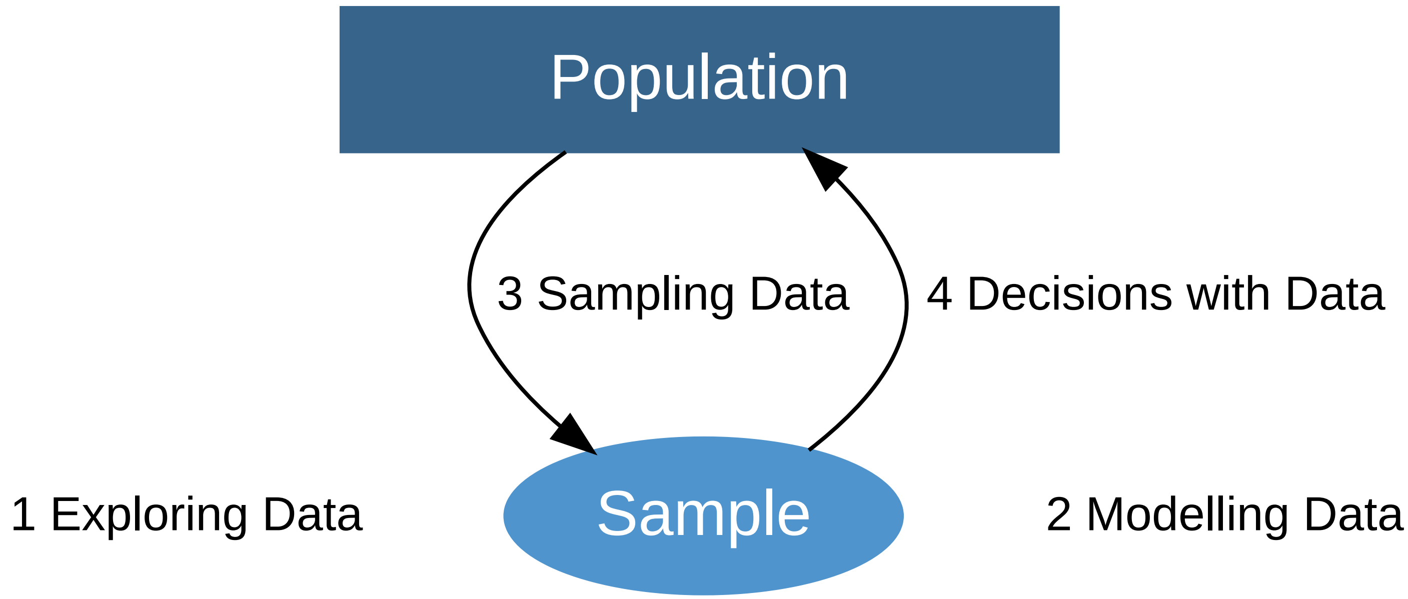
More Chance

Sampling Data | Understanding Chance

Unit Overview



Module3 Sampling Data

Understanding Chance

What is chance?

Chance Variability

How can we model chance variability by a box model?

Sample Surveys

How can we model the chance variability in sample surveys?

More Chance

Data Story

Why did the Chevalier de Mere lose money?

Why did the Chevalier de Mere lose money?

Image

- The Chevalier de Méré was a 17th century gambler, who played 2 games:
  - GameA: Toss a die 4 times. Win = at least 1 “Ace”.
  - GameB: Toss a pair of dice 24 times: Win = at least 1 double-Ace.
  - Note: an “Ace” means “1”.

He reasoned:

Game	1 roll	# rolls	Win
A	P(1 Ace) = 1/6	4	P(at least 1 Ace) = 4x1/6 = 2/3
B	P(double-Ace) = 1/36	24	P(double-Ace) = 24*1/36 = 2/3

Why did he lose money?

Making lists

Making lists

- For simple chance problems, a good way to start is:

1. Write a list of all outcomes

2. Count which outcomes belong to the event of interest.

- This can lead to a simpler way to summarise the outcomes, or using a probability rule.
- This can also to generalising for more complicated problems.

