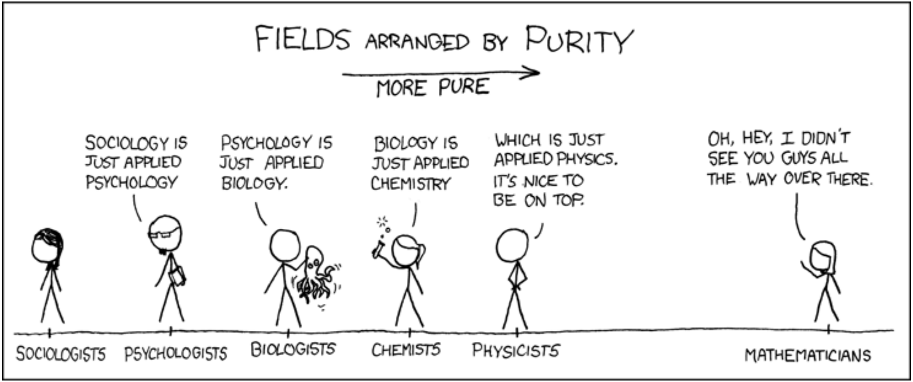
Discrete Mathematics

MATH1064, Lecture 1

Jonathan Spreer



Learning Activities

Lectures

Important concepts are explained and illustrated with examples.
Slides are posted on course website roughly one week before lectures.

Textbook: Discrete Mathematics and Its Applications by Kenneth H. Rosen, 7th Edition

Practice classes

Some course content or problems are discussed from a different viewpoint to the lectures (either in class or on Zoom). The classes are smaller than the lectures so you can participate more actively.

Tutorials

You will work through problems in groups at white boards (either in class or on Zoom). Tutorials give the opportunity to solve mathematical problems on your own.

Learning Activities – Zoom classes

Practice classes on Zoom

The lecturer will share their screen and work through mathematical problems. You are welcome to ask questions at any point.

Tutorials on Zoom

The form of Zoom tutorials will depend on the tutor and the rate of participation. The tutor may choose from a variety of applications to share mathematical thoughts and working with you. Example applications:

- Padlet
- EquatIO
- Ed discussion forum (see below)
- Screen writing apps (in case a tablet is available)

Make sure you have a plan for communicating your writings through, for instance, a shared screen on Zoom.

Assessment

Weekly Online Quizzes (10%)

Released every Friday, 11:00am. Closed following Friday, 11:59pm.
Five submissions, best attempt counts.

Assignments (10%)

2 assignments, each worth 5%, due in weeks 4, 11

Quizzes (20%)

2 quizzes, each worth 10%, submission online on Thursdays in weeks 6, 10

Final Exam (60%)

The better mark principle means that a quiz counts if and only if it is better than or equal to your exam mark. Otherwise the final exam mark will be used for that portion of your assessment instead.

Online resources

Canvas

Everything you need (including links to all other websites):

- https://canvas.sydney.edu.au/courses/27245

Other websites:

- Unit Website: https://www.maths.usyd.edu.au/u/UG/JM/MATH1064/
- Ed (discussions): https://edstem.org/courses/4750/
- WeBWorK (online quizzes): https://www.maths.usyd.edu.au/webwork2/MATH1064/

COVID-19

There is a COVID-19 category on Ed with important posts. This information is likely to change on short notice, so check for updates

When on Campus, remember to

- social distance
- swipe your ID at every room you enter (even if door is open)
- observe maximum capacities of tutorial rooms and lecture theatres
- wear a mask (not mandatory)

Tutors will also take attendance and have to right to turn you away when classes are full

Your lecturer...

is a topologist...

... developing discrete algorithms...

...but this is irrelevant!

Office hours

Thursdays 10-12 on discussion board Ed. Post using category Office hours

Web-site

www.sydney.edu.au/science/people/jonathan.spreer.php

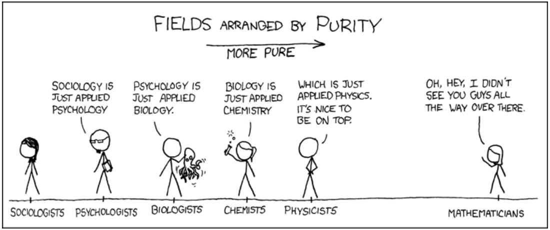
Email

MATH1064@sydney.edu.au (you can use this email to ask for a Zoom meeting)

What is this course about?

In this lecture, we will take a quick tour

of some of the key topics that we'll see this semester.



Logic and set theory, methods of proof

Modern mathematics uses the language of set theory
and the notation of logic.

You will learn to read and analyse sentences such as:

$$((p \wedge \neg q) \vee (p \wedge q)) \wedge q \equiv p \wedge q$$

Symbolic logic is the basis for many areas of computer science.
It helps us to formulate mathematical ideas and proofs
effectively and correctly!

Gödel's Incompleteness Theorem (1931)

There are true statements which we can't prove!

