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# COMP2823

## Lecture 2b: stacks and queues [GT 2.1]

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## Stacks and queues

These ADTs are restricted forms of List, where insertions and removals happen only in particular locations:

- stacks follow last-in-first-out (LIFO)
- queues follows first-in-first-out (FIFO)

So why should we care about a less general ADT?

- operations names are part of computing culture
- numerous applications
- simpler/more efficient implementations than Lists

# Stack ADT

Main stack operations:

– push(e): inserts an element, e
– pop(): removes and returns the last inserted element

Auxiliary stack operations:

– top(): returns the last inserted element without removing it
– size(): returns the number of elements stored
– isEmpty(): indicates whether no elements are stored

# Stack Example

| operation | returns | stack |
|---|---|---|
| push(5) | - | [5] |
| push(3) | - | [5, 3] |
| size() | 2 | [5, 3] |
| pop() | 3 | [5] |
| isEmpty() | False | [5] |
| pop() | 5 | [] |
| isEmpty() | True | [] |
| push(7) | - | [7] |
| push(9) | - | [7, 9] |
| top() | 9 | [7, 9] |
| push(4) | - | [7, 9, 4] |
| pop() | 4 | [7, 9] |

# Stack Applications

Direct applications

- Keep track of a history that allows undoing such as Web browser history or undo sequence in a text editor
- Chain of method calls in a language supporting recursion
- Context-free grammars

Indirect applications

- Auxiliary data structure for algorithms
- Component of other data structures

# Method Stacks

The runtime environment keeps track of the chain of active methods with a stack, thus allowing recursion

When a method is called, the system pushes on the stack a frame containing

- Local variables and return value
- Program counter

When a method ends, we pop its frame and pass control to the method on top

```
main() {
  int i ← 5;
  foo(i);
}
foo(int j) {
  int k;
  k ← j+1;
  bar(k);
}
bar(int m) {
  …
}
```

bar
PC = 1
m = 6

foo
PC = 3
j = 5
k = 6

main
PC = 2
i = 5

# Parentheses Matching

Each “(”, “{”, or “[” must be paired with a matching “)”, “}”, or “]”

- correct: ( )(( )){([( )])}
- correct: ((( )(( )){([( )])}))
- incorrect: )(( )){([( )])}
- incorrect: ({[ ])}
- incorrect: (

Scan input string from left to right:

- If we see an opening character, push it to a stack
- If we see a closing character, pop character on stack and check that they match

# Stack implementation based on arrays

A simple way of implementing the Stack ADT uses an array:

- Array has capacity N
- Add elements from left to right
- A variable t keeps track of the index of the top element

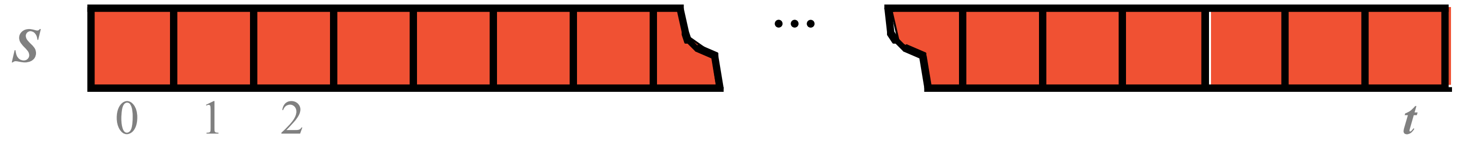
```
def size()
  return t + 1

def pop()
  if isEmpty() then
    return null
  else
    t ← t − 1
    return S[t + 1]
```

# Stack implementation based on arrays

- The array storing the stack elements may become full
- A push operation will then either grow the array or signal a "stack overflow" error.

```
def push(e)
  if t = N – 1 then
    return "stack overflow"
  else
    t ← t + 1
    S[t] ← e
```



