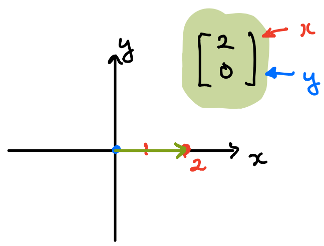
MATH1002 - LECTURE 1-D

VECTOR ADDITION

Recall from last lecture:

- vectors are characterised by their direction and magnitude.
- they can be represented geometrically
  - by arrows encoding the direction and magnitude (length)

  or algebraically



  - by columns of numbers, which give both the direction and magnitude by telling us how far to travel in each of the coordinate directions

- two arrows represent the same vector $\Leftrightarrow$ they have the same length and direction

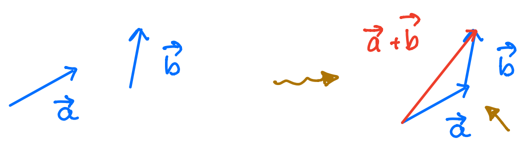
  (position doesn't matter)

Vector algebra

Given some vectors, there are two operations we can do to produce new vectors.

▸ In this lecture, we focus on vector addition.

Geometric definition: Given two vectors $\vec{a}$ & $\vec{b}$, we produce a new vector $\vec{a}+\vec{b}$ by "adding head to tail"



match tail of $\vec{b}$ to head of $\vec{a}$

- new arrow $\vec{a}+\vec{b}$ goes from tail of $\vec{a}$ to head of $\vec{b}$

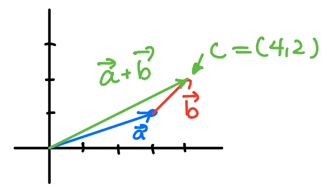
Algebraic definition: the vector sum of $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

is given by: $\vec{a} + \vec{b} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \end{bmatrix}$

Question: Why do these give the same result?

➤ Let's check with an example.

Let $\vec{a} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$



• From the geometric definition we see that

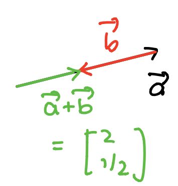
$$\vec{a} + \vec{b} = \vec{OC} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

• From the algebraic definition

$$\vec{a} + \vec{b} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3+1 \\ 1+1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}.$$

➥ They match !!

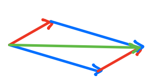
Exercise: Find $\vec{a} + \vec{b}$ algebraically and geometrically, where $\vec{a} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -2 \\ -1/2 \end{bmatrix}$



Parallelogram rule:

Consider two vectors $\vec{a}$ and $\vec{b}$

$\vec{b} + \vec{a}$



$\vec{a} + \vec{b}$

We conclude that $\boxed{\vec{a} + \vec{b} = \vec{b} + \vec{a}}$

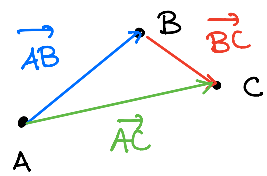
This is called commutativity

We can check algebraically as well:

Let $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

Then $\vec{a} + \vec{b} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \end{bmatrix} = \begin{bmatrix} b_1 + a_1 \\ b_2 + a_2 \end{bmatrix} = \vec{b} + \vec{a}$.

Recall displacement vectors



We see that $\vec{AB} + \vec{BC} = \vec{AC}$.

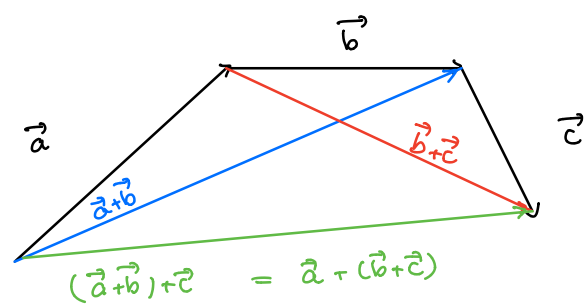
Associative law of vector addition: Given three vectors $\vec{a}, \vec{b}, \vec{c}$

$$\boxed{(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})}$$

Exercise: Draw a picture that gives a geometric reason for this to be true.

Verify algebraically for $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

- Geometrically:



- Algebraically:

$$(\vec{a} + \vec{b}) + \vec{c} = \begin{bmatrix} (a_1+b_1)+c_1 \\ (a_2+b_2)+c_2 \end{bmatrix} = \begin{bmatrix} a_1 + (b_1+c_1) \\ a_2 + (b_2+c_2) \end{bmatrix}$$

$$= \vec{a} + (\vec{b} + \vec{c})$$

↳ because of this, we don't usually write brackets when adding more than two vectors

"$\vec{a} + \vec{b} + \vec{c}$"

Summary of the lecture:

- The sum of two vectors $\vec{a}$, $\vec{b}$ is defined
  - geometrically by adding the vectors "tip to tail"
  - algebraically by adding the components
- Key properties of vector addition:
  - commutativity $\vec{a}+\vec{b}=\vec{b}+\vec{a}$
  - associativity $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$

You should be able to:

- add vectors by drawing arrows and by adding components.
- justify the fact that $\vec{a}+\vec{b}=\vec{b}+\vec{a}$.
- remember the (new) words commutativity and associativity.