Number theory

$1 + \cdots + 100$

Young Gauss had to add up all numbers from 1 to 100 in primary school. What did he do?

$$
\begin{array}{ccccccccccccc}
 & & 1 & + & 2 & + & \cdots & + & 98 & + & 99 & + & 100 \\
100 & + & 99 & + & 98 & + & \cdots & + & 2 & + & 1 & & \\
\hline
100 & + & 100 & + & 100 & + & \cdots & + & 100 & + & 100 & + & 100
\end{array}
$$

So

$$1 + \cdots + 100 = \frac{101 \cdot 100}{2}$$

This generalises to any number $n \in \mathbb{N}$!

$$
\begin{array}{ccccccccccc}
 & & 1 & + & 2 & + & \cdots & + & (n-1) & + & n \\
n & + & (n-1) & + & (n-2) & + & \cdots & + & 1 & & \\
\hline
n & + & n & + & n & + & \cdots & + & n & + & n
\end{array}
$$

So

$$1 + \cdots + n = \frac{(n+1)n}{2}$$

for every positive whole number $n \in \mathbb{N}$!
However, there are

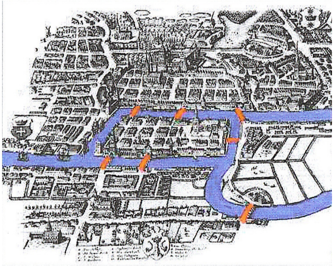
two leaps of faith!

The dots: We will introduce fancy notation to deal with them.
The equality of the two lines: We'll use a method called induction that will prove it cleanly and without any nagging doubts!

Graph theory

The Königsberg bridge problem

Find a route through the city that crosses each bridge exactly once and takes you back to the starting place.
(No turning around on bridges or swimming, please!)

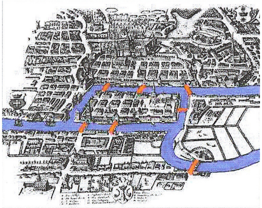


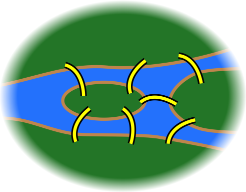
The Königsberg bridge problem

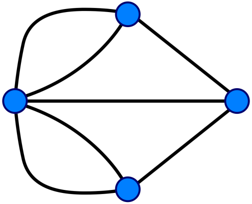
Find a route through the city that crosses each bridge exactly once and takes you back to the starting place.
(No turning around on bridges or swimming, please!)

Euler solved this in 1735 and invented graph theory.
I won't tell you (yet) whether the answer is positive or negative.

Here is a hint:







See https://youtu.be/nZwSo4vfw6c.

Counting and Probability

Both topics are extremely fundamental and we'll see many beautiful applications.
I'll only give an example for "counting" in this introduction.

The pigeonhole principle

If you have n pigeons sitting in k pigeonholes, and if $k < n$, then at least one of the pigeonholes contains at least two pigeons.

Why?
Suppose that each pigeonhole contains at most one pigeon.
Then you have at most as many pigeons as holes,
and hence at most k pigeons. So:

$$\text{number of pigeons} = n \leq k = \text{number of holes.}$$

But $k < n$ by hypothesis, and $n \leq k < n$ is not possible!

This method of reasoning is called the contrapositive.
Some statements are too difficult to prove directly!

If you have socks of three different colours in your drawer, what is the minimum number of socks you need to pull out in order to guarantee a matching pair?

1. 1 sock
2. 2 socks
3. 3 socks
4. 4 socks
5. 5 socks
6. 6 socks
7. 7 socks

True or False?

There are two people walking the Earth with exactly the same number of hairs on their head.

1. True.
2. False.

Functions

True or False?

A plane is coloured blue and red.
Is it always possible to find two points
of the same colour exactly 1 unit apart?

Each point has a colour (either red or blue). So if you pick your favourite point, p, in the plane, there is an assignment

$$p \longrightarrow \text{blue} \qquad \text{or} \qquad p \longrightarrow \text{red},$$

but not both! This is true for every point. So we have a function

$$\text{plane} \longrightarrow \{\text{blue}, \text{red}\}.$$

For the pigeonhole principle, you always need such a function

$$\text{pigeons} \longrightarrow \text{holes}$$

Relations

In the plane colouring example, each element of

$$\{\text{blue}, \text{red}\}$$

is associated with some (possibly bizarre) subset of the plane.

This is not a function, since at least one of the colours occurs at more than one point.

We will study such situations, where one element in set X
is associated with possibly more than one element of set Y,
later in this course!

. . . plus many more topics!

Have a look at
https://canvas.sydney.edu.au/courses/27245/pages/schedule
for an overview

Think about this:

True or False?

A plane is colored blue and red.
Is it always possible to find two points
of the same colour exactly 1 unit apart?

Post your answer on Ed using the category Lectures and sub-category Lecture 01