# Stack implementation based on arrays

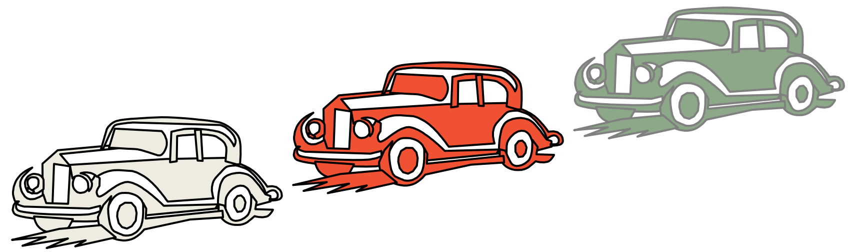
Performance

- The space used is $O(N)$
- Each operation runs in time $O(1)$

Qualifications

- Trying to push a new element into a full stack causes an implementation-specific exception or
- Pushing an item on a full stack causes the underlying array to double in size to make room for new element

# Queue ADT



Main queue operations:

- enqueue(e): inserts an element, e, at the end of the queue
- dequeue(): removes and returns element at the front of the queue

Auxiliary queue operations:

- first(): returns the element at the front without removing it
- size(): returns the number of elements stored
- isEmpty(): indicates whether no elements are stored

Boundary cases:

- Attempting the execution of dequeue or first on an empty queue signals an error or returns null

# Queue Example

| *Operation* | *Output* | *Queue* |
|---|---|---|
| enqueue(5) | – | (5) |
| enqueue(3) | – | (5, 3) |
| dequeue() | *5* | (3) |
| enqueue(7) | – | (3, 7) |
| dequeue() | *3* | (7) |
| first() | *7* | (7) |
| dequeue() | *7* | () |
| dequeue() | *null* | () |
| isEmpty() | *true* | () |
| enqueue(9) | – | (9) |
| enqueue(7) | – | (9, 7) |
| size() | *2* | (9, 7) |
| enqueue(3) | – | (9, 7, 3) |
| enqueue(5) | – | (9, 7, 3, 5) |
| dequeue() | *9* | (7, 3, 5) |

## Queue applications

Buffering packets in streams, e.g., video or audio

Direct applications

- Waiting lists, bureaucracy
- Access to shared resources (e.g., printer)
- Multiprogramming

