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Versuch: Lebensdauer des Positroniums

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Inhaltsverzeichnis

1 Experiment background and Theoretical principles

1.1 Positron-Electron Annihilation Process

The annihilation of a positron and an electron produces γ photons, primarily through two-photon and three-photon annihilation processes. Among these, two-photon annihilation is the most common. When the positron and electron are at rest, two photons are emitted in exactly opposite directions, each with an energy of 511 keV/c^2, satisfying energy and momentum conservation. In three-photon annihilation, the photons are emitted in a single plane with random energy distributions. Single-photon annihilation, due to its extremely small cross-section, is generally negligible.
The spin states of the leptons determine the annihilation modes:

- **Singlet state** ($I = 0$): The spins are anti-parallel, allowing two-photon annihilation.
- **Triplet state** ($I = 1$): The spins are parallel, requiring at least three photons to conserve angular momentum.

The cross-section for two-photon annihilation is given by:

$$\sigma_{2\gamma} = \pi r_0^2 \frac{c}{v}$$

where r_0 is the classical electron radius, c is the speed of light, and v is the relative velocity of the leptons. The cross-section increases significantly as the relative velocity decreases. In comparison, the cross-section for three-photon annihilation is only $\alpha/8$ of that for two-photon annihilation, where $\alpha \approx 1/137$ is the fine-structure constant.
The ratio of probabilities between three-photon and two-photon annihilation can be expressed as:

$$\frac{\sigma_{3\gamma}}{\sigma_{2\gamma}} = \frac{3}{8}\alpha \approx \frac{1}{372}$$

This indicates that the probability of three-photon annihilation is much smaller than that of two-photon annihilation. Considering statistical weights, the singlet state has a lifetime approximately one-fourth that of the triplet state.
This significant difference in annihilation probabilities provides an experimental basis for distinguishing between free positrons and positronium (a bound state of a positron and an electron), making it a critical tool for studying positron annihilation behaviors.

1.2 Positronium

1.2.1 Principle

The third task of the experiment involves determining the mean lifetime of positronium in plexiglass. In this task, positrons, which can form a bound state with electrons known as positronium, interact with the material to form positronium atoms.
In positronium, the positron replaces the hydrogen nucleus, and the energy levels are calculated similarly to the hydrogen atom. Since the positron's mass is approximately the same as the

electrons mass, the reduced mass of the positronium is about half of the electron's mass. The binding energy of positronium is 6.8 eV, roughly half of the hydrogen atom's binding energy.
Positronium exists in two spin configurations: the singlet state and the triplet state. The singlet state is called para-positronium, while the triplet state is called ortho-positronium. The average lifetime of para-positronium is approximately 1.25×10^{-10} s, which is similar to the lifetime of free annihilation, while the average lifetime of ortho-positronium is much longer, about 1.4×10^{-7} s. Therefore, ortho-positronium almost never decays directly from the excited state, but instead transfers to the ground state before decaying.
Experimental observations show that the positron is first decelerated by inelastic collisions with electrons, just like fast electrons. The deceleration is very rapid, occurring in approximately 10^{-12} s in solids. After this, the positron's energy decreases to the eV range, where annihilation and positronium formation become noticeable. In this energy range, no other inelastic processes occur except for positronium formation. To release an electron from an atom, the positron must have a minimum kinetic energy given by $E_{\text{kin}} = V - 6.8\,\text{eV}$, where V is the ionization energy of the atom. The maximum energy in this range is $E_{\text{max}} = E_a$, the lowest excitation energy of the atom. This energy gap is called the Ore-Gap, which represents the energy interval in which positronium formation occurs.

