

# Elements of Design for Containers and Solutions in the LinBox library

## Extended abstract

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**Abstract.** We develop in this paper design techniques used in the C++ exact linear algebra library LinBox. They are intended to make the library safer and easier to use, while keeping it generic and efficient.

First, we review the new simplified structure of the containers, based on our *founding scope allocation* model. Namely, vectors and matrix containers are all templated by a field and a storage type. Matrix interfaces all agree with the same minimal blackbox interface. This allows e.g. for a unification of our dense and sparse matrices, as well as a clearer model for matrices and submatrices. We explain the design choices and their impact on coding. We will describe several of the new containers, especially our sparse and dense matrices storages as well as their `apply` (*blackbox*) method and compare to previous implementations.

Then we present a variation of the *strategy* design pattern that is comprised of a controller–plugin system: the controller (solution) chooses among plug-ins (algorithms) and the plug-ins always call back the solution so a new choice can be made by the controller. We give examples using the solution `mul`, and generalise this design pattern to the library. We also show performance comparisons with former LinBox versions.

Finally we present a benchmark architecture that serves two purposes. The first one consists in providing the user with an easy way to produce graphs using C++. The second goal is to create a framework for automatically tuning the library (determine thresholds, choose algorithms) and provide a regression testing scheme.

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## 1 Introduction

This article follows several papers and memoirs on the LinBox<sup>6</sup> (*cf.* [11,15,2,7,8]) and builds upon them.

LinBox is a C++ template library for fast and exact linear algebra. It is designed with genericity and efficiency in mind. The LinBox library is under constant evolution, driven by new problems and algorithms, by new computing paradigms, new compilers and architectures. This poses many new challenges. To address this changes, we are incrementally updating the *design* of the library towards a 2.0 release.

Let’s start from a basic consideration: we show in the Table 1 the increase in the size<sup>7</sup> of LinBox and its dependencies in terms of “lines of code”. This in-

LinBox	1.0.0 <sup>†‡</sup>	1.1.0 <sup>†‡</sup>	1.1.6 <sup>‡</sup>	1.1.7 <sup>‡</sup>	1.2.0	1.2.2	1.3.0	1.4.0
loc ( $\times 1000$ )	77.3	85.8	93.5	103	108	109	112	135
FFLAS–FFPACK	n/a	n/a	n/a	1.3.3	1.4.0	1.4.3	1.5.0	1.8.0
loc	—	—	—	11.6	23.9	25.2	25.5	32.1
Givaro	n/a	n/a	3.2.16	3.3.3	3.4.3	3.5.0	3.6.0	3.8.0
loc	—	—	30.8	33.6	39.4	41.1	41.4	42.8
total	77.3	85.8	124	137	171	175	179	210

Table 1: Evolution of the number of lines of code (loc, in thousands) in LinBox, FFLAS–FFPACK and Givaro (<sup>†</sup>contains Givaro, <sup>‡</sup>contains FFLAS–FFPACK).

crease affects the library in several ways. First, it demands a stricter development model, and we are going to list some techniques we used. For instance, we have transformed FFLAS–FFPACK<sup>8</sup> (*cf.* [9]) into a new stand-alone header library, resulting in more visibility for the FFLAS–FFPACK project (Singular ?) but also in a better structuration and maintainability of the library, focusing the development areas more precisely. Also, a larger template library is harder to manage, there is more difficulty to trace, debug and write new code: techniques employed for easier development include reducing compile times, enforcing stricter warnings and checks, supporting for more compilers and more architectures, simplifying and automatising version number changes, automatising memory leak checks, setting up build-bots to check the code frequently,...

But this increase also forces the library to be more user friendly. For instance, we have: Developed an `auto-install.sh` script that installs automatically the

<sup>6</sup> See <http://www.linalg.org>.

<sup>7</sup> Using `sloccount`, available at <http://sourceforge.net/projects/sloccount/>.

<sup>8</sup> See <http://www.linalg.org/projects/fflas-ffpack/>.

latest stable or development versions of the trio, resolving the version dependencies; Facilitated the discovery of the BLAS/LAPACK libraries; Simplified and sped up the checking process while covering more of the library ([△dave ?](#)); Updated the documentation and created user and developer oriented docs; Added comprehensive benchmarking tools,...