Statistical Thinking

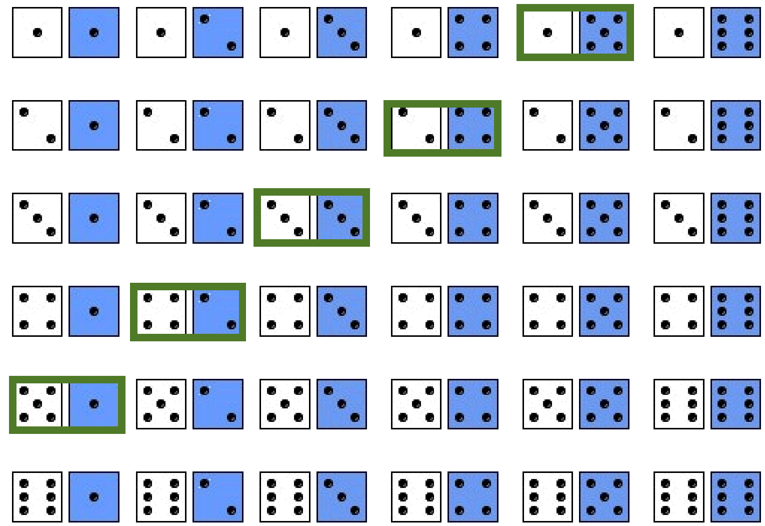
In what situations can ‘lists’ work or not?

Example

Two dice are thrown. What is the chance of getting a total of 6 spots?

Method1: Write a full list of outcomes and count the outcomes of interest.

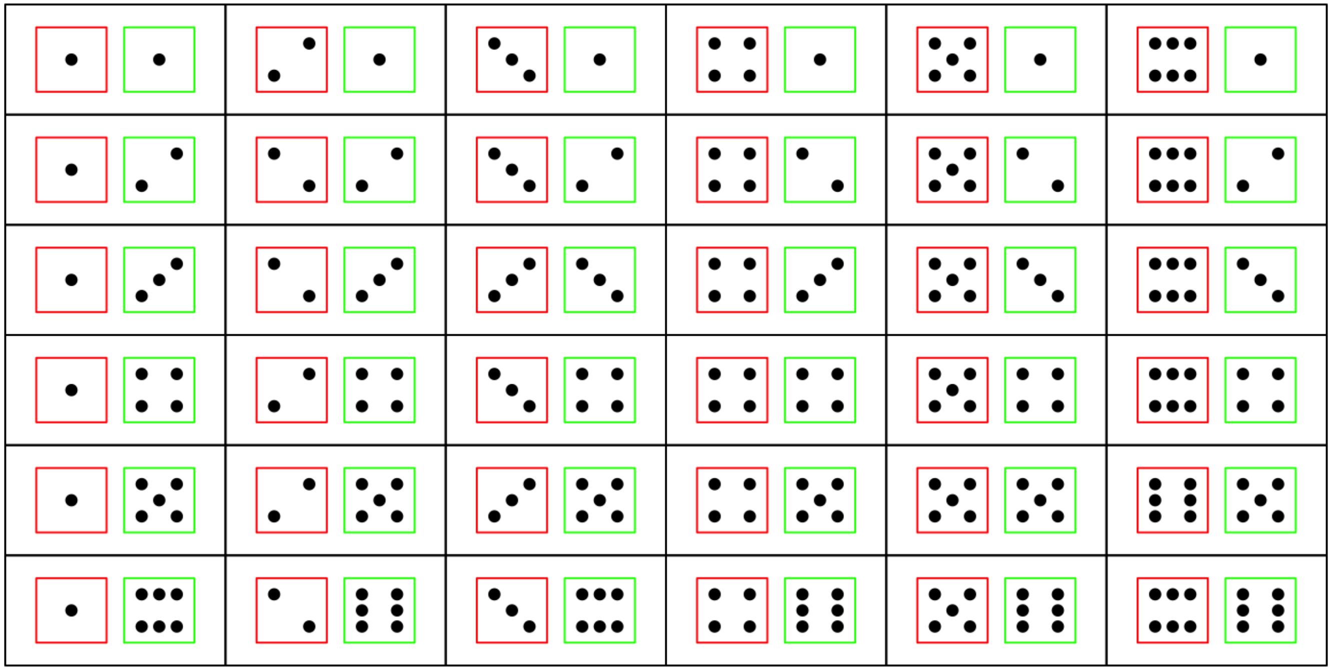
<img src=“figure/dice2.gif” width=“40%”+61403025863 height=“40%”