1.2.2 Detection

Detecting positronium requires more than just identifying the two- or three-photon decay processes, as these could also arise from free annihilation. There are two main approaches to distinguish positronium from free decay. One method involves analyzing the γ-spectrum, specifically the ratio of short-lived to long-lived decay components. In the case of free annihilation, this ratio follows a specific formula. For positronium, the decay channels correspond to different spin states: ortho-positronium (which is triplet state) decays three times more often than para-positronium (singlet state), as the latter is less likely to form. Thus, a higher frequency of decays from the long-lived component indicates positronium formation.
The second method is based on the pressure dependence of the lifetimes observed in gases. In such environments, the long-lived component, associated with ortho-positronium, does not change with pressure, whereas the short-lived component, linked to free annihilation, decreases as pressure increases. This difference provides insight into the nature of the positronium decay.
In solids, positronium behaves differently than in gases. The lifetime of positronium in solids is much shorter, and the decay is characterized by two-photon processes instead of three. This is because positronium in solid materials undergoes interactions with the surrounding electrons or molecular magnetic fields. These interactions can either convert ortho-positronium into para-positronium or even destroy it, resulting in a shorter lifetime for the triplet state. This phenomenon is known as the "pick-off" process, where a γ-quanta with a fixed energy is emitted. The average lifetime of ortho-positronium in solids, considering these interactions, can be described by the following relation:

$$\frac{1}{\tau_{\text{Ortho}}} = \frac{1}{\tau_0} + \frac{1}{\tau_P} + \frac{1}{\tau_K}$$

where τ_0 represents the intrinsic lifetime, and τ_P and τ_K account for the effects of the "pick-off" and conversion processes, respectively. These processes are influenced by the properties of the solid material and can be quantified through appropriate models.

1.2.3 Measurement

The measurement of the mean lifetime of positrons requires time-tagged events, and the β^+decay of ^{22}Na provides an effective method for this purpose. During the transition, a γ photon with

an energy of 1.276 MeV is emitted, and this process occurs on an extremely short timescale (approximately 10^{-13} seconds), which can be used as a time marker for the generation of positrons. The deceleration time of positrons in matter is also very short (approximately 10^{-12} seconds), making it a reliable time marker for the formation of positronium.
Using this time-tagging method, the entire process from positron generation to annihilation can be accurately correlated, allowing the statistical determination of its lifetime distribution.

2 Experimental setup

2.1 Analog-to-Digital Converter

An Analog-to-Digital Converter (ADC) transforms an analog signal into a digital signal through sampling, quantization, and encoding. The signal is sampled at a rate $f_s \geq 2f_{\max}$, where $f_{\max}$ is the maximum signal frequency. Quantization maps the sampled values to discrete levels with a step size

$$\Delta = \frac{V_{\text{ref,max}} - V_{\text{ref,min}}}{2^N}$$

, introducing a maximum quantization error of $E_q = \frac{\Delta}{2}$. Finally, the quantized values are encoded into binary form, with the digital output given by $D = \text{round}\left(\frac{x[n]-V_{\text{ref,min}}}{\Delta}\right)$. These steps enable ADCs to represent continuous signals in a discrete digital format.

2.2 Time-to-Pulseheight-Converter

A time-to-pulse height converter, hereinafter referred to as TPC, is an analog instrument in which the time difference between two signals is converted into an analog signal. In the following, only the function of the TPC used is described. When the start signal arrives, a capacitor is charged with a constant current source. When the stop signal arrives, the time can be determined by the voltage across the capacitor, which is proportional to the voltage across the capacitor:

$$U_{\text{C}} = \frac{Q}{C} = \frac{It}{C}$$

2.3 Discriminator

The discriminator only emits a logical pulse if the signal is between two adjustable thresholds. It is used in the experiment to distinguish between the events of positron-electron annihilation and the de-excitation of the excited state. It is connected to a zero-crossing trigger to minimize the walk of the signal. The zero-rossing trigger always triggers at the same time in relation to the signal. For this purpose, the input signal is split. One part is delayed by a fixed, small time which must be less than the rise time and amplified by a small amount. The other signal is inverted. Both are superimposed. The first zero crossing of the signal is selected as the trigger time because, unlike the trigger time of the discriminator, it always takes place at the same time as the signal. Because the zero-crossing trigger triggers at every signal it is conected to the discriminator through an AND-Gate. The resulting signal can be used as the start sgnal for the TPC

2.4 Time delay

Switches can be used to connect coaxial cables of different lengths to the signal line of the stop signal. The signal is delayed by the extended travel time of the signal through the cable. The operation is similar to the binary system and can be set up to $63\,ns$.

