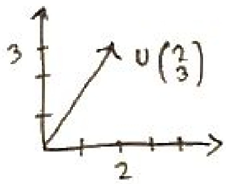
W1 L1 :-

Vectors → Mag & Direction.

$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ - 2 component vectors with components u_1 & u_2.



here the components are real numbers

$u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}$ → n component vectors

→ ordered n tuple.

Suppose we measure the biological parameters

n components → n parameters we measure

For eg:-

u_1 → Blood Pressure

u_2 → Blood glucose level.

u_3 → Pulse rate

u_4 → Heart rate

u_5 → SPO_2 Level

$U = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{pmatrix}$ → Collection of 5 parameters

> Collect these 5 Parameter from 100 patients

	u_1	u_2	u_3	u_4	u_5
P_1					
P_2					
$\vdots$					
P_{100}					

→ [Data Matrix] $_{100 \times 5}$

→ Parameter

↓ patients

Field $\mathbb{F}$:-

Non-empty collection of elements with the operations - $(+, \cdot)$ field addition, f multiplication - such that the following properties hold.

Properties :-

(i) There exists $0 \in \mathbb{F}$ such that for any element $a, b \in \mathbb{F}$, $a+0 = 0+a = a$

(ii) for any two elements $a, b \in \mathbb{F}$ $a+b \in \mathbb{F}$

(iii) ~~for~~ There exists for $\forall a \in \mathbb{F}$ the additive inverse $(-a) \in \mathbb{F}$ such that $a + (-a) = 0$ [(-a) Additive inverse of a]

(iv) There exists the element $1 \in \mathbb{F}$
$$a \cdot 1 = 1 \cdot a = a$$

(v) For $a, b \in \mathbb{F}$ $a \cdot b \in \mathbb{F}$

(vi) $\forall$ nonzero element $a \in \mathbb{F}$ then there exist a unique element $a^{-1} \in \mathbb{F}$ such that $a \cdot a^{-1} = a^{-1} \cdot a = 1$
[a^{-1} is multiplicative inverse of a]

For example :- as $\mathbb{R}$ is applicable in all properties. (set of Real numbers)

$\therefore$ R ($\mathbb{R}$) is a field.

& Also :- $\mathbb{N}$ is a field. (wrong)

b/c $a = 2 \in \mathbb{F}(\mathbb{N})$

but $a^{-1} = 1/2 \notin (\mathbb{N})$

✓ Can we make integers as a IF

eg:- Consider a set of inters modulas
→ Set of ~~so~~ remainder when divided by 5 (R_5)

$(R_5) : \{0, 1, 2, 3, 4\}$

$+ = \oplus_5$:- Add mod 5

$\cdot = \odot_5$:- Multiplication mod 5

$\oplus$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

1 & 4 are additive invers of each other

0	1	2	3	4	→ a
0	4	3	2	1	→ a^{-1}

• Additive inverse exisit for every element in R_5

$\odot_5$	1	2	3	4
1	$\boxed{1}$	2	3	4
2	2	4	$\boxed{1}$	3
3	3	$\boxed{1}$	4	2
4	4	3	2	$\boxed{1}$

• R_5 is closed under multip.

now will check ⑥ property which fails for $\mathbb{N}$

(iv) $1 \in R_5$ → multiplicative I. exisist

(v) Pass see the table

(vi) $\forall\, a \in R_5$ there is unique integer a^{-1} mod 5 such that $a \odot_5 a^{-1} \equiv 1$ mod 5.
$\equiv$ congruance

$(a)_5$	1	2	3	4
$(a^{-1})_5$	1	3	2	4

∴ Set 'R_5' is a field with operation $\oplus_5$ & $\odot_5$

- Set of integers mod any integers

$\Rightarrow R_n \rightarrow$ Is R_n for any

Is n a field ? $\rightarrow$?

n :- Integer

✓ R_6 is not a prime

↳ b/c no invers for (2, 3, 4)

$\odot$	1	2	3	4	5
2	2	4	0	2	4
3	3	0	3	0	3
4	4	2	0	4	2

} No one here

So, Rp (wher p is prime), the set of integers mod p is always a field.

so for R_2 $\{0, 1\}$

$\oplus_2$	0	1
0	0	1
1	1	0

$\oplus_2 \rightarrow$ XOR gate

$0 \oplus_2 0 \Rightarrow 0$ $\quad 1 \oplus 1 \Rightarrow 0$

$0 \oplus 1 \Rightarrow 1$ $\quad 1 \oplus 0 \Rightarrow 1$

Same = 0

Diff = 1

∴ There is just 2 digits in binary. (0, 1).

1 1 0

$2^2 \; 2^1 \; x \Rightarrow 6 \rightarrow$ we us 2^n b/c $\Rightarrow$ we are dividing till the reminder with 0.

