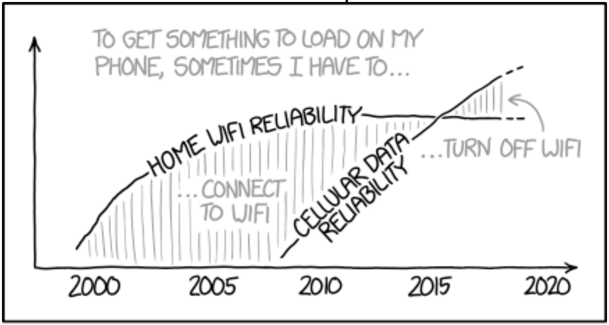
Discrete Mathematics

MATH1064, Lecture 31

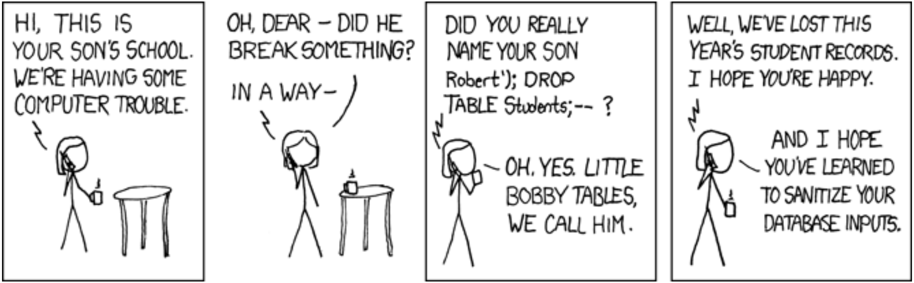
Jonathan Spreer



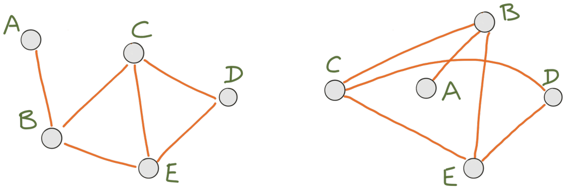
IT SEEMS WEIRD FROM A NETWORKING POINT OF VIEW, BUT SOMETIME IN THE LAST FEW YEARS THIS FLIPPED FOR ME.

Extra exercises for Lecture 31

Section 10.2: Problems 1–10



Graph theory



A graph G consists of two finite sets:

- a non-empty set $V(G)$ of vertices;
- a (possibly empty) set $E(G)$ of edges, where each edge is associated with a set $\{v, w\} \subseteq V(G)$.
  The vertices v and w are called the endpoints of the edge.

The graph above has five vertices: $V(G) = \{A, B, C, D, E\}$.
There are six edges, and their endpoints are:

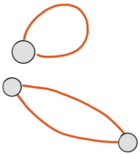
$$\{A, B\},\ \{B, C\},\ \{B, E\},\ \{C, E\},\ \{C, D\},\ \{D, E\}.$$

Exercise

Draw the graph G with vertices a, b, c, d, e, f and edges ab, bc, cd, de, ce, ac, cf, ff.

Things to note

- A graph G is defined purely in terms of sets $V(G)$ and $E(G)$. The drawing is just a visual aid.
- Finding a "good" drawing for a complex graph is difficult!
- Sometimes it is useful to allow the vertex set $V(G)$ to be empty, or infinite! Not in MATH1064.
- An edge may have endpoints $\{v, v\} = \{v\}$. We call such an edge a loop.
- Two edges may have the same endpoints $\{v, w\}$. We call these parallel edges.



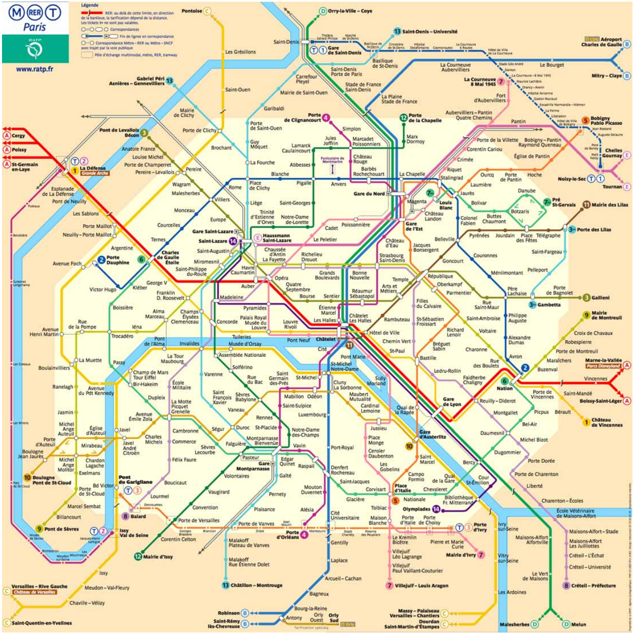
A graph with no loops or parallel edges is called a simple graph.

