Discrete Mathematics

MATH1064, Lecture 26

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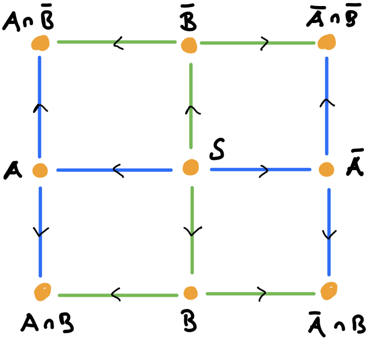
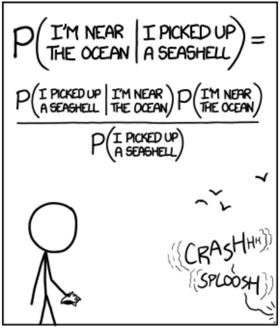


Extra exercises for Lecture 26

Section 7.3: Problems 5–10, 13–15



STATISTICALLY SPEAKING, IF YOU PICK UP A SEASHELL AND *DON'T* HOLD IT TO YOUR EAR, YOU CAN PROBABLY HEAR THE OCEAN.

Let's make a deal

Suppose you're on a game show, and you're given the choice of three doors: behind one door is a car; behind the others, goats.

You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?"

Question: Is it to your advantage to switch your choice?

Let's make a deal

Question: Is it to your advantage to switch your choice?

Some games with one equation

Let E and F be two events with $p(E) > 0$ and $p(F) > 0$.
The complement $\overline{F} = S \setminus F$ gives a partition

$$E = (E \cap F) \cup (E \cap \overline{F})$$

and hence:

$$p(E) = p(E \cap F) + p(E \cap \overline{F}).$$

Then

$$\begin{aligned} p(E) &= p(E \cap F) + p(E \cap \overline{F}) \\ &= p(E \mid F) \cdot p(F) + p(E \mid \overline{F}) \cdot p(\overline{F}) \end{aligned}$$

We also have

$$p(F \mid E) \cdot p(E) = p(F \cap E) = p(E \cap F) = p(E \mid F) \cdot p(F)$$

and so

$$p(F \mid E) = \frac{p(F)}{p(E)} \cdot p(E \mid F)$$

Bayes' theorem

We had:

$$p(E) = p(E \mid F) \cdot p(F) + p(E \mid \overline{F}) \cdot p(\overline{F})$$

$$p(F \mid E) = \frac{p(F)}{p(E)} \cdot p(E \mid F)$$

Hence:

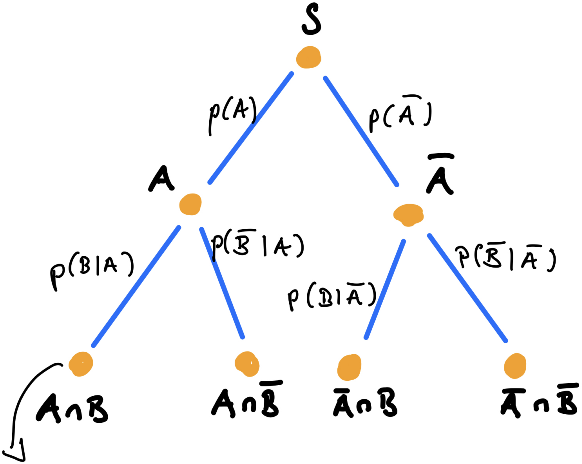
Bayes' theorem

Suppose E and F are events from a sample space S with $p(E) > 0$ and $p(F) > 0$. Then:

$$p(F \mid E) = \frac{p(F)}{p(E \mid F) \cdot p(F) + p(E \mid \overline{F}) \cdot p(\overline{F})} \cdot p(E \mid F)$$