2.5 Detector choice

In this experiment, we choose the fast plastic scintillators combined with photo-multiplier tubes (PMTs) as detectors. Compared to the inorganic crystal scintillators used in Compton scattering experiments, organic materials were chosen for this study based on the following considerations: First, there is a fundamental difference in detection mechanisms between the two types of scintillators. Plastic scintillators have a lower average nuclear charge number (approximately 2.7), so gamma photons primarily interact with them via Compton scattering rather than the photoelectric effect. In contrast, inorganic scintillators, such as NaI(Tl) or $CeBr_3$, possess a higher nuclear charge number (around 32), which significantly enhances the likelihood of the photoelectric effect, resulting in higher light yields and better energy resolution.
Second, the choice of detector aligns with the experimental objectives. In Compton scattering experiments, the focus lies on studying the scattering characteristics of gamma photons, making energy resolution critical; thus, $CeBr_3$ scintillators with high light yields are preferred. Conversely, in this positron annihilation experiment, the primary goal is to measure the time distribution of gamma photons produced during annihilation to distinguish between positronium states and free annihilation signals. Time resolution is therefore the top priority. Fast plastic scintillators, with their exceptionally short fluorescence decay time (approximately 8 nanoseconds), far outperform inorganic scintillators , which typically exhibit decay times around 250 nanoseconds. This makes plastic scintillators particularly well-suited for applications requiring nanosecond-level time resolution.
Furthermore, the combination of plastic scintillators with PMTs generates fast and stable electronic signal pulses. This characteristic is especially advantageous for detecting high-frequency events in positron annihilation experiments, effectively avoiding issues such as signal pile-up caused by longer response times and ensuring the accuracy and reliability of experimental data. In summary, the superior time resolution and efficient signal response of fast plastic scintillators make them the ideal choice for this experiment.

2.6 Measurement principle

In the experiment, the time difference is calibrated by setting start and stop signals. The start signal typically comes from a known radiation source, such as γ photons from ^{22}Na , while the stop signal is determined by detecting another event related to the start signal, such as γ photons from annihilation. The time measurement records the time difference between the start and stop signals. The signals reach the TPC and generate a pulse spectrum, which is converted by the ADC of the MCA. However, there is a certain delay in the time measurement, so a calibration method is required to ensure the accuracy of the time measurement.

3 experimental execution,evaluation, error calculation and discussion of the measurement results

3.1 ^{22}Na-spectrum and adjustment of the discriminator

First, the discriminator is set to a low energy threshold and a spectrum is recorded 3.1. The Compton continuum of the γ decay and that of the $\beta+$ decay, which is lower in energy, can be seen. In comparison to the spectrum measured with the anorganic scintillator used in the Compton-effect experiment no photo peaks can be seen. This is because the electrons are bound weaker. The discriminator is now set so that only events of the γ decay can be seen. To do this, the spectrum is reset and the lower energy threshold of the discriminator is set so that no new events occur in the blue range of 3.1. The setup can now be used for the decay measurement.

3.2 decay measurement

In the experiment ^{22}Na is used. The emission of the *gamma*- photon can be estimated as the time the positronium is formed, because the mean lifetime of the exited state is at least three orders of magnitude lower than the mean lifetime of the positronium. The time it takes for the positronium to decelerate is at least two orders of magnitude lower. So this is is a good approximation in the context of the experiment.
The output of the energy discriminator is now connected to the TPC which in turn is connected to the ADC which now displays a time spectrum instead of an energy spectrum. The time spectrum is now recorded for approx. one hour 3.2b. The spectrum represents the superposition of the decays of the paraprositronium and the orthopositronium, each of which decays exponentially with the decay time τ. This can be clearly seen in the logarithmic representation 3.2a. Two linear areas can be recognized. The values before the start event according to the time calibration are shown in orange. Since the measured values have a certain uncertainty due to uncertainties in the time measurement and signal processing, values with negative time also occur. As these

