THE UNIVERSITY OF SYDNEY

MATH1002 LINEAR ALGEBRA

Semester 1 | **Tutorial Exercises for Week 2** | 2021

There are thirteen teaching weeks. There will be a set of preparatory exercises and a set of tutorial exercises each week, except for Week 1. Each set of exercises covers the material from the lectures in the previous week. This is the first set of tutorial exercises; it is based on the lectures from Week 1. Short answers are provided to selected exercises. **Solutions to all exercises will be available from the course webpage at the end of each week.** Questions labelled with an asterisk are suitable for students aiming for a credit or higher.

Tutorial Exercises:

1. Let $\mathbf{a} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

(i) Draw these vectors in standard position in $\mathbb{R}^2$.

(ii) Compute the vectors $\mathbf{a}+\mathbf{b}$, $\mathbf{b}+\mathbf{c}$ and $\mathbf{a}-\mathbf{c}$. How can these results be obtained geometrically?

(iii) Draw the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ with their tails at the point $(2,-1)$.

2. Let $\mathbf{a} = [0, 2, 1]$, $\mathbf{b} = [1, 2, \frac{1}{3}]$ and $\mathbf{c} = [-1, -\frac{1}{2}, 5]$.

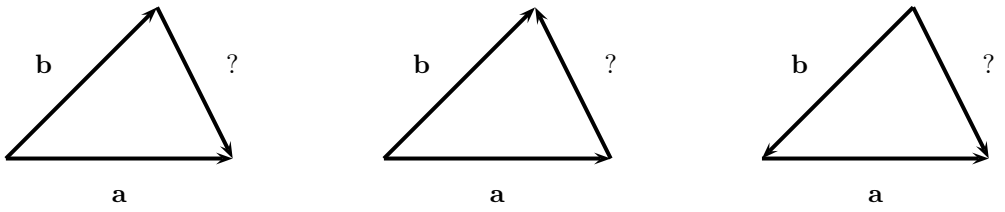
(i) Draw these vectors in standard position in $\mathbb{R}^3$.

(ii) Compute the vectors $2\mathbf{a}+3\mathbf{b}$ and $-\mathbf{a}+4\mathbf{b}-\mathbf{c}$.

3. If the vector $\mathbf{v}$ has length 2, find the length of the vector $\mathbf{u}$ in each of the following cases.

(i) $\mathbf{u} = 3\mathbf{v}$ (ii) $\mathbf{u} = \frac{1}{2}\mathbf{v}$ (iii) $\mathbf{u} = -3\mathbf{v}$ (iv) $\mathbf{v} = 3\mathbf{u}$

4. In each diagram below, find the unknown vector in terms of $\mathbf{a}$ and $\mathbf{b}$.



5. Solve for $\mathbf{x}$ in terms of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ in each case.

(i) $\mathbf{v}+\mathbf{x} = \mathbf{u}-\mathbf{w}$

(ii) $\mathbf{v}-\mathbf{x} = \mathbf{w}-\mathbf{u}$

(iii) $2\mathbf{v}+\mathbf{x} = 2\mathbf{w}-2\mathbf{u}-\mathbf{x}$

6. A balloon experiences two forces, a buoyancy force of 8 newtons vertically upwards and a wind force of 6 newtons acting horizontally to the right. Calculate the magnitude and direction of the resultant force.

7. * Prove the associative law for vector addition: for all vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in $\mathbb{R}^n$,

$$(\mathbf{a}+\mathbf{b})+\mathbf{c} = \mathbf{a}+(\mathbf{b}+\mathbf{c}).$$

8. Express $2\mathbf{a}-3\mathbf{b}$ in terms of $\mathbf{u}$ and $\mathbf{v}$, and simplify, when $\mathbf{a} = \mathbf{u}+\mathbf{v}$ and $\mathbf{b} = 3\mathbf{u}-2\mathbf{v}$.

9. Let $ABCDEF$ be a regular hexagon and put $\mathbf{a} = \overrightarrow{AB}$ and $\mathbf{b} = \overrightarrow{BC}$. Find vector expressions in terms of $\mathbf{a}$ and $\mathbf{b}$ for the displacements $\overrightarrow{CD}$, $\overrightarrow{DE}$, $\overrightarrow{EF}$ and $\overrightarrow{FA}$.

10. A plane travels 20km in the direction 30° north of east and then 10 km southeast. Use trigonometry and your calculator to find the final distance and direction of the aircraft from the starting position.

11. * Prove the following distributive laws:

(i) For all scalars c and all vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, $c(\mathbf{u}+\mathbf{v}) = c\mathbf{u} + c\mathbf{v}$.

(ii) For all scalars c and d and all vectors $\mathbf{u}$ in $\mathbb{R}^n$, $(c+d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$.

Short Answers to Selected Exercises:

1. $\mathbf{a}+\mathbf{b} = \begin{bmatrix}5\\2\end{bmatrix}$, $\mathbf{b}+\mathbf{c} = \begin{bmatrix}2\\3\end{bmatrix}$ and $\mathbf{a}-\mathbf{c} = \begin{bmatrix}3\\-1\end{bmatrix}$.

2. $2\mathbf{a}+3\mathbf{b} = [3, 10, 3]$ and $-\mathbf{a}+4\mathbf{b}-\mathbf{c} = [5, \frac{13}{2}, -\frac{14}{3}]$.

3. (i) 6 (ii) 1 (iii) 6 (iv) 2/3

4. From left to right, $\mathbf{a}-\mathbf{b}$, $\mathbf{b}-\mathbf{a}$ and $\mathbf{a}+\mathbf{b}$.

5. (i) $\mathbf{x} = \mathbf{u}-\mathbf{v}-\mathbf{w}$ (ii) $\mathbf{x} = \mathbf{u}+\mathbf{v}-\mathbf{w}$ (iii) $\mathbf{x} = -\mathbf{u}-\mathbf{v}+\mathbf{w}$

6. 10 newtons, 53° to the horizontal towards the right.

8. $2\mathbf{a}-3\mathbf{b} = -7\mathbf{u}+8\mathbf{v}$

9. $\overrightarrow{CD} = \mathbf{b}-\mathbf{a}$, $\overrightarrow{DE} = -\mathbf{a}$, $\overrightarrow{EF} = -\mathbf{b}$, $\overrightarrow{FA} = \mathbf{a}-\mathbf{b}$.

10. Final distance 25 km, final direction 7° north of east.