Why graphs?

Graphs turn up in many places,
some expected and some unexpected.

For example, they can represent:

- Road maps, computer networks, airline route maps
- Social networks (e.g., Facebook), text networks
- Trust networks between cryptographic keys
- Chess (vertices are board states, edges are legal moves)
- Assembly diagrams for geometric objects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Paris
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Légende
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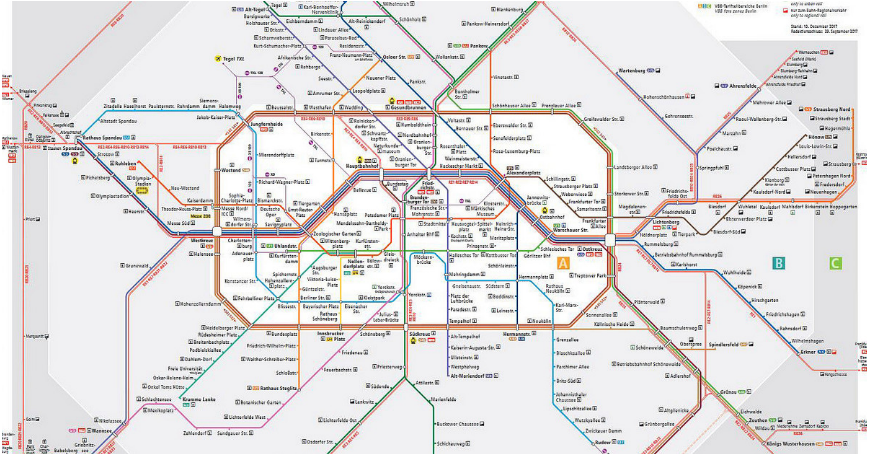
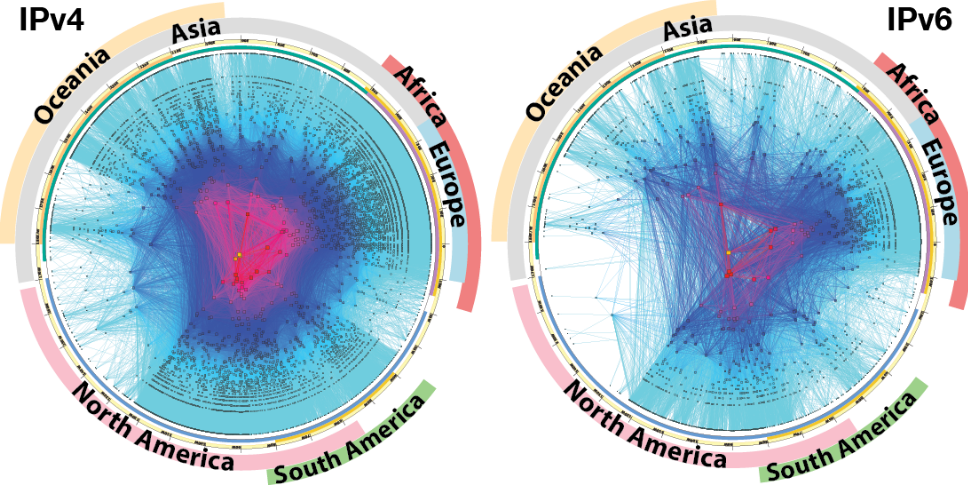
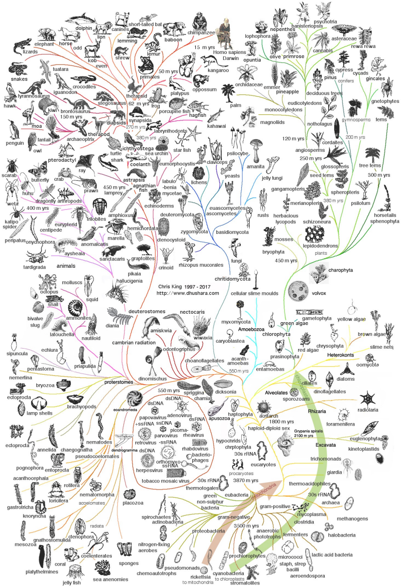
Orry-la-Ville – Coye
Saint-Denis – Université
Aéroport Charles de Gaulle
Mitry – Claye
Le Bourget
Gabriel Péri Asnières – Gennevilliers
Saint-Denis
Gare de Saint-Denis
Basilique de St-Denis
Saint-Denis Porte de Paris
Stade de France Saint-Denis
La Plaine Stade de France
La Courneuve Aubervilliers
La Courneuve 8 Mai 1945
Fort d'Aubervilliers
Aubervilliers – Pantin Quatre Chemins
Bobigny Pablo Picasso
Cergy
Poissy
St-Germain en-Laye
La Défense Grande Arche
Pont de Levallois Bécon
Porte de Clichy
Porte de Clignancourt
Porte de la Chapelle
Chelles Gournay
Noisy-le-Sec
Tournan
Pré St-Gervais
Mairie des Lilas
Porte des Lilas
Gare du Nord
Gare de l'Est
Gare Saint-Lazare
Saint-Lazare
Haussmann Saint-Lazare
Charles de Gaulle Étoile
Porte Dauphine
Opéra
Gallieni
Gambetta
Mairie de Montreuil
Châtelet
Châtelet Les Halles
Hôtel de Ville
République
Nation
Marne-la-Vallée
Vincennes
Boissy-Saint-Léger
Château de Vincennes
Gare de Lyon
Gare d'Austerlitz
Montparnasse
Gare Montparnasse
Boulogne Pont de St-Cloud
Pont du Garigliano
Balard
Pont de Sèvres
Issy Val de Seine
Mairie d'Issy
Porte d'Orléans
Châtillon – Montrouge
Place d'Italie
Olympiades
Bibliothèque Fr. Mitterrand
Porte d'Ivry
Mairie d'Ivry
Villejuif – Louis Aragon
Créteil – Préfecture
Versailles – Rive Gauche Château de Versailles
Saint-Quentin-en-Yvelines
Robinson
Saint-Rémy lès-Chevreuse
Massy – Palaiseau Versailles – Chantiers
Dourdan Saint-Martin-d'Étampes
Malesherbes
Melun


