MATH 100 2 - LECTURE 1-B

week 1 ↑ ↖ 2 of 5 or 6

INTRODUCTION: WHAT IS LINEAR ALGEBRA?

A warm-up problem:

Suppose that you run an ice cream factory, where you make 3 kinds of ice cream:

flavour	base ingredients per batch
Vanilla	2 eggs, 1 cup milk, 2 cups cream
Chocolate	1 egg, 1 cup milk, 2 cups cream
Strawberry	1 egg, 2 cups milk, 1 cup cream.

- Each day, your suppliers give you different amounts of eggs, milk, and cream.

  ↳ You decide how many batches of each flavour to make in order to use up your ingredients.

Pause your video & think about this exercise:

> On Monday, you get 850 eggs, 850 cups of milk, and 400 cups of cream.
> How many batches of each flavour should you make?

Strategy: Let v = # of batches of vanilla

c = # of batches of chocolate

s = # of batches of strawberry.

Then

- the number of eggs used is $2v + c + s = 350$ (1)
- cups of milk: $v + c + 2s = 350$ (2)
- cups of cream: $2v + 2c + s = 400$ (3)

▶ Solve this "system of linear equations" to find v, c, s.

e.g. (3) − (1): $\boxed{c = 50}$

then (1) $\Rightarrow$ $2v + s = 350 - c = 300$

(2) $\Rightarrow$ $v + 2s = 350 - c = 300$

Now (1) − 2(2): $(2-2)v + (1-4)s = 300 - 2 \cdot 300$

i.e. $-3s = -300$

$\Rightarrow \boxed{s = 100}$

So (2) $\Rightarrow$ $v + 2 \cdot (100) = 300$

$\Rightarrow \boxed{v = 100}$

Note: We should always check that our solution does satisfy the original equations!

- Pause your video and do that now.

- What if you are given 400 eggs, 500 cups of milk, and 400 cups of cream?
- What about X eggs, Y cups of milk, and Z cups of cream?

↦ observation: It is possible to find formulas for v, c, s in terms of X, Y, Z.

- So we don't need to solve the system of equations every morning at the factory, we can just plug X, Y, Z into our formulas.

(of course, we might get negative numbers out for v, c, s, but this is a more complicated problem...)

In this course we will learn efficient and general methods for solving problems like these, even in more complicated situations:

- more flavours
  ($\Rightarrow$ more variables $v, c, s, p, l, \ldots$)
- more ingredients
  ($\Rightarrow$ more equations)

So what is linear algebra?

Linear: relating to straight (not curved!) lines, and planes, and higher-dimensional analogues.

Algebra: derived from the Arabic term

al-jabr الجبر

- originally: "the restoring of broken parts"
- al-Khwarizmi (~820 CE):
  ↦ moving a value from one side of an equation to another.

Linear algebra: • the study of vector spaces and the linear transformations between them.

• vector spaces are sets of vectors ← weeks 1 – 5

• linear transformations are nice functions which are essentially matrices ← weeks 6 – 12.

Applications: geometry, the study of symmetry, computer graphics & 3d animation, financial trading, engineering, physics, biology, ...

Notation: • $\mathbb{R}$ is the set of all real numbers. We call them scalars.

• $\in$ means "is an element of"

e.g. $3 \in \mathbb{R}$, $-2 \in \mathbb{R}$, $\sqrt{5} \in \mathbb{R}$

apple $\notin \mathbb{R}$.

• if P and Q are statements, then

$P \Rightarrow Q$ means: "P implies Q"

"if P, then Q"

$P \Leftrightarrow Q$ means "P if and only if Q".

e.g. "I am a frog" $\Rightarrow$ "I am an animal"

$2x + 3 = 7 \Leftrightarrow x = 2$.

⚠ WARNING: Use "$\Rightarrow$" only when it is immediately clear that one statement follows from the previous one.

• Avoid using "$\therefore$" or "$\because$" for now –

it's best to practice using words to explain your reasoning.

Summary of the lecture:

Linear algebra is the study of lines, planes, and higher dimensional analogues (vector spaces)

It will give us tools to solve systems of linear equations efficiently.

You should be able to:

- extract equations from word problems
- manipulate linear equations by hand (e.g. add one equation to another, isolate variables & substitute)
- use the notation $\mathbb{R}$, $\in$, $\Rightarrow$, $\Leftrightarrow$