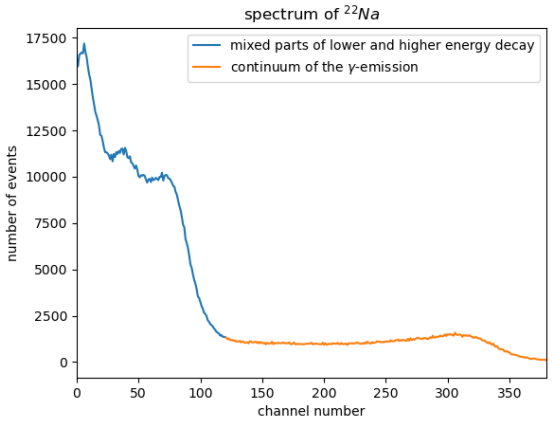
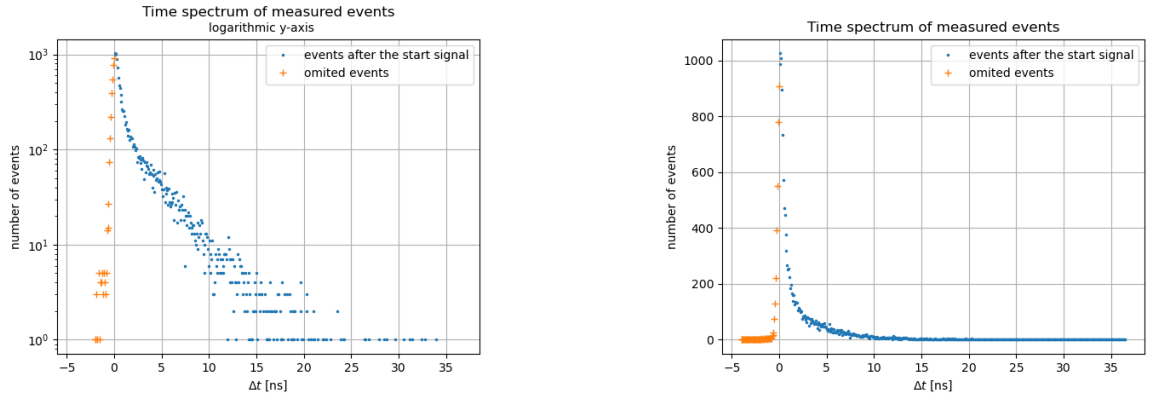


Abbildung 3.1: Spectrum of ^{22}Na with plastic scintillator

values only occur up to approx. $-0.5\,ns$ and are not uniformly distributed, the influence of random coincidences is therefore ruled out. The scatter is comparable to the scattering in the time calibration.



(a) logarithmic axis

(b) linear axis

Abbildung 3.2: Data of the decay

The function $Ae^{-\frac{t}{\tau_1}} + Be^{-\frac{t}{\tau_2}} + C$ is given as the function for the decay. However, as the values are scattered due to the time measurement, the function of the apparatus has to be taken into account also. In section 3.3 and 3.4 this distribution can be seen which is in good approximation a normal distribution. Therefore the exponential distribution is convolved with a normal distribution which results in the exponentially modified Gaussian distribution[1][2]

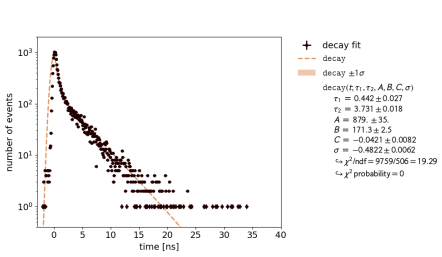
$$\frac{\lambda}{2} e^{\frac{\lambda}{2}(2\mu+\lambda\sigma^2-2t)} \operatorname{erfc}\left(\frac{\mu+\lambda\sigma^2-t}{\sqrt{2}\sigma}\right)$$

The constant C has no physical meaning, but simplifies the fit.
For the fit, the normalizing factor was omitted and the decay constant λ was used instead of τ, both of which simplify the fit.

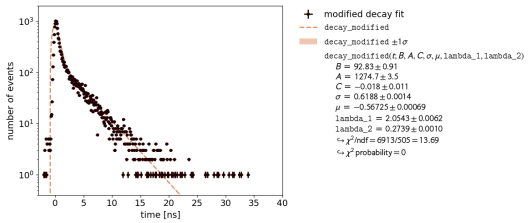
$$Ae^{\frac{\lambda_1}{2}(2\mu+\lambda_1\sigma^2-2t)} \operatorname{erfc}\left(\frac{\mu+\lambda_1\sigma^2-t}{\sqrt{2}\sigma}\right) + Be^{\frac{\lambda_2}{2}(2\mu+\lambda_2\sigma^2-2t)} \operatorname{erfc}\left(\frac{\mu+\lambda_2\sigma^2-t}{\sqrt{2}\sigma}\right) + C$$

