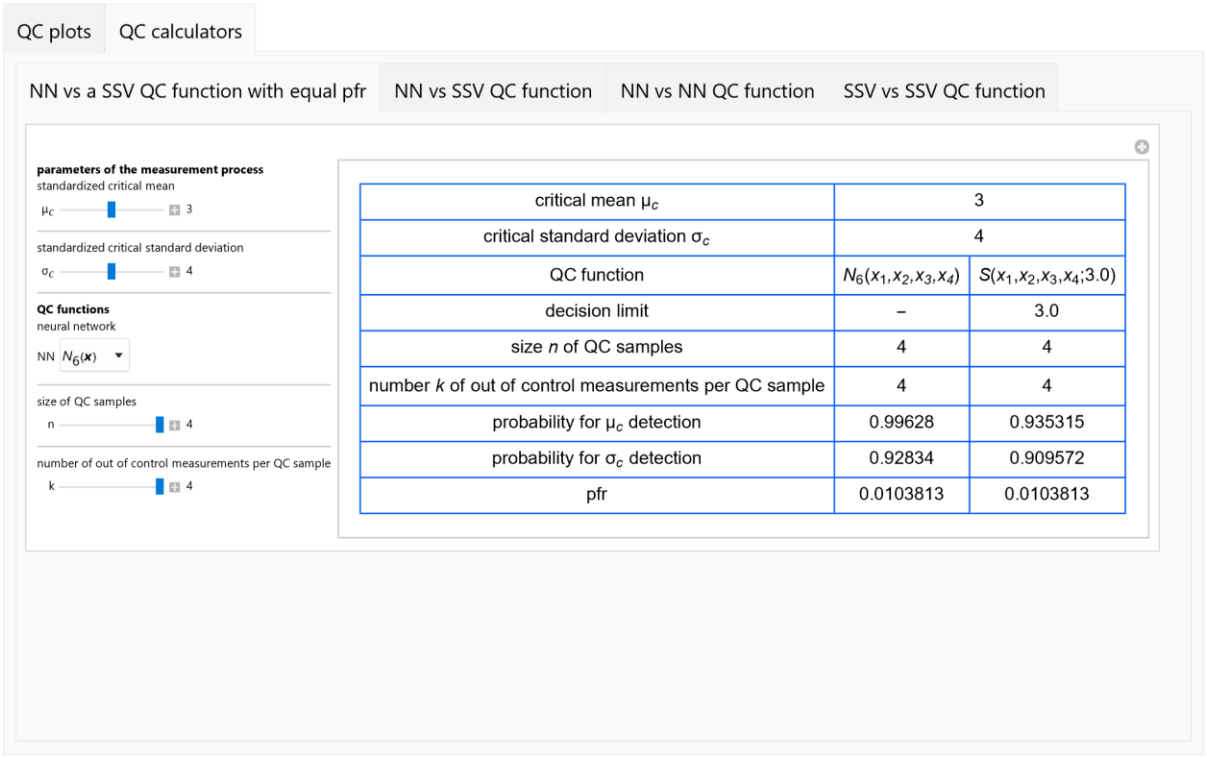
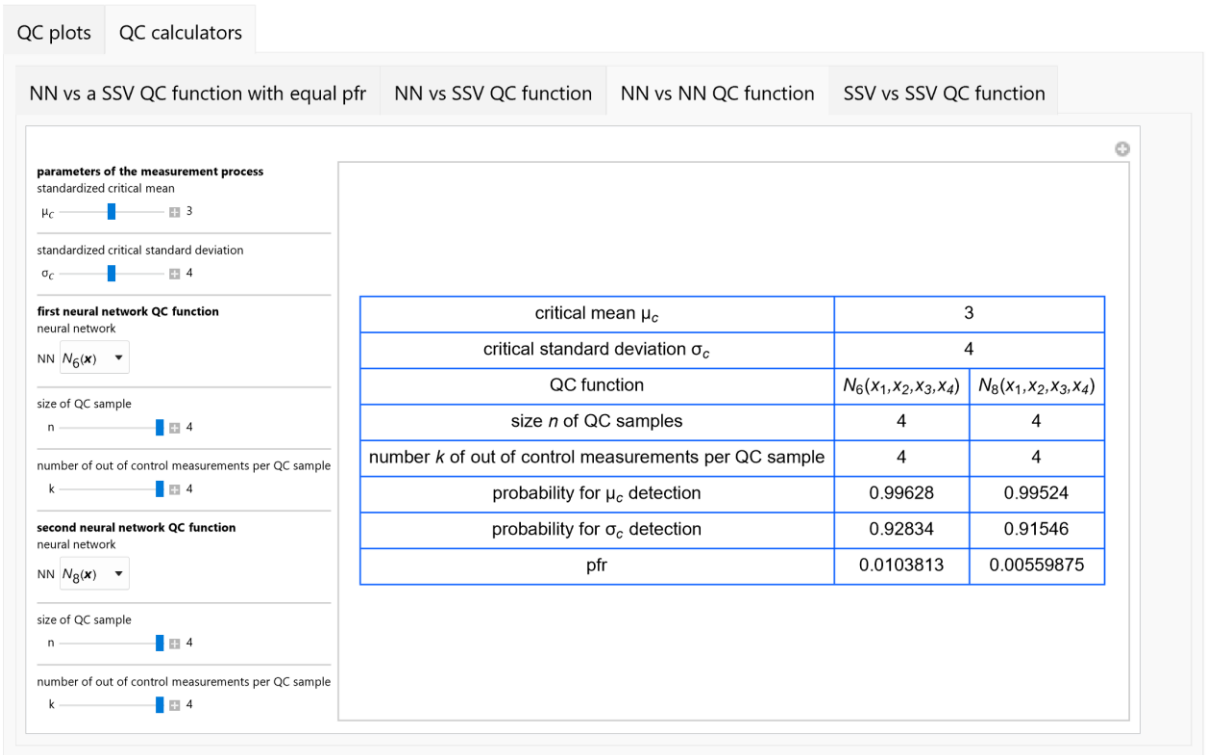