$\frac{6}{2^n} \Rightarrow \frac{6}{2^2} \Rightarrow \frac{6}{4} \Rightarrow 2 (mod)$

$\frac{2}{2^n} \Rightarrow \frac{2}{2^1} \Rightarrow 0 (mod) \Rightarrow$ Ans $\Rightarrow 2^2 + 2^1 + 2^0$

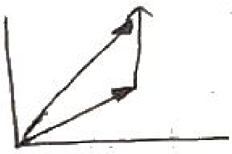
$\Rightarrow \underline{1 \; 1 \; 0}$

Vector Space V — set of (collection of) vector where you perform ① Vector addition ② Scalar multip.

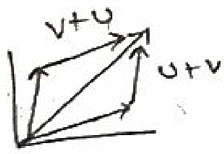
Let $\mathbb{F}$ be any field. Let V be a non empty collection of object called the vectors.

V is a vector space over $\mathbb{F}$ if rules for adding two vector space, and scalar multiplication exist such that V is closed vector addition & scalar multiplication.

(1) for u & $v \in V$
$u + v \in V$



(2) for $(\alpha \in \mathbb{F})(u \in V)$
$\alpha u \in V$



Properties :-

① u & $v \in V$ $\quad u + v = v + u$

② Associativity $\quad (u+v)+w = u + (v+w)$

② Scalar Multi $\quad (\alpha\beta)u = \alpha(\beta u)$

③ Distributivity :-
scalar ← $\alpha \in \mathbb{F}$ $\quad u, v \in V$
$\alpha(u+v) = \alpha u + \alpha v$.

④ Additive ~~Identity~~ Invers
$\forall\, v \in V \Rightarrow -v \in V$
such That $v + (-v) = 0$

⑤ Add Identity $\quad 0 \in V$ ST $0 + v = v \;\forall\, v \in$

⑥ Multiplicative Identity
$1 . v = v \quad \forall\, v \in V$

→ Construct a two component vector over the field F.

Let $\mathbb{F} = \mathbb{R}$

2 copies of $\mathbb{R}$

$$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \leftarrow \qquad u \in \mathbb{R}^2 \; (\mathbb{R} \times \mathbb{R})$$

u is a 2 component vector real vector

n copies of $\mathbb{R}$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \begin{array}{l} \rightarrow \mathbb{R} \times \mathbb{R} + \mathbb{R} + \mathbb{R} \\ \rightarrow (u \in \mathbb{R}^n) \end{array}$$

How do we do vector addition? :-

$u, v \in \mathbb{R}^2$ = v is defined over $\mathbb{R}$

$u \in \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$, $u_1, u_2 \in \mathbb{R}$ $\qquad v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ $\quad v_1, v_2 \in \mathbb{R}$

$$\underbrace{u + v = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}}_{\text{vector addition}} = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \end{pmatrix}$$

Addition as define in field.

- Scalar Multiplication :-

$\alpha \in \mathbb{R}$ $\qquad u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ $\quad u_1, u_2 \in \mathbb{R}$

$$\underbrace{\alpha . u = \alpha \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}}_{\text{Scalar Multiplication of vector}} = \begin{pmatrix} \alpha . u_1 \\ \alpha . u_2 \end{pmatrix}$$

Multiplication as defined in the field. $\mathbb{R}$

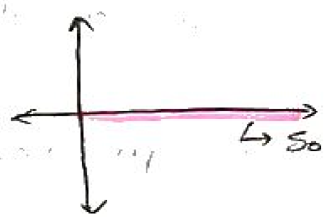
(ii) Examples for Vector spaces

1. Field $\mathbb{F}$ itself is a vector space
2. Any $\mathbb{R}^n$ for any finite n (or n) is a vector space over $\mathbb{R}$
3. Set of all polynomial of degree $\leq n$ & real coeffi... is a vector space over $\mathbb{R}$
4. Set of square metrices over $\mathbb{R}$ would be a vector space
5. Set of all real symetric matrix.
6. set of all continous functions of time t for t in $(-\infty, \infty)$ define over $\mathbb{R}$

eg: Consider $\mathbb{R}^2$

Subset of $\mathbb{R}^2$ $\Rightarrow$ $S_K = \left\{ \begin{pmatrix} x_1 \\ Kx_1 \end{pmatrix}, \begin{array}{l} x_1 \in \mathbb{R}, \\ K \in \mathbb{R} \end{array} \right\}$

$\Rightarrow$ (1) $\underset{(K=0)}{S_0} = \left\{ \begin{pmatrix} x_1 \\ 0 \end{pmatrix}, x_1 \in \mathbb{R} \right\}$



(A) Let u, v be vector $\in S_0$

$\Rightarrow u = \begin{pmatrix} u_1 \\ 0 \end{pmatrix}$ $v = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$

$u + v = \begin{pmatrix} u_1 \\ 0 \end{pmatrix} + \begin{pmatrix} v_1 \\ 0 \end{pmatrix} = \begin{pmatrix} u_1 + v_1 \\ 0 + 0 \end{pmatrix} = \begin{pmatrix} u_1 + v_1 \\ 0 \end{pmatrix} \in S_0$

S_0 is closed under vector add^n.