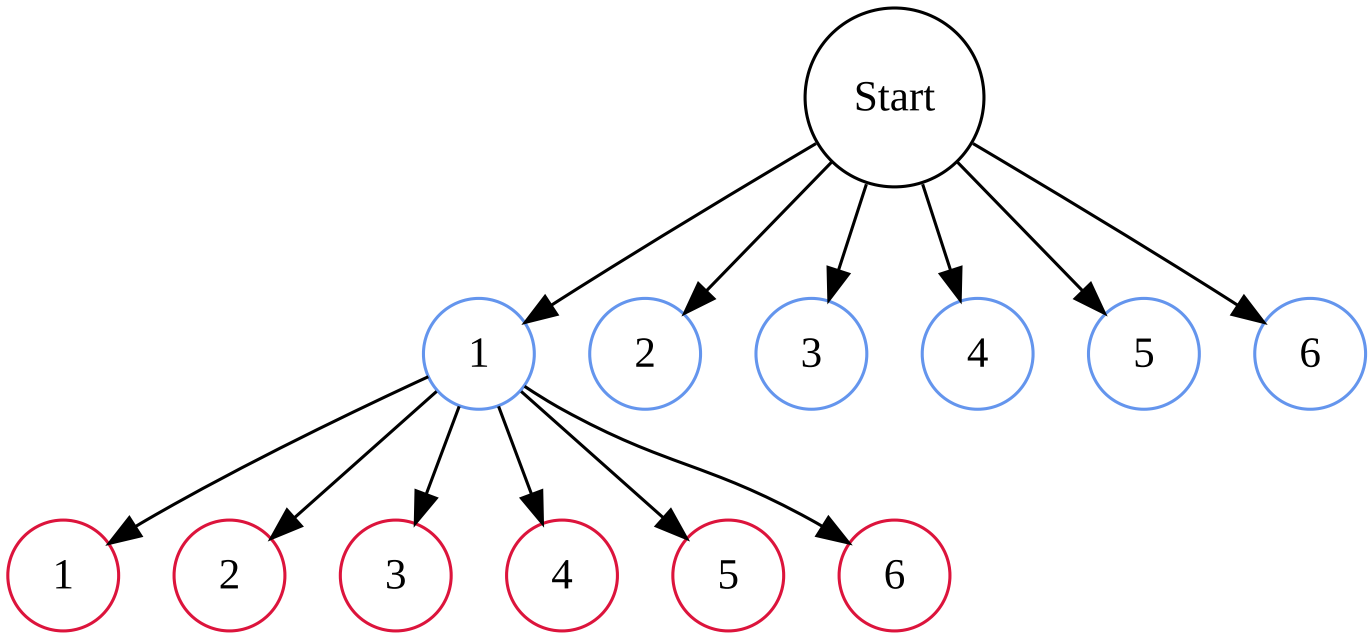
style=“margin:20px 20px”>

So the chance is 5/36 (approx 0.14).

```
library(TeachingDemos)
plot.dice( expand.grid(1:6,1:6), layout=c(6,6) )
```



Method2: Summarise in a tree diagram



The totals of 6 are (1,5), (2,4), (3,3), (4,2), (5,1) giving 5/36.

Method3: Simulate

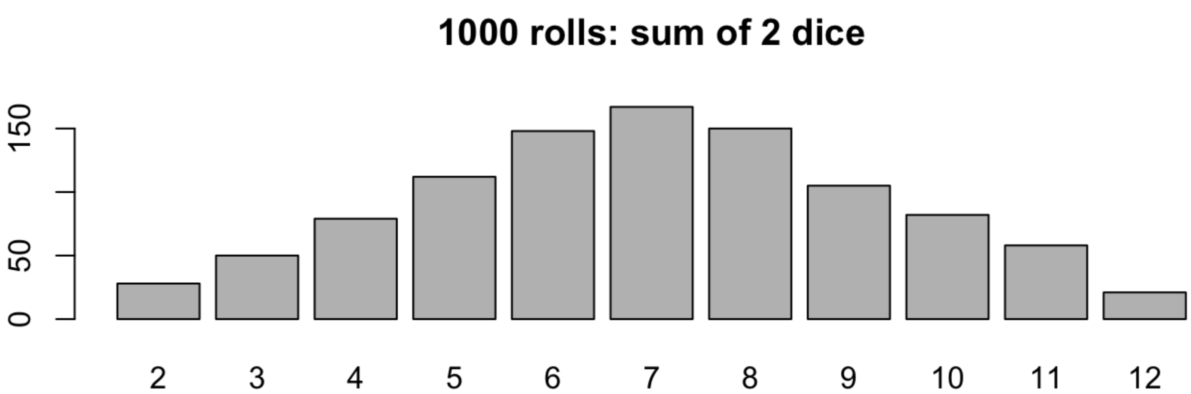
- Physically throw 2 dices x times and record the findings.
- Use an app and record the findings.
- Use R.