Alt-Tegel
Tegel TXL
Rathaus Spandau
Jungfernheide
Westend
Gesundbrunnen
Pankow
Schönhauser Allee
Hauptbahnhof
Friedrichstr.
Alexanderplatz
Ostbahnhof
Warschauer Str.
Ostkreuz
Westkreuz
Zoologischer Garten
Südkreuz
Hermannstr.
Wartenberg
Ahrensfelde
Strausberg Nord
Erkner
Königs Wusterhausen
Wannsee
Krumme Lanke
Rathaus Steglitz
Lichtenberg
A
B
C

CAIDA's IPv4 vs IPv6 AS Core AS-level Internet Graph

Archipelago July 2015




Chris King 1997 - 2017
http://www.dhushara.com
dolphin
lion
canines
short-tailed bat
chimpanzee
elephant
horse
odd
even
kob
lemming
shrew
baboon
lemur
Homo sapiens
Darwin
15 m yrs
lizards
snakes
tuatara
crocodiles
primates
50 m yrs
kangaroo
opossum
platypus
tyrannosaurus
iguanodon
stegosaurus
therapsid
62 m yrs
frog
porcupine fish
hagfish
hawk
kiwi
brontosaurus
150 m yrs
diapsids
synapsida
270 m yrs
moa
penguin
fantail
owl
therapod
archaeoptrix
380 m yrs
labrynthodonts
ichthyostega
sea urchin
star fish
turtle
shark
coelanth
eumorphocystis
pterodactyl
ray
scarab
butterfly
crab
huhu
prawn
450 m yrs
lamprey
astrapsis
agnathian fish
labulo -benia
mycelae
weta
dragonfly
arthropods
400 m yrs
trilobites
echinoderms
katipo spider
centipede
eurypterid
amphioxus
hemichordata
deuteromycota
peripatus
onychophora
anomalcaris
marella
ctenocystoid
aysheaia
sanctacaris
graptolites
crinoid
tardigrada
animals
pikaia
hallucigenia
molluscs
octopus
snail
squid
bivalve
slug
ammonites
latouchella
nautiloid
diania
deuterostomes
nectocaris
amiskwia
wiwaxia
cambrian radiation
odontogriphus
sipuncula
echiura
pentastoma
priapulida
nemertina
bryozoa
ectoprocta
lamp shells
brachyopods
proterostomes
dinomischus
550 m yrs
spriggina
charnia
dicksonia
dsDNA
ssDNA
papovavirus
adenovirus
dsRNA
retrovirus
parvovirus
picorna-
rheavirus
rhabdovirus
bacterio-phages
herpesvirus
orthomyxo-
tobacco mosaic virus
ectoproct
annelida
chaegognatha
nematodes
pseudocoelomates
pognophora
entoproct
acanthocephala
rotifera
nematomorpha
placozoa
gastrotricha
loricifera
acoelomates
knorhyncha
radiata
mesozoa