Developing generic and high-performance libraries is difficult. We can find a large literature on coding standards and software design references in (*cf.* [\[1,10,14,13,12\]](#)), and many internet sources and a lot of experience acquired by/from free software projects. Another motivation for developing a high-performance mathematical library is to make it available and easy for researchers and engineers that will use it for producing quality reliable results and quality research papers.

We are going to describe the advancement in the design of LinBox in the next three sections. We will first describe the new *container* framework in Section 2, then improve the *matrix multiplication* algorithms in Section 4 by contributing special purpose matrix multiplication plug-ins, and finally present the new *benchmark/optimisation* architecture (Section 5). [△develop this §](#) more later

## 2 Containers architecture

LinBox is mainly conceived around the RAII concept with re-entrant function (Resource Acquisition Is Initialisation), introduced by [\[13\]](#). We also follow the founding scope allocation model (or *mother model*) from [\[8\]](#) which ensures that the memory used by objects is allocated in the constructor and freed only at its destruction. The gestion of the memory allocated by an object is then exclusively reserved to it.

LinBox essentially uses matrix and vectors over fields as data objects (containers). The fragmentation of the containers into various matrices and blackboxes needed to be addressed and simplified. The many different matrix and vector types with different interfaces needed to be reduced into only two (possibly essentially one in the future) containers: **Matrix** and **Vector**.

### 2.1 General Interface for Matrices

Firstly, in order to allow operations on its elements, a container is parametrized by a field (*cf.* Listing 1.1), not the element type; this is also more general. The storage type is given by another template parameter that can default to *e.g.* dense BLAS type matrices (a stride and a leading dimension or an increment).

```

5  template< class _Field, class _Storage = denseDefault >
    class Matrix ;

    template< class _Field, class _Storage = denseDefault >
    class Vector ;

```

Listing 1.1: Matrix and vector classes in LinBox.

In the mother model, we need types that own or and types that share some memory. The `SubMatrix` and `SubVector` types share the memory while `Matrix` and `Vector` own it. The common interface shared by all matrices is the `BlackBox` interface described in the following paragraphs.

**Input/Output.** Our matrix all read and write from MatrixMarket format (ref, link). Adding extra comments ? (for instance the `init` field function in GF(q) needs a polynomial...) We can adapt the header to suit our needs. In particular write matrix in CSR fashion (saving roughly 1/3 space over COO)

**Accessing Elements.** The function `setEntry(...)` can be used to populate/-grow the matrix (from some `init()` until a `finish()` is emitted). The function `setEntry` can be (very) costly (for some sparse formats for instance) (Dave ?)

- `refEntry` that retrieves a reference to an entry may be difficult to implement or inefficient (compressed fields, sparse matrices)
- `getEntry` may be specialized, in all cases, there is a solution for this operation (can always be implemented from `imply`, cf. later.
- `clearEntry` can be used to zero out an entry, especially for a sparse matrix, if this is allowed (possibly not for structured matrices).
- iterators may be difficult to implement (but a lot of code relies on them...). Do we want only `const` iterators ?

**Apply method.** This is essential in the `BlackBox` interface, and we'll described it in Sections 2.2 and 4.

**Rebind** Rebind from one field to the other (if possible, using some default homeomorphism)

**Other** Conversion mechanisms are added to the interface when convenient, for instance all sparse matrix formats can convert to/from CSR format. This 'star' mechanism can simplify the code (to the expense of memory).

## 2.2 The apply method

The `apply` method (left or right) is arguably the most important feature in the matrix interface and the `LinBox` library. It performs what a linear application is defined for: apply to a vector (and by extension a block of vectors, *i.e.* a matrix).

We propose the new interface (Listing 1.2), where `_In` and `_Out` are vector or matrices, and `Side` is either `Tag::Right` or `Tag::Left`, wether the operation  $y \leftarrow A^\top x$  or  $y \leftarrow Ax$  is performed.

```

// y = A.x
template< class _In, class _Out >
_Out& apply(_Out &y, const _In& x, enum Side) ;

5 // y = alpha y + beta A.x
template< class _In, class _Out >
```

```
_Out& applyAcc(_Out &y, const Element& alpha, const _In& x,
               const Element& beta, enum Side) ;
```

Listing 1.2: Apply methods.

