MATH 1002 - LECTURE 1-C.

VECTORS

<u>Recall</u>: We denote the collection of all real numbers by $\mathbb{R}$

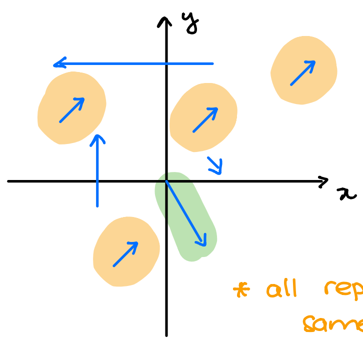
Real numbers are also called scalars

<u>Definition</u>: Vectors are quantities that are characterised by their magnitude (length) and direction.

<u>Examples</u>: displacement, velocity

<u>Non-examples</u>: distance, speed, mass ← these are scalars

We draw vectors as arrows

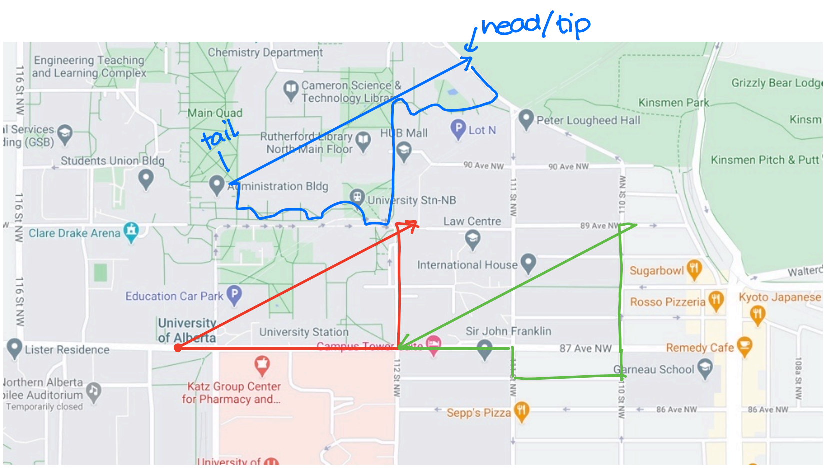


* all represent the same vector

! <u>IMPORTANT</u>:

Vectors are completely determined by their length and direction, not their position.

* vectors with tail at the origin are "in standard position"



Person A goes for a run.

Their <u>displacement</u> is given by a vector

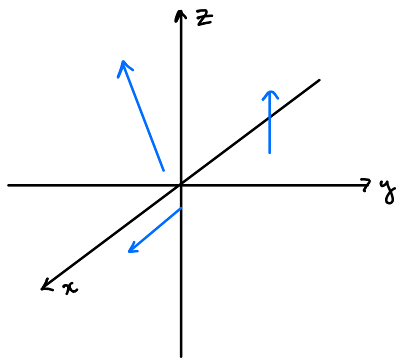
Person B goes for a run starting and ending at different places.

But their displacement is the same.

Person C has a different displacement vector, because it points in the opposite direction

- The set of all vectors in the plane is denoted $\mathbb{R}^2$

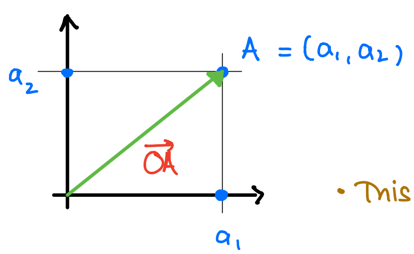
- We can also consider arrows representing vectors in 3D space



The set of all such vectors is denoted by $\mathbb{R}^3$.

Algebraic notation

1) If $A = (a_1, a_2)$ is a point in the plane, we consider the vector from the origin $O = (0,0)$ to A.