For $\alpha \in \mathbb{R}$ $\alpha \cdot \begin{pmatrix} u_1 \\ 0 \end{pmatrix} = \alpha \begin{pmatrix} u_1 \\ 0 \end{pmatrix} \in S_0$

$\therefore$ S_0 is closed under scalar multiplication

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \in S_0$

$\hookrightarrow$ S_0 is a VS over $\mathbb{R}$

- Same is true for $S_1 \rightarrow S_1$ is vs under $\mathbb{R}$
- & S_{-1} is also vs is under $\mathbb{R}$

From the above observation:-

$\Rightarrow$ Any line passing through the origin is vector space over ~~space~~ $\mathbb{R}$

$\Rightarrow$ Any Subset of a vector space V which by itself is a vector space with operations as defined in V is called a vector subspace of V.

(i) Every ~~sub~~ set is subset of itself
$\therefore$ Any $\mathbb{R}^n$ is a trivial subspace of $\mathbb{R}^n$

(ii) Set containg only the zero vector is a vector subspace $\Rightarrow$ TRIVIAL SUBSPACE

(iii) On any $\mathbb{R}^n$, a plane passing through the origin is a vector subspace.

Some questions :-

① What happens is we add multiple vectors in a vector space? $\Rightarrow$ Combining vectors

② Suppose we want to transform an entire vector space, what is the strategies to study the effect of the transformation on vectorspace

Linear Combination of vectors :-

Suppose $u_1, u_2 \ldots u_K$ are K vectors with n-components each.

$$\Rightarrow \quad u_i = \begin{pmatrix} u_{i1} \\ u_{i2} \\ \vdots \\ u_{in} \end{pmatrix} \quad \text{for } i = 1, 2, 3 \ldots K$$

Let $\alpha_1, \alpha_2 \ldots \alpha_K$ are scalars.

u_i for $i = 1, 2, \ldots K$ are real vectors &

α_i for $i = (1, 2, \ldots K$ are real

The vector

$$\boxed{v = \alpha_1 u_1 + \alpha_2 u_2 + \ldots u_K \alpha_K}$$

is called the linear combination of the K-vectors $u_1, u_2 \ldots u_K$.

eg:- Suppose $u_1, u_2 \ldots u_K$ are the standard unit vectors

$$u_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix} \quad u_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix} \ldots \quad u_K = \begin{pmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}$$

Kth comp.

eg:- Suppose $u_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ & $u_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \alpha_1 u_1 + \alpha_2 u_2$$
$$= v_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + v_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

eg:- $\begin{pmatrix} 3 \\ 4 \end{pmatrix} = 3\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 4\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Suppose $\alpha_1 = \alpha_2 \dots = 1$

$\alpha U_1 + \alpha_2 U_2 \dots \alpha_K U_K = v$

$v = 1 u_1 + 1 U_2 + \dots U_K$.

$v = U_1 + U_2 \dots + U_K$

↳ Sum of the vectors.

eg:- $u_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $u_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$v = \alpha_1 U_1 + \alpha_2 U_2$ $(\alpha_1 = \alpha_2 = 1)$

$v = U_1 + U_2$

$v = 1\begin{pmatrix} 1 \\ 2 \end{pmatrix} + 1\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 2+1 \\ 1+2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

- $\alpha_1 = \alpha_2 \dots = \alpha_n = 1/n$

$v = \frac{1}{n}(U_1) + \frac{1}{n}(U_2) \dots + \frac{1}{n}(U_n)$

$v = \frac{1}{n}(U_1 + U_2 + U_3 \dots U_n)$.

↳ Average of vector.

- If the cofficient or the scalars add up to 1, we call such combination as the affine combination

- Suppose the coefficients in affine combination are all non-negative, we call this combination as
  (i) Convex combination. (ii) Weighted Avarage.

Suppose $\alpha_1 = \alpha_2 = \cdots \alpha_{i-1} = 0$, $\alpha_i = 1$, $\alpha_{i+1} \cdots = \alpha_k = 0$

The linear combination $\Rightarrow \alpha_1 u_1 + \cdots + \alpha_k u_k$.
$\hookrightarrow (u_i)$

⇒ Suppose we want to study the effect of a transforming on a vector space, what stratigy do we adopt to do this?