gnathostomulida
ctenophora
coelenterates
platyhelmines
coral
jelly fish
sea anemonies
sponges
chemoautotrophs
pseudomonads
rickettsia
to mitochondria
to chloroplasts
cyanobacteria
stromatolites
nitrogen-fixing aerobes
spirochaetes
actinobacteria
proteobacteria
green non-sulphur bacteria
eubacteria
gram-negative
gram-positive
3500 m yrs
anaerobic phototrophs
fermenters
prochlorophytes
micrococci
staph, strep bacilli
aeroendospora
lactic acid bacteria
halobacteria
methanogens
mycoplasma
clostridia
archaea
30s rRNA
thermoacidophiles
thermotogales
3870 m yrs
procaryotes
eucaryotes
giardia
trichomonads
Excavata
kinetoplastids
euglenophyta
Grypania spiralis
2100 m yrs
haploid-diploid sex
1800 m yrs
acritarch
haptophyta
hypochrids
chrplophyta
apusozoa
foramenifera
Rhizaria
radiolaria
Alveolates
sporozoans
ciliates
dinoflagellates
diatoms
oomycota
Heterokonts
chrysophyta
brown algae
slime nets
yellow algae
gametophyta
green algae
chlorophyta
red algae
prasinophyta
entamoebas
acanth-amoebas
550 m yrs
choanoflagellates
caryoblastea
Amoebozoa
myxomycota
cellular slime moulds
chritidomycota
volvox
charophyta
fungi
rhizopus mucorales
zygomycota
basidiomycota
euascomycetes
ascomycetes
rusts
lichens
yeasts
claviceps
psilocybe
amanita
jelly fungi
herbacious lycopods
schizoneura
lepidodendrons
plants
mosses
bryophyta
450 m yrs
gangamopteris
merianopteris
380 m yrs
psilotum
horsetails
sphenophyta
spheropteris
seed ferns
glossopteris
300 m yrs
tree ferns
angiosperms
250 m yrs
cordaites
120 m yrs
200 m yrs
nothofagus
gymnosperms
ferns
gnetophytes
magnolids
monocotyledons
eudicotyledons
deciduous trees
palm
kahawai
pineapple
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orchidaceae
pinus
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cypress
cycads
gincales
opuntia
olive
primrose
cannabis
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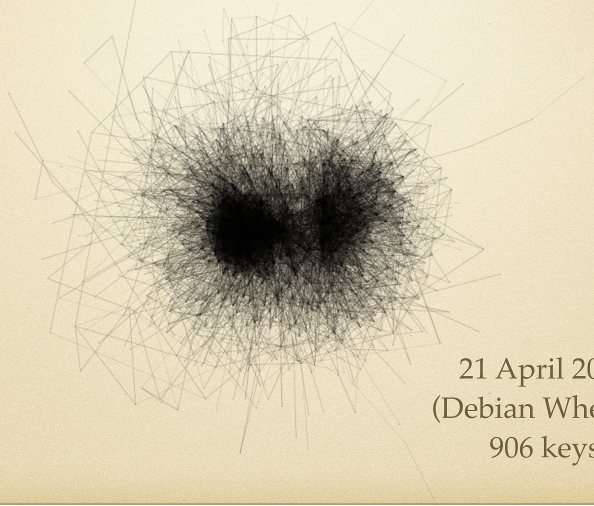
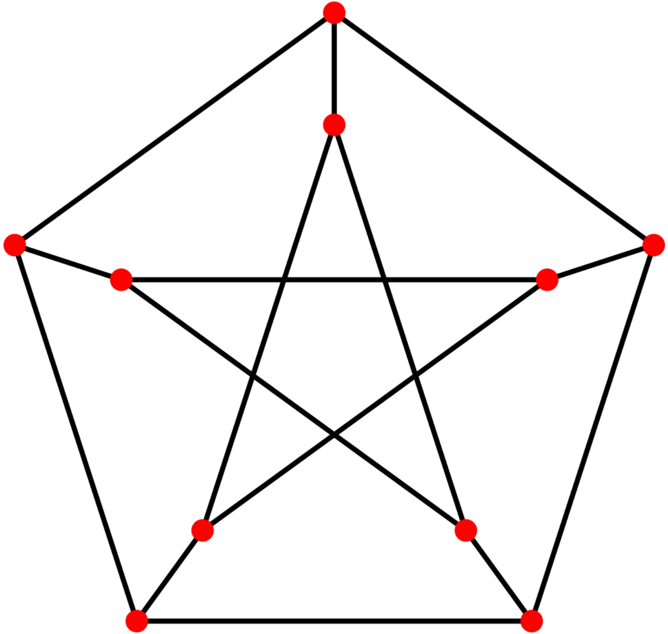
asteraceae
lophophora
nepenthes
banisteriopsis
psychotria
mitochondria

