$$I = \lim_{\varepsilon \to 0} \int_{\varepsilon}^{1} \frac{\cos(\log(x)/x)}{x} dx$$

$$z = \frac{1}{x} \quad dx = -\frac{1}{z^2} dz$$

$$= -\lim_{\varepsilon \to 0} \int_{\frac{1}{\varepsilon}}^{1} \frac{\cos(\log(\frac{1}{z}) \cdot z)}{\frac{1}{z}} \frac{1}{z^2} dz$$

$$= \int_{1}^{\infty} \frac{\cos(\log(z) \cdot z)}{z} dz$$

$$x \log(x) = y \rightarrow dy = (\log(x) + 1) dx$$

$$I = \int_1^{\infty} \frac{\cos(x \log(x))}{x} dx = \int_{y(1)}^{y(\infty)} \frac{\cos(y)}{x \cdot (\log(x) + 1)} dy$$

$$= \int_0^{\infty} \frac{\cos(y)}{y + x(y)} dy$$

$$I = \int_0^{\infty} \frac{\cos(y)}{y + x(y)} dy$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos(z)}{z + x(z)} dz + \lim_{n \to \infty} \sum_{k=1}^{n} \int_{-\frac{\pi}{2} + k\pi}^{\frac{\pi}{2} + k\pi} \frac{\cos(y)}{y + x(y)} dy$$

$$y = z + k\pi$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos(z)}{z + x(z)} dz + \lim_{n \to \infty} \sum_{k=0}^{n} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos(z + k\pi)}{z + k\pi + x(z + k\pi)} dz$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos(z)}{z + x(z)} dz + \lim_{n \to \infty} \sum_{k=0}^{n} (-1)^k \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos(z)}{z + k\pi + x(z + k\pi)} dz$$

$$f(x) = x \log(x) - y \qquad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = \log(x) + 1 \qquad = x_n - \frac{x_n \log(x_n) - y}{\log(x_n) + 1}$$

$$= \frac{x_n + y}{\log(x_n) + 1}$$