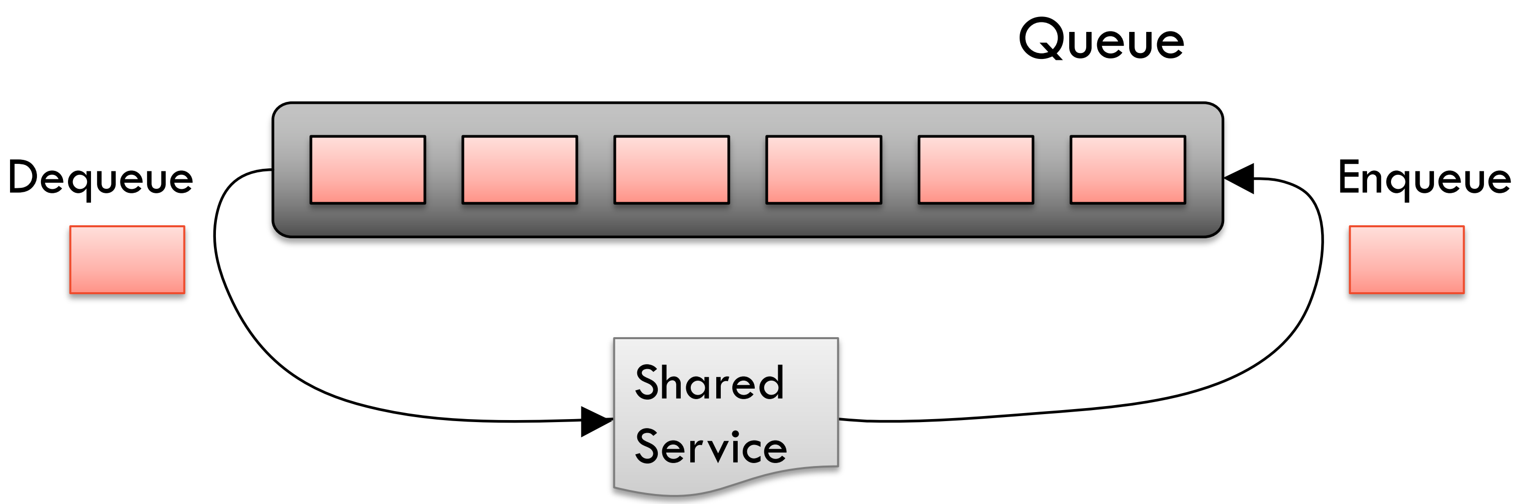
Indirect applications

- Auxiliary data structure for algorithms
- Component of other data structures

# Queue application: Round Robin Schedulers

Implement a round robin scheduler using a queue Q by repeatedly performing the following steps:

1. e ← Q.dequeue()
2. Service element e
3. Q.enqueue(e)



# Queue implementation based on arrays

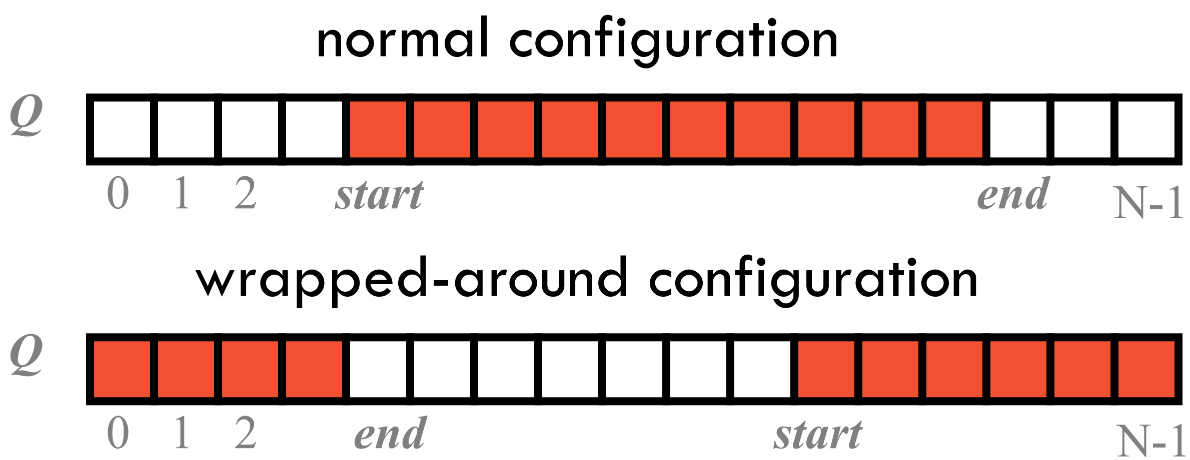
Use an array of size N in a circular fashion

Two variables keep track of the front and size

- start : index of the front element
- end : index past the last element
- size : number of stored elements

These are related as follows end = (start + size) mod N,

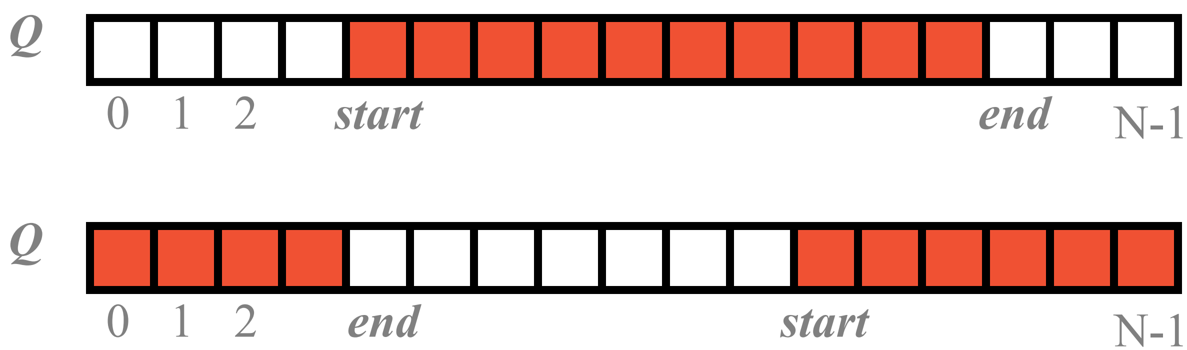
so we only need two, start and size



# Queue Operations: Enqueue

Return an error if the array is full. Alternatively, we could grow the underlying array as dynamic arrays do

```
def enqueue(e)
  if size = N then
    return “queue full”
  else
    end ← (start + size) mod N
    Q[end] ← e
    size ← size + 1
```