This method is important for two reasons: first it is the building block of the **BlackBox** algorithms (for instance Wiedemann and block-Wiedemann); second the matrix multiplication is a basic operation in linear algebra that needs to be extremely efficient (this is the matter of Section 4).

### 3 Examples of Containers

First, we have the dense containers that follow some BLAS conventions (row major ordering, leading dimension or increment,...) and are based on `std::vector` (inheriting for instance the iterators). In the same fashion, we have permutations matrices that follow the compressed LAPACK format or more traditional representations.

We have many kinds of structured or compound matrices (Hankel, stacked, add, sub,...) All other matrices (including the special case of Permutations) : the same Structured matrices, add, sub, stacked,...

Finally, the sparse matrices are very important containers and require particular interest because they are the basis of our **BlackBox** algorithms. Sparse matrices are usually problematic because the notion of *sparsity* is too general *vs.* the specificity of real world sparse matrices: the algorithms have to adapt to the shape of the sparse matrices —which is not really the case for the dense case. Getting the best performance for an sparse matrix as an **BlackBox** is not a challenging task. There is a huge literature on SpMV(Sparse Matrix Vector multiplication) and on sparse matrix formats, some of which are becoming standard (COO, CSR, BCSR, SKY,...). In [4] we developped some techniques to improve the SpMVoperation in LinBox. Just like the BLAS numerical routines, we would also like to take advantage of existing high performance numerical libraries (come back to this later).

The addition of standard matrix format is driven by the availability of numerical routines and the expectation of better performance in the SpMVoperation. However, legacy LinBox sparse matrix formats (based on STL structures such as `map`, `deque`, `pair`,...) can be more convenient for elimination techniques.

## 4 Improving LinBox matrix multiplication

Efficient matrix multiplication is key to LinBox library.

### 4.1 Plug-in structure

We propose the following design pattern (the closest design pattern to our knowledge is the *strategy* one, see also [6, Fig 2.]. The main advantage of this design

pattern is that the modules always call back the controller so that the best choice is always chosen. Besides modules can be easily added as *plug-ins*. An analogy can be drawn with dynamic systems—once the controller sends a correction to the system, it receives back a new measure that allows for a new correction.

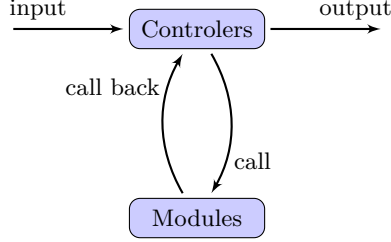


Fig. 1: Controller–Module design pattern

For instance, we can write the standard cascade algorithms in that model:

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**Algorithm 1:** Algo: controller

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**Input:**  $A$  and  $B$ , denses, with resp. dimensions  $n \times k$  and  $k \times n$ .  
**Input:**  $H$  Helper  
**Output:**  $C = A \times B$   
**if**  $\min(m, k, n) < H.\text{threshold}()$   
**then**  
  | **BaseCase**  $K()$  ;  
  | **Algo**( $C, A, B, K$ ) ;  
**else**  
  | **RecursiveCase**  $H()$  ;  
  | **Algo**( $C, A, B, H$ )  
**end**  
**return**  $C$  ;

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**Algorithm 2:** Algo: recursive module

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**Input:**  $A, B, C$  as in controller.  
**Input:**  $H$ , **RecursiveCase** Helper  
**Output:**  $C = A \times B$   
 Cuts  $A, B, C$  in  $S_i, T_i \dots$   
 ...  
 $P_i = \text{Algo}(S_i, T_i, H)$   
 ...  
**return**  $C$

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Fig. 2: Conception of a recursive controlled algorithm

This method allows for the reuse of modules and ensures efficiency. It is then possible to adapt to the architecture, the available modules, the resources. The only limitation is that the choice of the module should be done fast.

On top of this design, we have Methods/Helpers that allow (preferred) selection of algorithms and cut short in the strategy selection of Figure 1.

△ timing old fgemm/plugin fgemm with no noticeable change ?