Figure 2: The plot of the difference of the probabilities for rejection $P_{N,6}(2,2,0,\sigma)$ and $P_{N,8}(2,2,0,\sigma)$ of the $N_6(\mathrm{x}_1,\mathrm{x}_2)$ and $N_8(\mathrm{x}_1,\mathrm{x}_2)$ QC functions, with the settings shown at the left.



critical mean μ_c	3	
critical standard deviation σ_c	4	
QC function	$N_6(x_1,x_2,x_3,x_4)$	$S(x_1,x_2,x_3,x_4;3.0)$
decision limit	–	3.0
size *n* of QC samples	4	4
number *k* of out of control measurements per QC sample	4	4
probability for μ_c detection	0.99628	0.935315
probability for σ_c detection	0.92834	0.909572
pfr	0.0103813	0.0103813

Figure 3: The table presenting the probabilities for rejection of the critical mean and standard deviation of the $N_6(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4)$ and $\mathrm{S}_6(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4) = S(\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4;3.0)$ QC functions.



critical mean μ_c	3	
critical standard deviation σ_c	4	
QC function	$N_6(x_1,x_2,x_3,x_4)$	$N_8(x_1,x_2,x_3,x_4)$
size *n* of QC samples	4	4
number *k* of out of control measurements per QC sample	4	4
probability for μ_c detection	0.99628	0.99524
probability for σ_c detection	0.92834	0.91546
pfr	0.0103813	0.00559875

Figure 4: The table presenting the probabilities for rejection of the critical mean and standard deviation of the $N_4(\mathrm{x}_1,\mathrm{x}_2)$ and $N_8(\mathrm{x}_1,\mathrm{x}_2)$ QC functions, with the settings shown at the left.

Details

Four 1-D CNNs were designed, trained, and tested with datasets of *n* -tuples of simulated standardized normally distributed QC measurements for 1 ≤ *n* ≤ 4, as described in [1].

