算法简化描述

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Chapter 1

非线性方程求根

Chapter 2

线性方程组求解

Chapter 3

微分方程数值解

3.1 龙格库塔 (Runge Kutta)

3.1.1 四阶龙格库塔

$$\frac{dy}{dx} = f(x, y), y(0) = y_0 \tag{3.1}$$

Algorithm 1 四阶龙格库塔

Require: f, y_0, h, n

for $i = 0, 1, \ldots$ **do**
- $k_1 = f(x_i, y_i)$
- $k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h)$
- $k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h)$
- $k_4 = f(x_i + h, y_i + k_3 h)$
- $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$

求解微分方程组.

Chapter 4

最优化算法

4.1 直接搜索

指无需对目标函数求导的搜索方法, 比如 [4].

4.1.1 单纯形法 (Simplex Search Method)

4.2 梯度法 (Gradient Method)

4.2.1 梯度法 (Gradient Descent Method)

梯度法或称梯度下降法.

Algorithm 2 梯度法

Require: $x_0 \in \mathbf{R}^n$

for $k = 0, 1, \ldots$ **do**

$\quad x_{k+1} = y_k - t_k \nabla f(y_k)$

4.2.2 加速邻近梯度法 (Accelerated Proximal Gradient Method, APG)

此方法由 Nesterov[2] 首先提出.

Algorithm 3 加速邻近梯度法 [2, 1]

Require: $x_0 \in \mathbf{R}^n, y_0 = x_0, \theta_0 = 1, q \in [0, 1]$

for $k = 0, 1, \ldots$ **do**

$\quad x_{k+1} = y_k - t_k \nabla f(y_k)$

$\quad \theta_{k+1}^2 = (1 - \theta_{k+1})\theta_k^2 + q\theta_{k+1}, \theta_{k+1} \in (0, 1)$, 求解 θ_{k+1}

$\quad \beta_{k+1} = \theta_k(1 - \theta_k)/(\theta_k^2 + \theta_{k+1})$

$\quad y_{k+1} = x_{k+1} + \beta_{k+1}(x_{k+1} - x_k)$

其中, $\theta_{k+1} \in (0,1)$ 见文献 [1] 91 页. 当 $q=1$ 时, 该算法为梯度法.

4.2.3 自适应重启加速梯度法

加速邻近梯度法 (算法3) 迭代到一定程度时, 外推系数 β_k 趋于 0, 算法退化成梯度法. 使用该算法自适应的重置参数, 能够保持加速邻近梯度法的快速收敛.

Algorithm 4 自适应重启加速梯度法 [3]

Require: $x_0 \in \mathbf{R}^n, y_0 = x_0, \theta_0 = 1$

1: **for** $j = 0, 1, \ldots$ **do**

2: 取 $q = 0$ 执行算法3, 直到 $f(x_k) > f(x_{k+1})$ 时停止 (或可以在 $\nabla f(y_{k-1})^T(x_k - x_{k-1})$ 时停止, 两者选其中一种)

3: $x_0 = x_k, y_0 = x_k, \theta_0 = 1$

Chapter 5

随机搜索

5.1 遗传算法 (Genetic Algorithm, GA)

5.2 粒子群算法 (Particle Swarm Optimization, PSO)

Chapter 6

数据分析

6.1 聚类

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