This infrastructure also forces/invites to modularise the code. For instance, a lot of work has been done in FFLAS–FFPACK to factor code in modules (addition,

scaling, initialisation, . . . ). Not only this permits to write code with hardly more line than it takes for pseudo-code listings in [5] (compared to  $\approx 2.5\times$  on some routines before) but also it automatically brings performance, because we can the separately improve on these modules. Also, this reduces the lines of code, hence the probability for bugs, and eases the tracing/traking of bugs, allows for more unit tests. Modularising the code comes at almost no cost because we may add  $O(1)$  operations that 1) don't cost much compared to  $O(n^2)$  or more complexity of the modules; 2) allow early decisions and terminations by testing against 0,  $\pm 1$  or checking the leading dimensions and increments; 3) allow better code (AVX, SSE, copy-cache friendly operation-copy back, representation switching, . . . )

## 4.2 New algorithms/infrastructure

We introduce now several new algorithms that improve on matrix multiplication in various ways: reducing memory consumption, introducing new efficient algorithms, using graphics capabilities, generalizing the BLAS to integer routines.

*New algorithms: low memory* The routine `fgemm` in FFLAS uses the classic schedules for the multiplication and the product with accumulation (*cf.* [5]), but we also implement the lower memory routines therein. The difficulty consists in using the part of the memory contained in a sub-matrix of the original matrix. It is two-fold. – First we have to use some part of a memory that has already been allocated to the input matrices, therefore we cannot free and reallocate part of it. – Second, several of these algorithms are meant for square matrices.

*New algorithms: Bini.* In [3], we use Bini's approximate matrix multiplication formula to derive a new algorithms that is more efficient that the Strassen–Winograd implementation in `fgemm` by  $\approx 5 - 10\%$  on sizes 1 500–3 000. This is a cascade of Bini's algorithm and Strassen–Winograd algorithm and/or the naïve algorithm (using BLAS). The idea is to analyse precisely the error term in the approximate formula and make it vanish.

*Integer BLAS* pascal

*Polynomial Matrix Multiplication* Pascal

*OpenCL* dave

*Sparse Matrix–Vector Multiplication* The issue in the SpMVoperation is taking advantage of existing numerical routines. There is a Sparse BLAS specification, but it is not widely implemented (Intel's MKL does, there is a cuSPARSE library in the Nvidia Cuda toolkit). Contrary to dense BLAS representation, there is no way of viewing a sparse submatrix in most of the standard formats, so we cannot apply the same techniques as those in FFLAS for  $Z/pZ$ . However, we develop hybrid structures that represent a matrix  $A$  as a sum  $A_1 + A_2 + \dots + A_k$ . For instance, the number of non zeros in each row of each  $A_i$  will not overflow

a dot product and numerical sparse BLAS can be used. Besides, the format of each submatrix is independant and we can create some ELL+CSR format for instance. Besides, sparse matrices from many domains have only  $\pm 1$  as non zero entries and we take advantage of that (no multiplication, accumulation further delayed, no need to store the data).

Contrary to the BLAS case, the apply using the transpose of  $A$  can be very inefficient compared to the apply on  $A$ . In this case, we create a helper function that (if memory allows) just stores  $A^\top$  in an optimised hybrid format too. Setting up the hybrid representation of  $A$  usually costs the order of 5 to 10 non optimised SpMV operations: the conversion is performed only for larger matrices that will be used repeatedly for the SpMV operation and remains constant.

#### *Using conversions*

- double- $\rightarrow$ float
- using flint for integer matmul is faster, even with conversion. Need better CRA implementation (but with the plugins, we can do without our faulty code and just use flint).
- implementation of Toom-Cook for  $\text{GF}(q)$
- when does spmv choose to optimise ?
- transition to benchmarking

## 5 Benchmarking

Benchmarking was introduced in LinBox for several reasons. First, It would give the user a user-friendly way for producing quality graph with no necessary knowledge of a graphing library like gnuplot<sup>9</sup> or provide the LinBox website with automatically updated tables and graphs. Second, it would be used for regression testing. Finally, it would be used for selecting default method, threshold. A lot of libraries do some automatic tuning at installation (fftw, ATLAS, NTL, ...).

What do we do differently ? Selection between "larger" algorithms, takes more time. Interpolation.

### 5.1 Graph/Table creation

Our plotting mechanism is based on two structures: `PlotStyle` and `PlotData`. The `PlotGraph` structure uses the style and data to manage the output. We allow plotting in standard image formats, html and  $\text{\LaTeX}$  tables, but also in raw csv or xml. The last raw formats allow for file exchange, data comparisons and extrapolation. [△](#)dave benchmark formats discussion ?