For $a \in \{1,2,3,4,6,8,12,16\}$, $n = \#\boldsymbol{x}$ *(*that is *n* equal to the cardinality of $\mathbf{x}$*) and* $1 \leq n \leq 4$, each 1-D CNN $N_{\alpha}(\boldsymbol{x})$ consists of:

1. The input layer, with *n* nodes.
2. Ten parallel arrays of five convolutional layers each. Every convolutional layer has *n* output channels and kernels of size 1x1.
1. A catenating layer, which catenates the ten parallel arrays.
2. A pooling layer, performing 1-D pooling with kernels of size 2x1.
3. Ten convolutional net layers having *n* output channels and kernels of size 1x1.
4. A pooling net layer, performing 1-D pooling with kernels of size 2x1.
5. Four linear net layers with output vectors of decreasing size.
6. A softmax net layer, normalizing the exponential of the output vector of the fourth linear layer.
7. The output layer which is a decoder with two nodes.

Let C be the probability for rejection of the SSV QC function $S(\boldsymbol{x}; l)$ applied to a n -tuple $\boldsymbol{x}$ of QC

measurements, for $n = \#\boldsymbol{x}$, $1 \leq n \leq 4$, $1 \leq k \leq n$, where k of them are distributed as $\mathcal{N}(\mu, \sigma^2)$ and (n -

k *)* as $\mathcal{N}(0,1)$. Then:

$$P_S(n,k,\mu,\sigma;l) = 1 - \frac{1}{2^k}\operatorname{erf}\left(\frac{1}{\sqrt{2}}\right)^{n-k}\left(\operatorname{erf}\left(\frac{\mu+l}{\sigma\sqrt{2}}\right) - \operatorname{erf}\left(\frac{\mu-l}{\sigma\sqrt{2}}\right)\right)^k \text{ for } 0 > k \leq n$$

Given that $P_S(n,k,0,1;l) = p$, to define l, we solve the following equation:

$$P_S(n,k,0,1;l) = p \Rightarrow l = \sqrt{2}\operatorname{erf}^{-1}\left((1-p)^{\frac{1}{n}}\right)$$

Therefore:

$$P_{S,a}(n,k,\mu,\sigma) = P_S\left(n,k,,\mu,\sigma;\sqrt{2}\operatorname{erf}^{-1}\left(\left(1 - P_{N,a}(n,n,0,1)\right)^{\frac{1}{n}}\right)\right)$$

The program *Intelligent Quality* compares each designed CNN $N_a(\boldsymbol{x})$:

1. To SSV QC functions $S_a(\boldsymbol{x})$ with equal pfr, applied to samples of the same size.
2. To SSV QC functions $S(\boldsymbol{x}; l)$ with user defined *pfr.*
3. To other designed CNN $N_a(\boldsymbol{x})$.

In addition, SSV QC functions $S(\boldsymbol{x}; l)$ with user defined *pfr* can be compared to each other.

Conclusion

The program *Intelligent Quality* can be used as an educational or laboratory tool to evaluate and compare the designed CNN and SSV QC functions.

References

1. Chatzimichail RA, Hatjimihail AT. Quality Control Using Convolutional Neural Networks Applied to Samples of Very Small Size. Technical Report XXI. Drama: Hellenic Complex Systems Laboratory, 2022. Available at: https://www.hcsl.com/Documents/hcsltr21.pdf

Abbreviations

QC: quality control
NN: neural network(s)
CNN: convolutional neural networks
SSV: statistical single-value (QC function)

