Discrete Mathematics

MATH1064, Lecture 36

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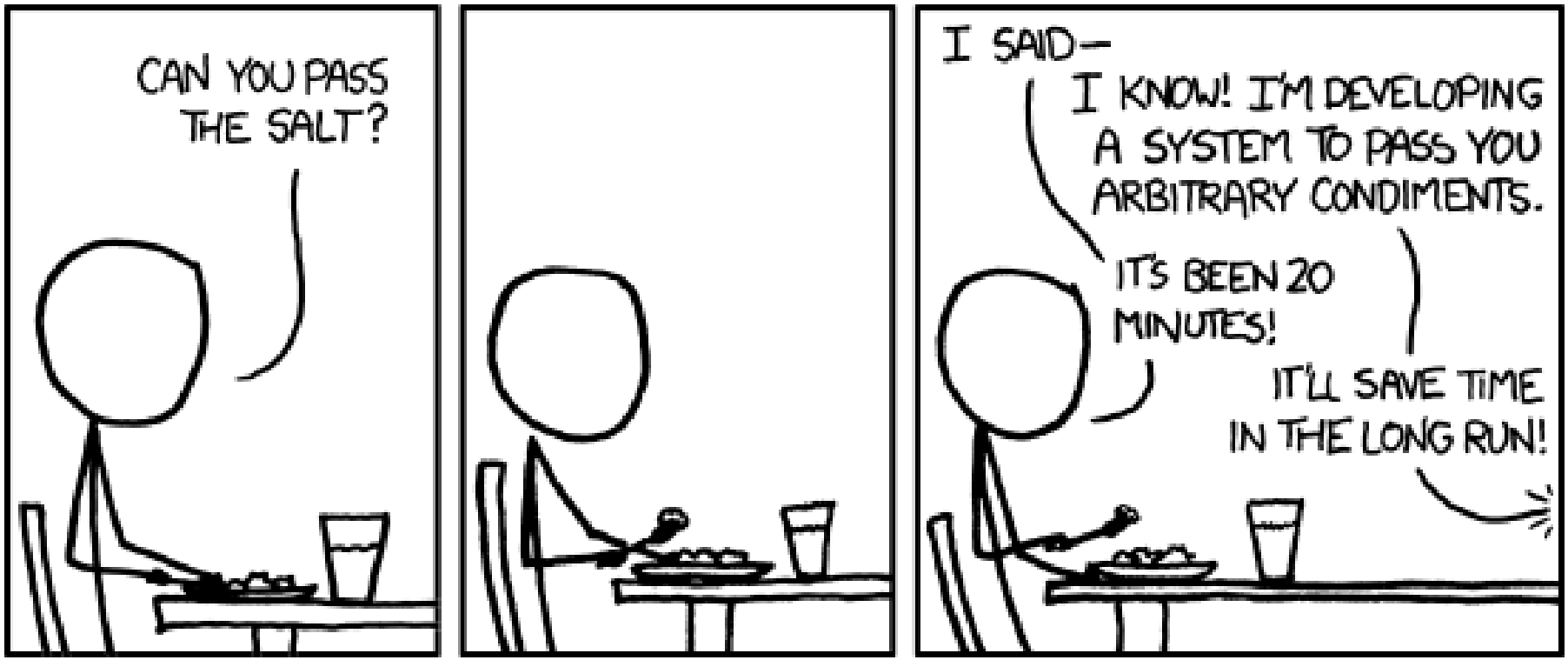
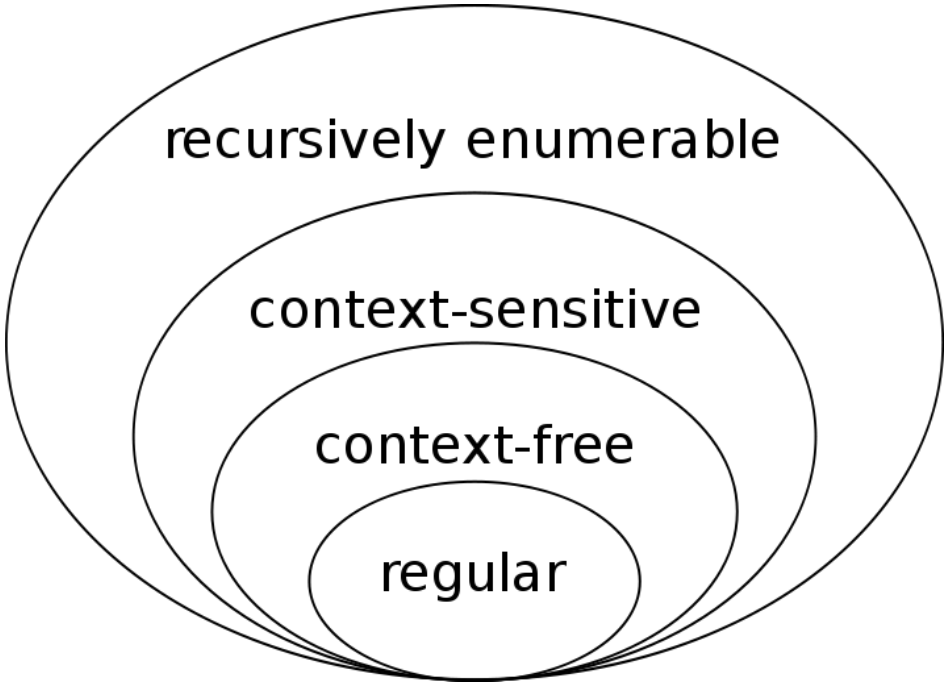