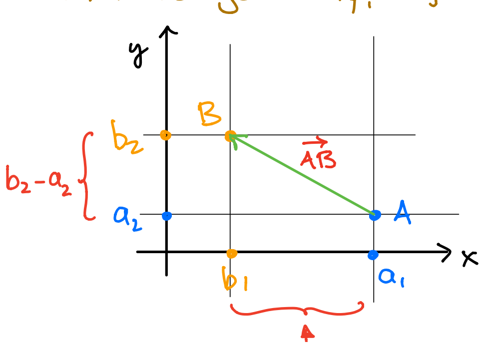


We write $\overrightarrow{OA} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ (or sometimes $[a_1, a_2]$ to save space)

- This notation tells us that the vector direction is
  - a_1 in the x-direction
  - a_2 in the y-direction

Definitions:
- $\overrightarrow{OA}$ is the position vector of A.
- the scalars a_1, a_2 are the components of $\overrightarrow{OA}$.

2) More generally, if $A = (a_1, a_2)$ and $B = (b_1, b_2)$, then we consider the arrow from A to B.



- We say that A is the tail and B is the head/tip

$b_1 - a_1$ (<0 in this example)

the arrow determines a vector

$$\vec{AB} = \begin{bmatrix} b_1 - a_1 \\ b_2 - a_2 \end{bmatrix}$$

"the displacement vector from A to B"

Note: We can also consider position vectors and displacement vectors in $\mathbb{R}^3$

- the difference is that our vectors will have three components $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

Recall: Different arrows can represent the same vector!

- two vectors $\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ are equal

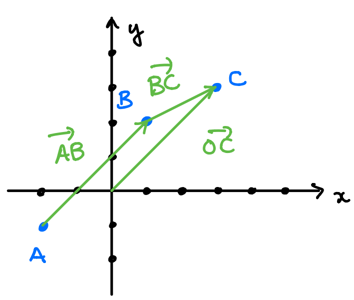
$\Longleftrightarrow$ they have the same components.

i.e. $a_1 = b_1$, $a_2 = b_2$.

Example: Let $A = (-2,-1)$, $B = (1,2)$, $C = (3,3)$.

Which of the vectors $\vec{AB}$, $\vec{BC}$, and $\vec{OC}$ are equal?

Exercise: Draw A, B, C on the grid below, and then draw the arrows $\vec{AB}$, $\vec{BC}$, $\vec{OC}$



! Pause your video and do the exercise now.

So it looks like $\vec{AB} = \vec{OC}$ and $\vec{BC}$ is different.

Let's check algebraically.

$$\vec{AB} = \begin{bmatrix} 1 - (-2) \\ 2 - (-1) \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} ; \quad \vec{OC} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

:)

$$\vec{BC} = \begin{bmatrix} 3 - 1 \\ 3 - 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

different.

Exercise Are the vectors $\vec{BC}$ and $\vec{OA}$ equal?

Solution: they are not: $\vec{OA} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

- It is parallel to $\vec{BC}$ and has the same magnitude, but it points in the opposite direction.

Notation:
- If we don't have a head and a tail specified (e.g. $\vec{AB}$) we usually write $\vec{a}, \vec{b}, \vec{u}, \vec{v}$ etc to denote vectors.

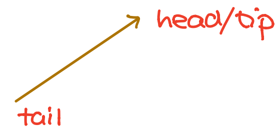
  (Others may use $\underset{\sim}{a}, \underset{\sim}{b}$ etc.)
- In typed material, the letters will be in bold

  $\mathbf{a}, \mathbf{b}, \mathbf{u}, \mathbf{v}$ etc.

Summary of the lecture

- Vectors are characteristed by direction and magnitude (but not position)

  - can be represented by arrows



    e.g. $\vec{OA}$, $\vec{AB}$

    position vector   displacement vector

    ! but different arrows can represent the same vector.

  - can also be written as columns of numbers

    called the <u>components</u> of the vector

    e.g. $\begin{bmatrix} -1 \\ 2 \end{bmatrix} \in \mathbb{R}^2$ $\begin{bmatrix} 7 \\ 1/2 \\ -3 \end{bmatrix} \in \mathbb{R}^3$

You should be able to:

- Sketch points and coordinates
- Identify arrows in standard position
- Write down position and displacement vectors
- Identify visually or algebraically whether two vectors are equal.