Debian's web of trust



21 April 2013
(Debian Wheezy)
906 keys

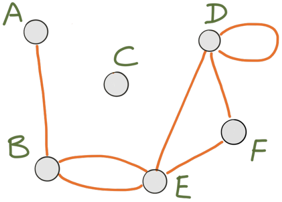


More terminology

An edge e and a vertex v are incident if v is an endpoint of e.

Vertices u, v are adjacent if there is an edge with endpoints $\{u, v\}$.
A vertex u is adjacent to itself if there is a loop with endpoints $\{u\}$.

The degree of a vertex v is the number of edges incident with v, where we count each loop twice. We write this as $\deg(v)$.



Informally, $\deg(v)$ counts the "ends of edges" that meet v.

$\deg(A) =$, $\deg(B) =$, $\deg(D) =$, $\deg(C) =$.

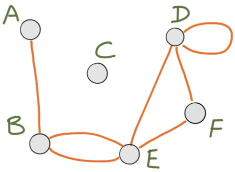
The handshake theorem

Theorem

Let G be a graph with n vertices $V(G) = \{v_1, \ldots, v_n\}$. Then:

$$\sum_{i=1}^{n} \deg(v_i) = \deg(v_1) + \ldots + \deg(v_n) = 2 \cdot |E(G)|.$$

In particular, in any graph, the sum of all vertex degrees must be even.



Example: The graph above has $|E(G)| = 7$ edges.

$$\sum \deg(v_i) = 1 + 3 + 0 + 4 + 4 + 2 = 14 = 2 \cdot |E(G)|$$

The handshake theorem: $\sum_{i=1}^{n} \deg(v_i) = 2 \cdot |E(G)|$.

Question: Is there a graph with seven vertices, having degrees 2, 3, 3, 4, 4, 5 and 6 respectively?

Corollary

In any graph, the number of vertices of odd degree is even.

Directed graphs

Let G be a digraph (directed graph), and $v \in V(G)$.

The in-degree $\deg^-(v)$ is the number of edges terminating in v.

The out-degree $\deg^+(v)$ is the number of edges starting in v.

We have the following version of the handshake theorem:

Theorem

Let G be a directed graph with n vertices $V(G) = \{v_1, \ldots, v_n\}$. Then:

$$\sum_{i=1}^{n} \deg^-(v_i) = \sum_{i=1}^{n} \deg^+(v_i) = |E(G)|.$$

A trip to the zoo

Complete graphs: The complete graph on n vertices is a simple graph with exactly one edge between any pair of vertices.

Cycles: A cycle C_n for $n \geq 3$ is a graph that looks like a loop:

A trip to the zoo

Wheels: You get a wheel W_n from the cycle C_n by adding a vertex that connects to each of the vertices:

Cubes: An n–cube Q_n consists of the edges of an n–dimensional cube:

Trees: Graphs without cycles.

Cactus graphs:

Make up your own special class of graphs!