After the fit, A and B are multiplied by the mean lifetime to later determine the ratio of decays. In addition the fit is performed with the given function to compare the results. The values less than zero are taken into account in the fit with a normal distribution, even if this is only a rough approximation. for determining the ratio A and B are also multiplied by the mean lifetime which this time is just an approximation as this only normalizes the exponential decay not the normal distribution on the left.
The error of the time calibration 3.3 and a standard deviation of the time calibration peaks, which is ~ 3.5 channels is assumed as the time error. This results in $\tau_1 = 0.4868 \pm 0.0015\,ns$ and $\tau_2 = 3.652 \pm 0.014\,n$ for the decay times, with the short-lived decay occurring more frequently than the long-lived decay. The value for orthopositronium is given as $\tau_{\text{ortho}} = 2-4\,ns$ which is in accordance with our values. The decay time of the free decay $\tau_{\text{free}} = \sim 0.5\,ns$ is in accordance with our values. The ratio of the decays, which is $\frac{A\tau_1}{B\tau_2} = 1.831 \pm 0.021$, is not within the range, however the magnitude of A is probably underestimated as can be seen in 3.3b. The lower mean lifetime decay should occur $60\% - 80\%$ of the time and the higher one $10\% - 30\%$ of the time which would mean the ratio should actually be around $2-8$. With some more time the fit could maybe be tweaked to get more accurate values. The decays of the paraprositronium, which represents

[1] `https://en.wikipedia.org/wiki/Exponentially_modified_Gaussian_distribution`
[2] s. Mathematica notebook



(a) regular Function



(b) exponentially modified Gaussian distribution

Abbildung 3.3: fits of the decay

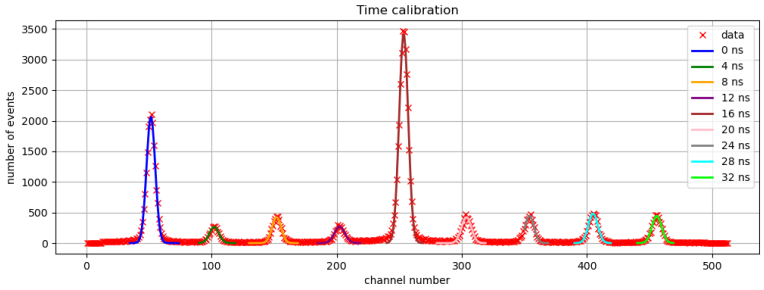


Abbildung 3.4: Data of the time calibration with fitted normal distributions

$\sim 10\%$ of the decays, could not be detected because their low mean lifetime. The values from the regular function aren't that different from the modified one, except for the ratio of decays. After multiplying with τ the Data obtained using the given function would indicate that the orthopositronium decays about 1.6 times more often then free decays. Also the goodness of fit is slightly worse. Though both fits have low probability with zero and the $\chi^2/ndf = 13.69$ of the ex-gauss fit is big. The errors for the time and number of events were probably underestimated.

3.3 Time calibration

The aim is to assign the appropriate delay to the channels. The measurement for time calibration was carried out with a low lower discriminator threshold. The measurement principle is that in electron-positron-annihilation with $S = 0$ two *gamma*-photons are sent out simultaneous at an angle of 180°. The time difference between start and stop signal can then be used to calibrate the channels.
For time calibration, this delay is increased in $4\,ns$ steps to $32\,ns$. A normal distribution is fitted to the resulting distributions. A fit can then be used to interpolate from the known delay and the expectation value of the distributions. The standard deviation divided by the square root of the number of events in the range of one standard deviation is used as the error of the channels. The processing of the signals, especially the start signal, delays them. The end signal is therefore constantly delayed by $2\,ns$. By carrying out the time calibration, the positioning of the sample and the associated differences in travel times are also taken into account. They would only be relevant if the transit time exceeds the delay of the stop signal (too far to the right), but this is unrealistic and cannot occur in the experiment. The walk of the start signal at the discrete trimmer is negligible due to the use of a zero-crossing trigger. The resulting small time delay is taken into account by the time calibration.
The fit 3.5 is good the probability is high with 0.96 and the $\chi^2/ndf = 0.2843$ is also good. The obtained line can now be used to convert the channel to the time in 3.4 and 3.2.