Types of grammars: The Chomsky Hierarchy



Type 0: no restrictions

Type 1: context-sensitive

$$L = \{0^n 1^n 2^n \mid n > 0\} \text{ has a context-sensitive grammar}$$

Type 2: context-free

$$L = \{0^n 1^n \mid n > 0\} \text{ has a context-free grammar}$$

Type 3: regular

$$L = \{0^n 1^m \mid m, n > 0\} \text{ has a regular grammar}$$

Types of grammars (Details)

Type	Restrictions on productions
0	No restrictions
1	Either $lAr \to lwr$ where $A \in N$ non-terminal, and $l, r, w \in V^*$ arbitrary words over V, $w \neq \emptyset$; or $S \to \emptyset$ and S cannot be the right hand side of another production.
2	$A \to w$ where $A \in N$ non-terminal, and $w \in V^*$ arbitrary
3	$A \to aB$ or $A \to a$ where $A, B \in N$ non-terminal and $a \in T$ terminal; or $S \to \emptyset$.

Claim: $L = \{0^n 1^m \mid m, n > 0\}$ is of type 3 (regular)

Proof: $P = \{S \to 0A,\ S \to 0S,\ A \to 1A,\ A \to 1\}$

Claim: $L = \{0^n 1^n \mid n > 0\}$ is of type 2 (context-free)

Proof: $P = \{S \to 0S1,\ S \to 01\}$.

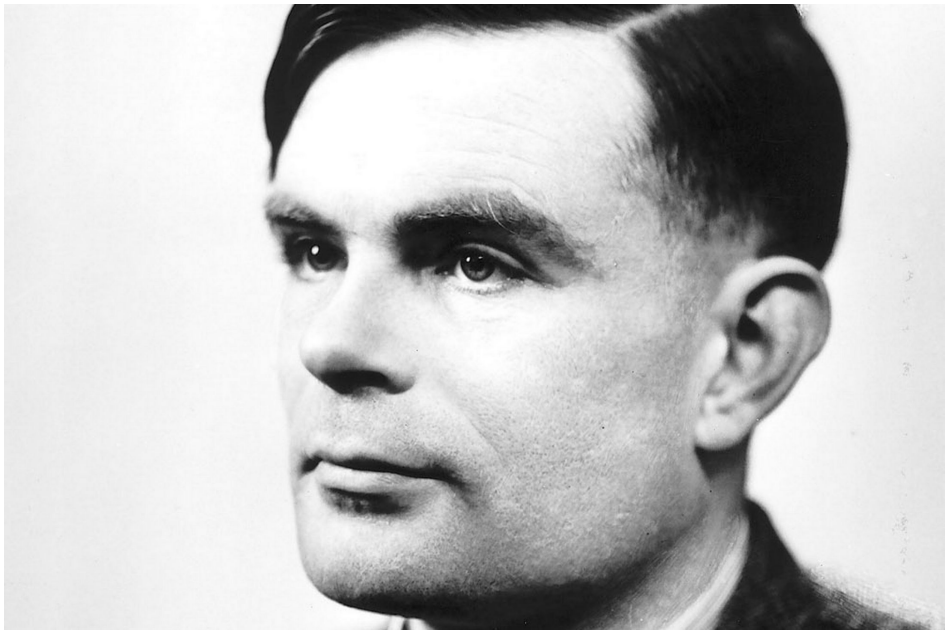
Types of grammars (Details)

Type	Restrictions on productions
0	No restrictions
1	Either $lAr \to lwr$ where $A \in N$ non-terminal, and $l, r, w \in V^*$ arbitrary words over V, $w \neq \emptyset$; or $S \to \emptyset$ and S cannot be the right hand side of another production.
2	$A \to w$ where $A \in N$ non-terminal, and $w \in V^*$ arbitrary
3	$A \to aB$ or $A \to a$ where $A, B \in N$ non-terminal and $a \in T$ terminal; or $S \to \emptyset$.

Claim: $L = \{0^n 1^n 2^n \mid n > 0\}$ is of type 1 (context-sensitive)

What's next?

- This concludes the material for MATH1064
- The story about models of computation continues in COMP2$\{0, 9\}$22



Alan Turing: A mathematician ;)

- All the other mathematical topics you have learned in this course will re-appear frequently (even though sometimes in disguise)


I
REVISION


I BELIEVE IN YOU!
GOOD LUCK