Notation

$\boldsymbol{x}$: *n*-tuple $(x_1, x_2, \dots, x_n)$ of QC measurements
$\#\boldsymbol{x}$: the cardinality of $\boldsymbol{x}$
s_c : the critical random error
m_c : the critical systematic error
$\mathcal{N}(\mu, \sigma^2)$: normal distribution with mean μ and standard deviation σ.
$N_a(\boldsymbol{x})$: NN QC function applied to a tuple $\boldsymbol{x}$ of QC measurements, for $a \in \{4,6,8,12,16\}$. The index α is related to the ratio of the simulated QC measurements of the training set distributed as $\mathcal{N}(0,1)$ and $\mathcal{N}(\mu, \sigma^2)$, for $0<|\mu|\leq 6$ or $1<|\sigma|\leq 7$
$S_a(\boldsymbol{x})$: SSV QC function applied to a tuple $\boldsymbol{x}$ of QC measurements, for $a \in \{4,6,8,12,16\}$; $N_a(\boldsymbol{x})$ and $S_a(\boldsymbol{x})$ have equal probabilities for false rejection.
$S(\boldsymbol{x}; m, s, l)$: SSV QC function with decision limits $m \pm l\, s$, applied to a tuple $\boldsymbol{x}$ of QC measurements. The function rejects an analytical run if: $\exists\, x_i \in \boldsymbol{x}: x_i < m - l\, s \vee x_i > m + l\, s$
$S(\boldsymbol{x}; l) = S(\boldsymbol{x}; 0, 1, l)$
$P_{N,a}(n, k, \mu, \sigma)$: probability for rejection of the NN QC function $N_a(\boldsymbol{x})$ applied to a *n* -tuple $\boldsymbol{x}$ of control measurements, *k* of them distributed as $\mathcal{N}(\mu, \sigma^2)$ and $n-k$ as $\mathcal{N}(0,1)$, for $a \in \{4,6,8,12,16\}$, and $0<|\mu|\leq 6$ or $1<|\sigma|\leq 7$ and
$P_{S,a}(n, k, \mu, \sigma)$: probability for rejection of the SSV QC function $S_a(\boldsymbol{x})$ applied to a *n* -tuple $\boldsymbol{x}$ of control measurements, *k* of them distributed as $\mathcal{N}(0,1)$ and $n-k$ as $\mathcal{N}(\mu, \sigma^2)$, for $a \in \{4,6,8,12,16\}$, and $0<|\mu|\leq 6$ or $1<|\sigma|\leq 7$.
$P_S(n, k, \mu, \sigma; l)$: probability for rejection of the SSV QC function $S(\boldsymbol{x}; l)$ applied on a *n* -tuple $\boldsymbol{x}$ of control measurements, *k* of them distributed as $\mathcal{N}(0,1)$ and $n-k$ as $\mathcal{N}(\mu, \sigma^2)$, for $a \in \{4,6,8,12,16\}$, and $0<|\mu|\leq 6$ or $1<|\sigma|\leq 7$.

The program interface

QC Plots

1. NN vs a SSV QC function with equal *pfr*

1.1. Input.

1.1.1. Plot options

1.1.1.1. x-axis

1.1.1.1.1. The standardized standard deviation (σ).

1.1.1.1.2. The standardized mean (μ).

1.1.1.2. The type of the plot:

1.1.1.2.1. The probabilities for rejection of the QC functions

1.1.1.2.2. Their difference.

1.1.1.2.3. Their relative difference.

1.1.1.2.4. Their ratio.

1.1.2. QC parameters

The following are defined:

1.1.2.1. The type of neural network $N_a(\boldsymbol{x})$, a $\in \{1,2,3,4,6,8,12,16\}$.

1.1.2.2. The size *n* of the QC samples ($1 \le n \le 4$).

1.1.2.3. The number *k* of the out of control QC measurements per sample ($1 \le k \le n$).

1.2. Output

1.2.1. Table of QC parameters

1.2.1.1. The decision limit *l* of the SSV QC function $\mathrm{S}(\boldsymbol{x}; l) = \mathrm{S}_a(\boldsymbol{x})$.

1.2.1.2. The probability *pfr* for false rejection of the $N_a(\boldsymbol{x})$ and $\mathrm{S}_a(\boldsymbol{x})$.

1.2.2. Plots of:

1.2.2.1. The probabilities for rejection $P_{N,a}(n, k, \mu, \sigma)$ and $P_{S,a}(n, k, \mu, \sigma)$ of the $N_a(\boldsymbol{x})$ and $\mathrm{S}_a(\boldsymbol{x})$.