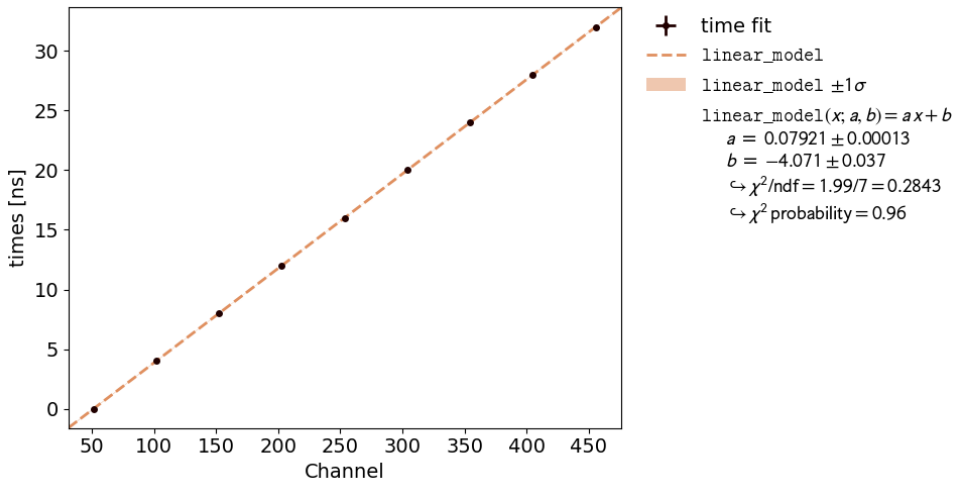


Abbildung 3.5: Fit for the time calibration

3.4 speed of light

The settings are the same as in 3.3. The detector that sends the stop signal is now moved 10 cm away from the sample after each measurement. Normal distributions are again fitted to the spectra.
Now the speed of light is determined from the travel time. The exact positioning of the sample and the left detector are irrelevant as they remain constant. A straight line is plotted against the distance over the travel time. The uncertainty of the distance is estimated to be $3\,mm$, the uncertainty of the runtime is the standard deviation of the peaks divided by the square root of the number of events in the range of a standard deviation and the small uncertainty of the time calibration.

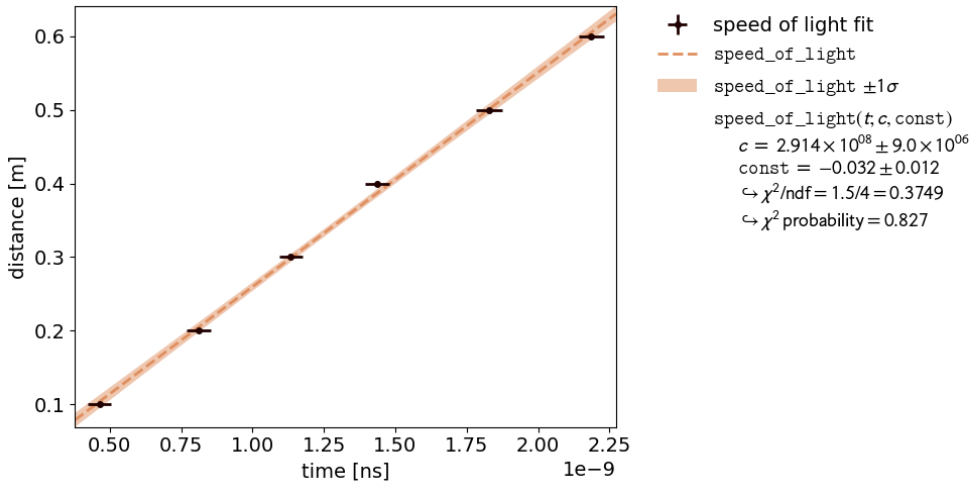


Abbildung 3.6: speed of light fit

The fit 3.6 is good with a probability of 0.827 and $\chi^2/ndf = 0.3749$. The fit yields $c = (2.914 \pm 0.09) \cdot 10^8 \frac{m}{s}$. The literature value[3] $c_{\text{lit}} = 299792458 \frac{m}{s}$ is within the range of uncertainty. The value could be determined more accurately with more careful positioning and longer measurements. Especially at large distances, only a few gammas hit the scintillator so those measurements would benefit from longer times.

[3] https://physics.nist.gov/cgi-bin/cuu/Value?c