Simulate in R

```
set.seed(1)
totals=sample(1:6, 1000, rep = T)+sample(1:6, 1000, rep = T)
table(totals)
```

```
## totals
##   2   3   4   5   6   7   8   9  10  11  12
##  28  50  79 112 148 167 150 105  82  58  21
```

```
barplot(table(totals), main="1000 rolls: sum of 2 dice")
```



So the (simulated) chance of getting a total of 6 is 148/1000 = 0.148, which is very close to the exact answer of 5/36.

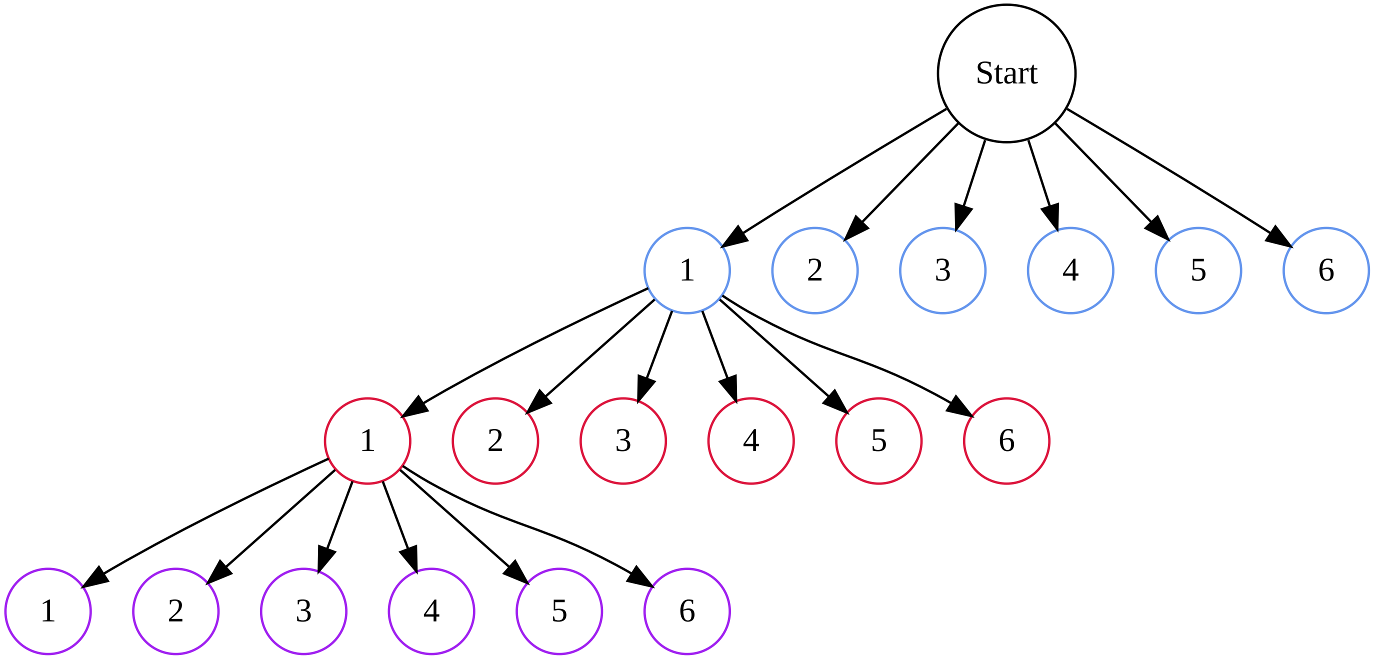
Example

Three dice are thrown. What is the chance of getting a total of 6 spots?

Method1: Write a list manually

- Total outcomes is 6x6x6 = 216
- Totals of 6: (1,1,4) (1,2,3) (1,3,2) (1,4,1) (2,1,3) (2,2,2) (2,3,1) (3,1,2) (3,2,1), (4,1,1)
- So exact chance of getting total of 6 is 10/216 (approx 0.05).