1.2.2.2. Their difference.

1.2.2.3. Their relative difference.

1.2.2.4. Their ratio.

2. NN vs SSV QC function

2.1. Input.

2.1.1. Plot options

2.1.1.1. x-axis

2.1.1.1.1. The standardized standard deviation (σ).

2.1.1.1.2. The standardized mean (μ).

2.1.1.2. The type of the plot:

2.1.1.2.1. The probabilities for rejection.

2.1.1.2.2. Their difference.

2.1.1.2.3. Their relative difference.

2.1.1.2.4. Their ratio.

2.1.2. QC parameters

2.1.2.1. The neural network $N_a(\boldsymbol{x})$, a ∈ {1,2,3,4,6,8,12,16}

2.1.2.2. The size *n* of the QC samples (1 ≤ *n* ≤ 4), for the $N_a(\boldsymbol{x})$.

2.1.2.3. The number *k* of the out of control QC measurements per sample (1 ≤ *k* ≤ *n*), for the $N_a(\boldsymbol{x})$.

2.1.2.4. The size *n* of the QC samples (1 ≤ *n* ≤ 4), for the SSV QC function $\mathrm{S}(\boldsymbol{x}; l)$.

2.1.2.5. The number *k* of the out of control QC measurements per sample (1 ≤ *k* ≤ *n*), for the $\mathrm{S}(\boldsymbol{x}; l)$.

2.1.2.6. The probability *pfr* for false rejection of the $\mathrm{S}(\boldsymbol{x}; l)$.

2.2. Output

2.2.1. Table of QC parameters

2.2.1.1. The decision limit *l* of $\mathrm{S}(\boldsymbol{x}; l)$.

2.2.1.2. The probabilities *pfr* for false rejection of the $N_a(\boldsymbol{x})$ and $\mathrm{S}_a(\boldsymbol{x})$.

2.2.2. Plots of:

2.2.2.1. The probabilities for rejection $P_{N,a}(n, k, \mu, \sigma)$ and $P_S(n, k, \mu, \sigma; l)$ of the $N_a(\boldsymbol{x})$ and $\mathrm{S}(\boldsymbol{x}; l)$.

2.2.2.2. Their difference.

2.2.2.3. Their relative difference.

2.2.2.4. Their ratio.

3. NN vs NN QC function

3.1. Input.

3.1.1. Plot options

3.1.1.1. x-axis

3.1.1.1.1. The standardized standard deviation (*σ*).

3.1.1.1.2. The standardized mean (*μ*).

3.1.1.2. The type of the plot:

3.1.1.2.1. The probabilities for rejection.

3.1.1.2.2. Their difference.

3.1.1.2.3. Their relative difference.

3.1.1.2.4. Their ratio.

3.1.2. QC parameters

For each of the two NN QC functions $N_a(\boldsymbol{x})$.:

3.1.2.1. The type of neural network $N_a(\boldsymbol{x})$, $a \in \{1,2,3,4,6,8,12,16\}$.

3.1.2.2. The size *n* of the QC samples (1 ≤ *n* ≤ 4), for the $N_a(\boldsymbol{x})$.

3.1.2.3. The number *k* of the out of control QC measurements per sample (1 ≤ *k* ≤ *n*), for the $N_a(\boldsymbol{x})$.

3.2. Output

3.2.1. Table of QC parameters

3.2.1.1. The probability *pfr* for false rejection of each of the $N_a(\boldsymbol{x})$.

3.2.2. Plots of:

3.2.2.1. The probabilities for rejection $P_{N,a}(n, k, \mu, \sigma)$ of each of the $N_a(\boldsymbol{x})$.

3.2.2.2. Their difference.

3.2.2.3. Their relative difference.

3.2.2.4. Their ratio.

4. Single-value vs SSV QC function

4.1. Input.

4.1.1. Plot options

4.1.1.1. x-axis

4.1.1.1.1. The standardized standard deviation (σ).

4.1.1.1.2. The standardized mean (μ).

4.1.1.2. The type of the plot:

4.1.1.2.1. The probabilities for rejection of the QC functions.

4.1.1.2.2. Their difference.

4.1.1.2.3. Their relative difference.

4.1.1.2.4. Their ratio.

4.1.2. QC parameters

For each of the two SSV QC functions $\mathrm{S}(\boldsymbol{x}; l)$:

4.1.2.1. The size *n* of the QC samples ($1 \le n \le 4$).

4.1.2.2. The number *k* of the out of control QC measurements per sample ($1 \le k \le n$).

4.1.2.3. The probability *pfr* for false rejection.

4.2. Output

4.2.1. Table of QC parameters

4.2.1.1. The decision limits *l* of $\mathrm{S}(\boldsymbol{x}; l)$.

4.2.1.2. The probabilities *pfr* for false rejection of each of the $\mathrm{S}(\boldsymbol{x}; l)$.

4.2.2. Plots of:

4.2.2.1. The probabilities for rejection $P_S(n, k, \mu, \sigma; l)$ of each of the $\mathrm{S}(\boldsymbol{x}; l)$.

4.2.2.2. Their difference.

4.2.2.3. Their relative difference.

4.2.2.4. Their ratio.

QC Calculators

5. NN vs a SSV QC function with equal *pfr*

5.1. Input.

5.1.1. Parameters of of the measurement process

5.1.1.1. The standardized critical mean (m_c).

5.1.1.2. The standardized critical standard deviation (s_c).

5.1.2. QC parameters

The following are defined:

5.1.2.1. The type of the neural network $N_a(\boldsymbol{x})$, a $\in \{1,2,3,4,6,8,12,16\}$.

5.1.2.2. The size *n* of the QC samples ($1 \le n \le 4$).

5.1.2.3. The number *k* of the out of control QC measurements ($1 \le k \le n$).

5.2. Output

5.2.1. Table of QC parameters

5.2.1.1. The decision limit *l* of the SSV QC function $\mathrm{S}(\boldsymbol{x}; l) = \mathrm{S}_a(\boldsymbol{x})$.

5.2.1.2. The size *n* of the QC samples.

5.2.1.3. The number *k* of the out of control QC measurements per sample.

5.2.1.4. The probabilities for critical standard deviation s_c detection of the $N_a(\boldsymbol{x})$ and $\mathrm{S}_a(\boldsymbol{x})$.

5.2.1.5. The probabilities for critical mean m_c detection of the $N_a(\boldsymbol{x})$ and $\mathrm{S}_a(\boldsymbol{x})$.

5.2.1.6. The probability *pfr* for false rejection of of the $N_a(\boldsymbol{x})$ and $\mathrm{S}_a(\boldsymbol{x})$.

6. NN vs SSV QC function

6.1. Input.

6.1.1. Parameters of of the measurement process

6.1.1.1. The standardized critical standard deviation (s_c)

6.1.1.2. The standardized critical mean (m_c)

6.1.2. QC parameters

6.1.2.1. The type of the neural network $N_a(\boldsymbol{x})$, a $\in \{1,2,3,4,6,8,12,16\}$

6.1.2.2. The size *n* of the QC sample ($1 \le n \le 4$), for the $N_a(\boldsymbol{x})$.

6.1.2.3. The number *k* of the out of control QC measurements per sample ($1 \le k \le n$), for the $N_a(\boldsymbol{x})$.

6.1.2.4. The size *n* of the QC samples ($1 \le n \le 4$), for the SSV QC function $\mathrm{S}(\boldsymbol{x}; l)$.

6.1.2.5. The number *k* of the out of control QC measurements per sample ($1 \le k \le n$), for the $\mathrm{S}(\boldsymbol{x}; l)$.

