

Initial proof:

$$\begin{array}{c}
\frac{\frac{C \rightarrow C}{C \rightarrow C, A} \quad \frac{A \rightarrow A}{A \rightarrow C, A}}{(C \vee A) \rightarrow C, A} \vee_l \\
\frac{}{(C \vee A) \rightarrow (C \vee A)} \vee_r \\
\hline
\frac{}{(C \vee A) \rightarrow (C \vee A), (((\neg(C \vee A)) \wedge ((B \supset (C \vee A)) \supset (C \supset (C \vee A)))) \supset ((C \vee A) \wedge (\neg D)))} \\
\frac{}{((\neg(C \vee A)) \wedge ((B \supset (C \vee A)) \supset (C \supset (C \vee A)))) \supset ((C \vee A) \wedge (\neg D)), (C \vee A) \rightarrow (C \vee A)} \vee_l \\
\hline
\frac{}{(C \vee A) \rightarrow (C \vee A)} \text{cut}
\end{array}$$

Cut-free proof:

$$\begin{array}{c}
\frac{\frac{C \rightarrow C}{C, (C \vee A) \rightarrow (C \vee A), C, A, C, C, A, C, A} \quad \frac{A \rightarrow A}{A, (C \vee A) \rightarrow (C \vee A), C, A, C, C, A, C, A}}{(C \vee A), (C \vee A) \rightarrow (C \vee A), C, A, C, C, A, C, A} \vee_l \\
\frac{}{C, (C \vee A), (C \vee A), (C \vee A) \rightarrow (C \vee A), C, A, C, A, C, A, C, A, C, A, C, A} \\
\frac{}{A, (C \vee A), (C \vee A), (C \vee A) \rightarrow (C \vee A), C, A, C, A, C, A, C, A, C, A, C, A} \vee_l \\
\hline
\frac{}{(C \vee A), (C \vee A), (C \vee A), (C \vee A) \rightarrow (C \vee A), C, A, C, A, C, A, C, A, C, A, C, A} \\
\frac{}{(C \vee A), (C \vee A) \rightarrow (C \vee A), (C \vee A), C, (C \vee A), C, C, A} \vee_r \\
\frac{}{(C \vee A), (C \vee A) \rightarrow (C \vee A), (C \vee A), C, (C \vee A), C, (C \vee A)} \vee_r \\
\frac{}{(C \vee A), (C \vee A) \rightarrow (C \vee A), (C \vee A), C, (C \vee A), C, A} \vee_r \\
\frac{}{(C \vee A), (C \vee A) \rightarrow (C \vee A), (C \vee A), C, (C \vee A), (C \vee A)} \vee_r \\
\frac{}{(C \vee A), (C \vee A) \rightarrow (C \vee A), (C \vee A), (C \vee A), C, C, A} \vee_r \\
\frac{}{(C \vee A), (C \vee A) \rightarrow (C \vee A), (C \vee A), (C \vee A), C, (C \vee A)} \vee_r \\
\frac{}{(C \vee A), (C \vee A) \rightarrow (C \vee A), (C \vee A), (C \vee A), C, A} \vee_r \\
\frac{}{(C \vee A), (C \vee A) \rightarrow (C \vee A), (C \vee A), (C \vee A), (C \vee A)} \vee_r \\
\hline
(C \vee A) \rightarrow (C \vee A)
\end{array}$$