THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Solutions to Logic – Week 2 Practice Class

MATH1064: Discrete Mathematics for Computing

Exercises: Section 1.1: Questions 1, 3, 5, 11, 12, 40, 41, 48, 49; Section 1.2: Questions 1, 5, 6, 15, 34, 37 Section 1.3: Questions 1, 5, 7, 10, 16–28

Here is a list of **problems** for the contact class. Try to solve them before you go to class!

1. Let $r =$ “Bruce has red hair,” $u =$ “Bruce has a ute” and $f =$ “Bruce likes to eat figs.” Translate the following statements into symbolic from.

(a) Bruce does not like to eat figs.

(b) Bruce has red hair and does not have a ute.

(c) Bruce likes to eat figs, and he has red hair or a ute.

(d) It is not the case that Bruce has a ute or he has red hair.

(e) It is not the case that Bruce has a ute, or he has red hair.

(f) Bruce has a ute and red hair, or he has a ute and likes to eat figs.

Solution:

(a) $\neg f$

(b) $r \wedge \neg u$

(c) $f \wedge (r \vee u)$

(d) $\neg(u \vee r)$

(e) $\neg u \vee r$

(f) $(u \wedge r) \vee (u \wedge f)$

2. If p is the statement “it is raining” and q is the statement “it is hot,” translate the following into English sentences: (a) $p \wedge \neg q$; (b) $(p \vee q) \wedge \neg(p \wedge q)$.

Solution:

(a) It is raining and it is not hot.

(b) It is either raining or hot, but not both.

3. Construct a truth table to determine the truth values for $(p \vee q) \wedge \neg p$.

Solution:

p	q	$((p \vee q) \wedge \neg p)$
F	F	F
F	T	T
T	F	F
T	T	F

4. Construct a truth table to determine the truth values for $(p \vee q) \wedge \neg(p \vee r)$.

Solution:

p	q	r	$((p \vee q) \wedge \neg(p \vee r))$
F	F	F	F
F	F	T	F
F	T	F	T
F	T	T	F
T	F	F	F
T	F	T	F
T	T	F	F
T	T	T	F

5. Are the statement forms $p \wedge \neg q$ and $(p \vee q) \wedge \neg q$ logically equivalent?

Solution:

$$\begin{aligned}(p \vee q) \wedge \neg q &\equiv (p \wedge \neg q) \vee (q \wedge \neg q) \\ &\equiv (p \wedge \neg q) \vee \text{ (contradiction)} \\ &\equiv (p \wedge \neg q)\end{aligned}$$

Hence, the statements are equivalent.

6. Verify that $p \wedge (q \vee r)$ is not logically equivalent to $(p \wedge q) \vee r$.

Solution:

p	q	r	$(p \wedge (q \vee r))$	$((p \wedge q) \vee r)$
F	F	F	F	F
F	F	T	F	T
F	T	F	F	F
F	T	T	F	T
T	F	F	F	F
T	F	T	T	T
T	T	F	T	T
T	T	T	T	T

The two logical statements are not logical equivalent, because they take different values in the truth table (rows two and four).

7. Find a simpler statement that is logically equivalent to $p \wedge \neg(\neg p \vee \neg q)$.

Solution:

$$\begin{aligned} p \wedge \neg(\neg p \vee \neg q) &\equiv p \wedge (p \wedge q) \\ &\equiv (p \wedge p) \wedge q \\ &\equiv p \wedge q \end{aligned}$$

8. Using logical equivalences, show that the statement $(p \wedge q) \to (p \vee q)$ is a tautology.

Solution:

$$\begin{aligned} (p \wedge q) \to (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) \\ &\equiv (\neg p \vee \neg q) \vee (p \vee q) \\ &\equiv (\neg p \vee p) \vee (\neg q \vee q) \\ &\equiv \text{ (tautology) } \vee \text{ (tautology)} \\ &\equiv \text{ (tautology)} \end{aligned}$$

Here are two **puzzles** that you can think about during week 2; they are related to the first lecture. Feel free to ask your tutors or lecturer for more hints!

A Suppose each point of the plane is coloured either red or blue. Show that the four vertices of some rectangle are all of the same colour.

  Hint: Draw three parallel lines. What do you notice when you draw a common perpendicular to them?

B Suppose you label 10 points on a circle randomly with the numbers $1, \ldots, 10$, with each number used exactly once. Show that there are always 3 consecutive points whose labels sum to strictly more than 16.