6.1.2.6. The probability *pfr* for false rejection of the SSV QC function $\mathrm{S}(\boldsymbol{x}; l)$.

6.2. Output

6.2.1. Table of QC parameters

6.2.1.1. The decision limit *l* of $\mathrm{S}(\boldsymbol{x}; l)$.

6.2.1.2. The size *n* of the QC samples.

6.2.1.3. The number *k* of the out of control QC measurements per sample.

6.2.1.4. The probabilities for critical standard deviation s_c detection of both QC functions.

6.2.1.5. The probabilities for critical mean m_c detection of both QC functions.

6.2.1.6. The probabilities *pfr* for false rejection of the $N_a(\boldsymbol{x})$ and $\mathrm{S}(\boldsymbol{x}; l)$.

6.2.2. Plots of:

6.2.2.1. The probabilities for rejection $P_{N,a}(n, k, \mu, \sigma)$ and $P_S(n, k, \mu, \sigma; l)$ of the $N_a(\boldsymbol{x})$ and $\mathrm{S}(\boldsymbol{x}; l)$.

6.2.2.2. Their difference.

6.2.2.3. Their relative difference.

6.2.2.4. Their ratio.

7. NN vs NN QC function

7.1. Input.

7.1.1. Plot options

7.1.1.1. x-axis

7.1.1.1.1. The standardized mean (μ)

7.1.1.1.2. The standardized standard deviation (σ)

7.1.2. QC parameters

For each NN QC function:

7.1.2.1. The type of the neural network $N_a(\boldsymbol{x}), a \in \{1,2,3,4,6,8,12,16\}$

7.1.2.2. The size *n* of the QC samples ($1 \le n \le 4$), for the $N_a(\boldsymbol{x})$.

7.1.2.3. The number *k* of the out of control QC measurements per sample ($1 \le k \le n$), for the $N_a(\boldsymbol{x})$.

7.2. Output

7.2.1. Table of QC parameters

7.2.1.1. The size *n* of the QC samples.

7.2.1.2. The number *k* of the out of control QC measurements per sample.

7.2.1.3. The probabilities for critical standard deviation s_c detection of each of the $N_a(\boldsymbol{x})$.

7.2.1.4. The probabilities for critical mean m_c detection of each of the $N_a(\boldsymbol{x})$.

7.2.1.5. The probabilities *pfr* for false rejection of each of the $N_a(\boldsymbol{x})$.

8. Single-value vs SSV QC function

8.1. Input.

8.1.1. QC parameters

For each SSV QC function $\mathrm{S}(\boldsymbol{x}; l)$:

8.1.1.1. The size *n* of the QC samples ($1 \le n \le 4$).

8.1.1.2. The number *k* of the out of control QC measurements per sample ($1 \le k \le n$).

8.1.1.3. The probability *pfr* for false rejection.

8.2. Output

8.2.1. Table of QC parameters

8.2.1.1. The decision limits *l* of each of the $\mathrm{S}(\boldsymbol{x}; l)$.

8.2.1.2. The size *n* of the QC samples.

8.2.1.3. The number *k* of the out of control QC measurements per sample.

8.2.1.4. The probabilities for critical standard deviation s_c detection of each of the $\mathrm{S}(\boldsymbol{x}; l)$.

8.2.1.5. The probabilities for critical mean m_c detection of each of the $\mathrm{S}(\boldsymbol{x}; l)$.

8.2.1.6. The probabilities *pfr* for false rejection of each of the $\mathrm{S}(\boldsymbol{x}; l)$.

Source Code

Programming language: Wolfram Language

Availability: The updated source code is available at: https://www.hcsl.com/Tools/IntelligentQuality/IntelligentQuality.nb

Supplement

The trained CNN models, are available as Wolfram Language Objects at:
https://www.hcsl.com/Supplements/hcsltr22s.zip

Software Requirements

Operating systems: Microsoft Windows, Linux, Apple iOS
Other software requirements: Wolfram Player®, freely available at: https://www.wolfram.com/player/ or Wolfram Mathematica®.

System Requirements

Processor: Intel Core i7® or equivalent CPU
System memory (RAM): 16GB+ recommended.

Permanent Citation:

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