Independence and conditional probability

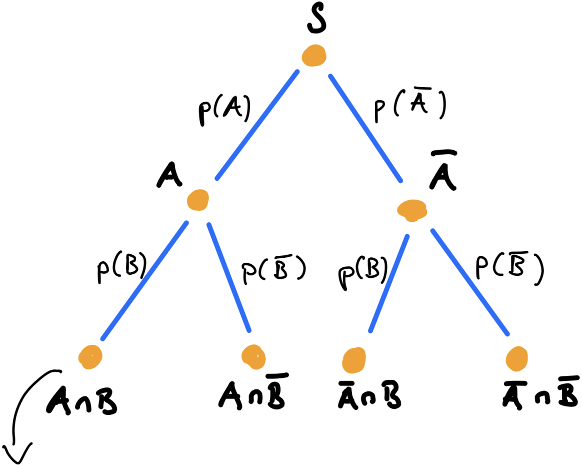
Decision tree for a pair of arbitrary events A and B



$$p(A \cap B) = p(A) \cdot p(B|A)$$

Independence and conditional probability

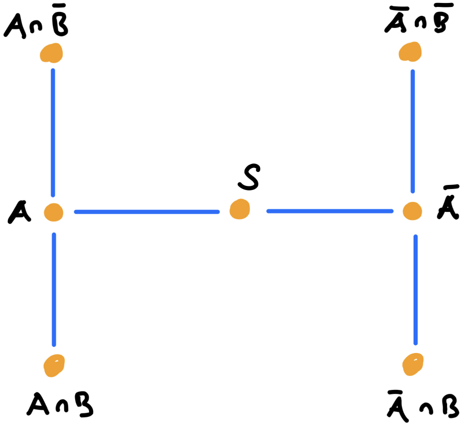
Decision tree for a pair of independent events A and B



$$p(A \cap B) = p(A) \cdot p(B)$$

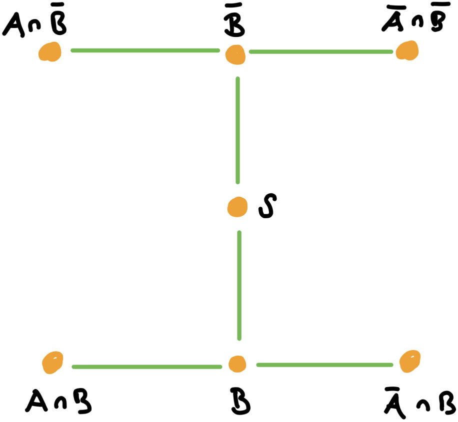
Independence and conditional probability

Bayes' theorem



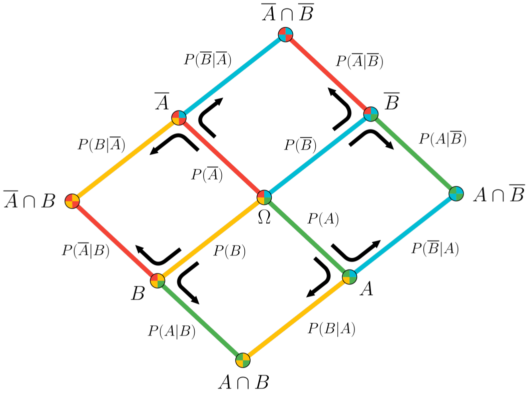
Independence and conditional probability

Bayes' theorem



Independence and conditional probability

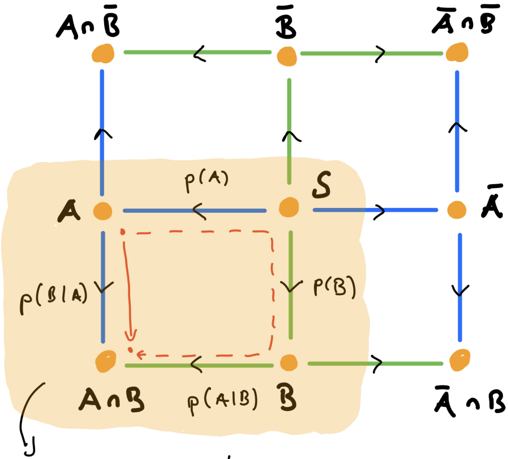
Bayes' theorem



$$P(A|B) \cdot P(B) = P(A \cap B) = P(B|A) \cdot P(A)$$

Independence and conditional probability

Bayes' theorem



$$p(B \mid A) = \frac{1}{p(A)} \cdot p(B) \cdot p(A \mid B)$$

Question

Suppose that one person in 10 000 people has a rare genetic disease. There is an excellent test for the disease;

99.9% of people with the disease test positive and only

0.02% who do not have the disease test positive.

1. What is the probability that someone who tests positive has the genetic disease?
2. What is the probability that someone who tests negative does not have the disease?

Question

1. What is the probability that someone who tests positive has the genetic disease?
2. What is the probability that someone who tests negative does not have the disease?