Method2: Summarise in a tree diagram

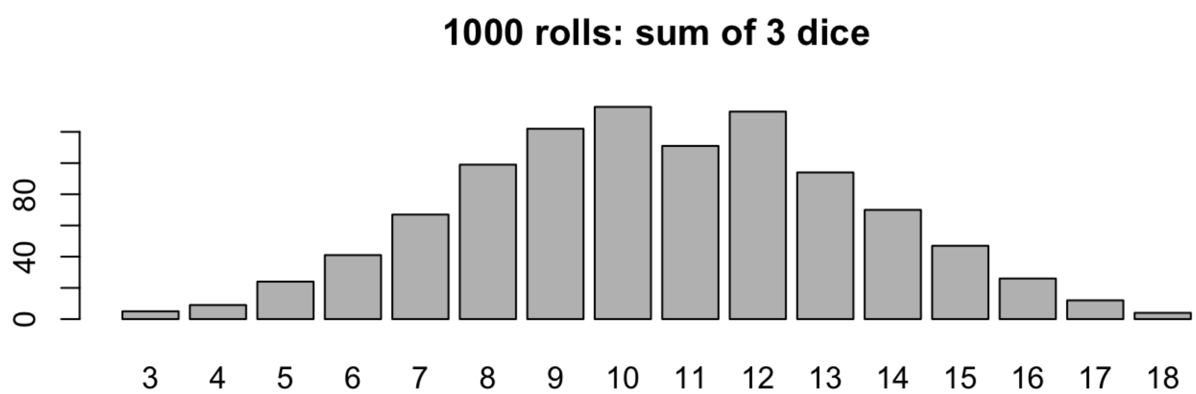


Method3: Simulate in R

```
set.seed(1)
totals=sample(1:6, 1000, rep = T)+sample(1:6, 1000, rep = T)+sample(1:6, 1000, rep = T)
table(totals)
```

```
## totals
##   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18
##   5   9  24  41  67  99 122 136 111 133  94  70  47  26  12   4
```

```
barplot(table(totals), main="1000 rolls: sum of 3 dice")
```



The (simulated) chance of getting a total of 6 is 41/1000 = 0.041.

🔗 App

Addition Rule

Addition Rule

Mutually exclusive

2 things are **mutually exclusive** when the occurrence of one event prevents the other.

Addition Rule

- If 2 things are **mutually exclusive** then the chance of at least 1 occuring is the sum of the individual chances.

Common FAQs

###1. What’s the difference between mutually exclusive and independence?

Term	Definition
Mutually exclusive	the ocurrence of Event1 prevents Event2 occuring
Independence	the ocurrence of Event1 does not change the chance of Event2

###2. When do I add and when do I multiply?

What	When	Formula	Condition
Addition Rule	P(At least 1 of 2 events occurs)	P(Event1) + P(Event2)	if mutually exclusive
Multiplication Rule	P(Both events occur)	P(Event1) x P(Event2)	if independent
		P(Event1) x P(Event2, given Event 1)	if dependent

Example

A die is rolled 6 times and a deck of cards is shuffled. Fill out the table, for the following questions.

What is chance that:

- the 1st roll is a 1 or the 6th roll is a 1?
- both the 1st and 6th rolls are 1s?
- the top card is the ace of spades or the bottom card is the ace of spades?
- both the top card and the bottom card are the ace of spades?

How the Chevalier de Mere stopped losing

How the Chevalier de Mere stopped losing

- The Chevalier de Mere got advice from the philosopher Blaise Pascal, who got advice from his friend Pierre de Fermat.

They reasoned:

Game	1 roll	# rolls	P(no Win)	P(Win)
A	P(not Ace) = 5/6	4	P(no Aces) = (5/6)^4	1-(5/6)^4 = 0.518
B	P(not double-Ace) = 35/36	24	P(no double-Aces) = (35/36)^24	1-(35/36)^24=0.49

- Considering the complement event, makes each of the complements mutually exclusive, so the solution follows easily.
- So it’s slightly better to play GameA.

Using simulation

```
experimentA <- function(){
  rolls <- sample(1:6, size = 4, replace = TRUE)
  condition <- sum(rolls == 6) > 0
  return(condition)
}
simsA <- replicate(100000, experimentA())
sum(simsA)/length(simsA)
```

```
## [1] 0.51541
```

```
gameB <- function(){
  first.die <- sample(1:6, size = 24, replace = TRUE)
  second.die <- sample(1:6, size = 24, replace = TRUE)
  condition <- sum((first.die == second.die) & (first.die == 1)) > 0
  return(condition)
}
simsB <- replicate(100000, gameB())
sum(simsB)/length(simsB)
```

```
## [1] 0.49275
```

Summary

Addition Rule

- 2 events are mutually exclusive when the occurrence of one event prevents the other.
- If 2 events are mutually exclusive then the chance of **at least 1 event** occuring is the **sum** of the individual chances.

Multiplication Rule

- 2 events are independent if the ocurrence of the first event does not change the chance of the second event.
- If 2 events are independent then the chance of **both** events occuring is the **multiplication** of the individual chances.

Key Words

mutually exclusive, addition rule, simulation