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<sup>9</sup> <http://www.gnuplot.info/>



## 5.2 Regression Testing

Saving graphs in raw format can enable automatic regression testing on the buildbots. For some determined matrices (of different shape and size) over a few fields, we can accumulate over time the timings for some of our solutions (rank, det, mul, ...). At each new release, when we update the documentation, we can check any regression on these base cases and automatically update the regression plots.  $\triangle$  We need to implement this framework (not difficult; anybody?).

## 5.3 Method Selecting

CPU throttling for ATLAS, FFLAS–FFPACK not reliable. XXX Default are provided, method can be selected via a benchmark (cf wino\_threshold)  
XXX howto

## References

1. A. Alexandrescu. *Modern C++ design: generic programming and design patterns applied*. C++ in-depth series. Addison-Wesley, 2001.
2. B. Boyer. *Multiplication matricielle efficace et conception logicielle pour la bibliothèque de calcul exact LINBOX*. PhD thesis, Université de Grenoble, June 2012.
3. B. Boyer and J.-G. Dumas. Matrix multiplication over word-size prime fields using Bini’s approximate formula. Submitted. <http://hal.archives-ouvertes.fr/hal-00987812>, May 2014.
4. B. Boyer, J.-G. Dumas, and P. Giorgi. Exact sparse matrix-vector multiplication on GPU’s and multicore architectures. In *Proceedings of the 4th International Workshop on Parallel and Symbolic Computation*, PASCO ’10, pages 80–88, New York, NY, USA, 2010. ACM.
5. B. Boyer, J.-G. Dumas, C. Pernet, and W. Zhou. Memory efficient scheduling of Strassen-Winograd’s matrix multiplication algorithm. In *Proceedings of the 2009 international symposium on Symbolic and algebraic computation*, ISSAC ’09, pages 55–62, New York, NY, USA, 2009. ACM.
6. V.-D. Cung, V. Danjean, J.-G. Dumas, T. Gautier, G. Huard, B. Raffin, C. Rapine, J.-L. Roch, and D. Trystram. Adaptive and hybrid algorithms: classification and illustration on triangular system solving. In J.-G. Dumas, editor, *Proceedings of Transgressive Computing 2006, Granada, España*, Apr. 2006.
7. J.-G. Dumas, T. Gautier, M. Giesbrecht, P. Giorgi, B. Hovinen, E. Kaltofen, B. D. Saunders, W. J. Turner, and G. Villard. LINBOX: A generic library for exact linear algebra. In *Proceedings of the 2002 International Congress of Mathematical Software, Beijing, China*. World Scientific Pub, Aug. 2002.
8. J.-G. Dumas, T. Gautier, C. Pernet, and B. D. Saunders. LinBox founding scope allocation, parallel building blocks, and separate compilation. In K. Fukuda, J. Van Der Hoeven, and M. Joswig, editors, *Proceedings of the Third International Congress Conference on Mathematical Software*, volume 6327 of *ICMS’10*, pages 77–83, Berlin, Heidelberg, Sept. 2010. Springer-Verlag.
9. J.-G. Dumas, P. Giorgi, and C. Pernet. Dense linear algebra over word-size prime fields: the FFLAS and FFPACK packages. *ACM Trans. Math. Softw.*, 35(3):1–42, 2008.

10. E. Gamma. *Design Patterns: Elements of Reusable Object-Oriented Software*. Addison-Wesley Professional Computing Series. Addison-Wesley, 1995.
11. P. Giorgi. *Arithmétique et algorithmique en algèbre linéaire exacte pour la bibliothèque LINBOX*. PhD thesis, École normale supérieure de Lyon, Dec. 2004.
12. D. Gregor, J. Järvi, M. Kulkarni, A. Lumsdaine, D. Musser, and S. Schupp. Generic programming and high-performance libraries. *International Journal of Parallel Programming*, 33:145–164, 2005. 10.1007/s10766-005-3580-8.
13. B. Stroustrup. *The design and evolution of C++*. Programming languages/C++. Addison-Wesley, 1994.
14. H. Sutter and A. Alexandrescu. *C++ Coding Standards: 101 Rules, Guidelines, And Best Practices*. The C++ In-Depth Series. Addison-Wesley, 2005.
15. W. J. Turner. *Blackbox linear algebra with the LINBOX library*. PhD thesis, North Carolina State University, May 2002.