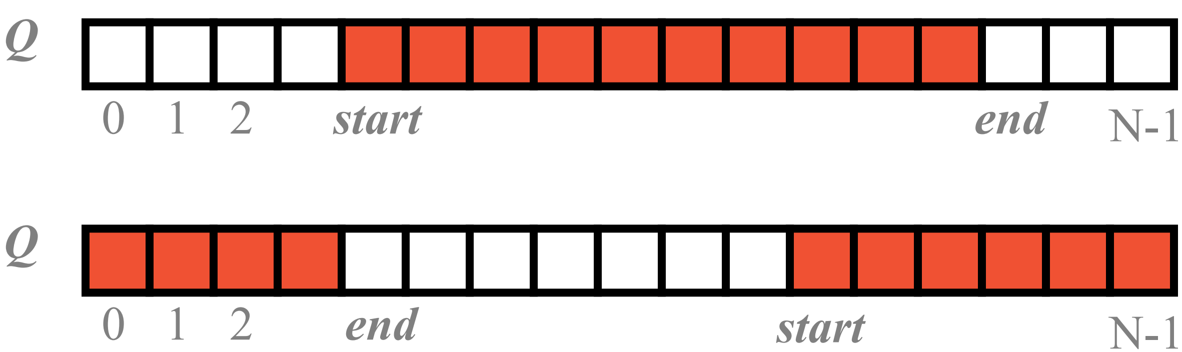


# Queue Operations: Dequeue

Note that operation dequeue returns error if the queue is empty

One could alternatively signal an error

```
def dequeue()
  if isEmpty() then
    return “queue empty”
  else
    e ← Q[start]
    start ← (start + 1) mod N
    size ← (size - 1)
    return e
```



# Stack implementation based on dynamic arrays

```
def push(e)
  # if we run out of space, double size of S
  if t = N - 1 then
    aux ← new array[2*N]
    for j in [0:N] do
      aux[j] ← S[j]
    S ← aux
    N ← 2*N
  t ← t + 1
  S[t] ← e
```

# Amortized analysis

Back to the array-based implementation of lists. Every time we run out of space we double the underlying array.

Recall that worst-case analysis of push takes $O(n)$ time since we may need to double the size of underlying array

Let $T(i)$ be the complexity of $i$-th operation

$$T(i) = \begin{cases} O(i) & \text{if } i = 2^k \\ O(1) & \text{if } i \neq 2^k \end{cases}$$

# Amortized analysis

Starting from the empty list, the total complexity is linear:

Let $T(i)$ be the complexity of the $i$-th operation

$$\begin{aligned}\Sigma_{i<n} T(i) &= \Sigma_{i\neq 2\text{^}k} O(1) + \Sigma_{k<\log n} O(2^k)\\ &= O(n) + O(2^{\log n})\\ &= O(n)\end{aligned}$$

So even though a single operation can take $O(n)$, we can amortize (average) that cost among $n$ operation.

# Amortized analysis definition

We say that a sequence of n operation has O(f(n)) amortized time complexity if in the worst-case the total amount of work done by the n operations is no more than O(n f(n))

For the dynamic array implementation using stack. If we double the size of the array then append takes O(1) amortized time.

# Improved space usage

Current solution is wasteful because if we pop too many items the underlying array may be much larger than the current size of the stack

What if we halve the size of S every time $t < N / 2$?

What if we halve the size of S every time $t < N / 4$?

# This week

Tutorial Sheets 1: Available on Ed

Quiz 1: Available on Canvas

Assignment 1: Available on Ed and Gradescope