Consider $u_1, u_2 \ldots u_n \rightarrow$ n component vectors.

Look at that linear combination of the u vectors that results in the n-comp zero vectors.

The linear combination of u-vectors is given by:-

$$\alpha_1 u_1 + \alpha_2 u_2 + \cdots + \alpha_n u_n = \emptyset_n = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}_n.$$

eg:- $u_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $u_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $u_3 = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

$$\therefore \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \alpha \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \alpha_3 \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} - 1 \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

If $\alpha_1 U_1 + \alpha_2 U_2 + \dots + \alpha_n U_n = O_n$

Such that not all α_i's are 0 we say that $U_1, U_2 \dots U_n$ are lin-early dependent.

* Any set of vectors that contains the zero vector is a linearly dependent set.

If $S = \{U_1, U_2, U_3 \dots U_K, 0\}$

then

$$\vec{0} = \alpha_1 U_1 + \alpha_2 U_2 + \dots \alpha_K U_K + \alpha_r 0.$$

$\alpha_1 = 0 = \dots \alpha_K$, $\alpha_r \Rightarrow$ Arbitary

Linearly Dep set $\Rightarrow$ Redundancy

- Let $U_1, U_2 \dots U_K$ be K n-component vector $\alpha_1, \alpha_2 \dots \alpha_K$ be scalers.

Look at the linear combination results in O_n

$$\alpha_1 U_1 + \alpha_2 U_2 + \alpha_3 U_3 \dots + \alpha_K U_K = O_n$$

- If the above l.c is that the only way to get the O_n is by making the scalars 0, then we say that $U_1 \dots U_K$ are linear independent vectors

$LI \Rightarrow$ all $\alpha = 0$.

Some observations :-

① A Linearly independent set cannot contain the On Vectors

② A single vector is always linear independent unless it is the zero vector.

③ Any subset of a linearly independent ~~sub~~ set of vector is always linearly independent.

④ Superset of LD vector is LD.

⑤ Two vectors are linearly independent if one is not the multiple of other.

Span of set of vectors :-

Let $u_1, u_2 \ldots u_k$ be K vector

Span $\Rightarrow$ Set of all possible LC of $u_1, u_2, \ldots u_k$.

span is a vector space.

Span of set of lin indep. vector

Let $v_1, v_2 \ldots v_n$ be a set of vectors

Span : $\{\alpha_1 v_1 + \alpha_2 v_2 \ldots + \alpha_n v_n \quad \alpha_i \in \mathbb{R} \text{ for } i = 1 - n\}$

↳ Smallest subspace of contain set of linear indep. vectors

for eg :- $v_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ & $v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\Rightarrow \{ \alpha_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \alpha_1, \alpha_2 \in \mathbb{R} \}$

(i) $\begin{pmatrix} 0 \\ 0 \end{pmatrix} \in$ Span $\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix} \& \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right)$

(ii) Span of $\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix} \& \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right)$ is closed under VA

(iii) span $\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix} \& \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right)$ is closed under SM.

(iv) Span $\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right)$ is a vector space

Basics :- Span of each base should be same.

A set of n linearly independ n-compon vectors is called a basis for the vector subspace that contains these n-linearly indep n-comp vectors.

Basis :- Sampling set for a vector space.

Suppose $u_1, u_2, \ldots u_k$ are lin indp vector & let

$$x = \alpha_1 u_1 + \cdots + \alpha_k u_k$$

Let x also have another representation in terms of $u_1, u_2 \ldots u_k$.

$$x = \beta_1 u_1 + \cdots + \beta_k u_k$$

1 - 2

$$\vec{x} - \vec{x} = \vec{0} = (\alpha_1 - \beta_1)\vec{u_1} \cdots + (\alpha_k - \beta_k)\vec{u_k}$$

Since $u_1 - u_k$ are linearly independent $\vec{0}$ has only one rep when

$$(\alpha_1 - \beta_1) = 0 = \cdots = (\alpha_k - \beta_k)$$

$$\therefore (\alpha_1 = \beta_1), (\alpha_2 = \beta_2) \ldots, (\alpha_k = \beta_k)$$

⇒ Any vector when expressed as a linear combinati of linearly indep set of vector has A 'UNIQUE' set of scalars.

⇒ Any vector in a vectorspace has a unique representation in terms of basic vetors.

eg :- $V = \mathbb{R}^2$

$$\beta = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$$

$$\begin{pmatrix} 3 \\ 3 \end{pmatrix} \Rightarrow \alpha_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\alpha_1 = 1$ & $\alpha_2 = 1 \rightarrow$ unique.