



美赛A题特点与切入

《美国数学建模竞赛》

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2016 MCM A Hot Bath 洗个热水澡



2016 MCM A: Hot Bath热水澡

- A person fills a bathtub with hot water from a single faucet and settles into the bathtub to cleanse and relax. Unfortunately, the bathtub is not a spa-style tub with a secondary heating system and circulating jets, but rather a simple water containment vessel. After a while, the bath gets noticeably cooler, so the person adds a constant trickle of hot water from the faucet to reheat the bathing water. The bathtub is designed in such a way that when the tub reaches its capacity, excess water escapes through an overflow drain.
- 热水从一个水龙头里灌满一个浴缸，水龙头安顿在浴缸中，人坐在浴缸里清洗和放松。不幸的是，这个浴缸并不是一个带有二次加热系统和循环射流的温泉式浴缸，而是一个简单的水容器。过了一段时间，洗澡水明显变凉了，于是人们不断地从水龙头里往浴缸里灌热水，以重新加热洗澡水。浴缸的设计是这样的，当浴缸达到其容量时，多余的水通过溢流排水沟排出。



2016 MCM A: Hot Bath热水澡

- Develop a model of the temperature of the bathtub water in space and time to determine the best strategy the person in the bathtub can adopt to keep the temperature even throughout the bathtub and as close as possible to the initial temperature without wasting too much water.
- 本题目的是：在空间和时间上开发一个浴缸保持水温的灌水模型，以确定最佳的策略，在浴缸的人可以选择保持温度与整个浴缸的水温尽可能接近其初始温度，并且没有浪费太多的水。



2016 MCM A: Hot Bath热水澡

- Use your model to determine the extent to which your strategy depends upon the shape and volume of the tub, the shape/volume/temperature of the person in the bathtub, and the motions made by the person in the bathtub. If the person used a bubble bath additive while initially filling the bathtub to assist in cleansing, how would this affect your model's results?
- 使用你的模型来确定你的策略在多大程度上实用，这取决于浴缸的形状和体积，浴缸里人的形状/体积/温度，以及浴缸里人的动作。如果这个人在最初使用浴缸时借助了泡泡浴添加剂来帮助清洁，这将如何影响你的模型的结果？



2016 MCM A: Hot Bath热水澡

- In addition to the required one-page summary for your MCM submission, your report must include a one-page non-technical explanation for users of the bathtub that describes your strategy while explaining why it is so difficult to get an evenly maintained temperature throughout the bath water.
- 除了提交MCM所需的一页摘要外，您的报告还必须包括一页浴缸用户的非技术说明，描述您的策略，同时解释为什么很难在整个浴缸中保持均匀的温度。



2017 MCM Managing The Zambezi River管理赞比西河



2017 Problem A: Managing The Zambezi River 管理赞比西河

- The Kariba Dam on the Zambezi River is one of the larger dams in Africa. Its construction was controversial, and a 2015 report by the Institute of Risk Management of South Africa included a warning that the dam is in dire need of maintenance. A number of options are available to the Zambezi River Authority (ZRA) that might address the situation.
- 赞比西河上的卡里巴水坝是非洲的大水坝之一。它的建设是有争议的，南非风险管理研究所的2015年的报告中：警告大坝急需维护。赞比西河管理局（ZRA）可提供若干选择以解决这一问题。



2017 Problem A: Managing The Zambezi River 管理赞比西河

- Three options in particular are of interest to ZRA:
- ZRA特别感兴趣的有三个选项：
- (Option 1) Repairing the existing Kariba Dam,
- 修复现有的卡里巴大坝，
- (Option 2) Rebuilding the existing Kariba Dam,
- 重建现有的卡里巴大坝，
- (Option 3) Removing the Kariba Dam and replacing it with a series of ten to twenty smaller dams along the Zambezi River.
- 拆除卡里巴水坝，并用沿赞比西河修建10-20个较小的水坝替代它。



2017 Problem A: Managing The Zambezi River 管理赞比西河

- There are two main requirements for this problem:
- 这个问题有两个主要要求：
- **Requirement 1** ZRA management requires a brief assessment of the three options listed, with sufficient detail to provide an overview of potential costs and benefits associated with each option. This requirement should not exceed two pages in length, and must be provided in addition to your main report.
- **要求1** ZRA的管理层需要对所列出的三个选项进行简要评估，并提供足够的详细信息，提供与每个选项相关的潜在成本和收益的概述。此要求的长度不应超过两页，除了主报告之外，还必须提供此页。



2017 Problem A: Managing The Zambezi River 管理赞比西河

- **Requirement 2** Provide a detailed analysis of Option (3) - removing the Kariba Dam and replacing it with a series of ten to twenty smaller dams along the Zambezi river. This new system of dams should have the same overall water management capabilities as the existing Kariba Dam while providing the same or greater levels of protection and water management options for Lake Kariba that are in place with the existing dam. Your analysis must support a recommendation as to the number and placement of the new dams along the Zambezi River.
- **要求2** 提供选项（3）的详细分析 - 拆除卡里巴水坝，并用沿赞比西河的10-20个较小的水坝替换它。这个新的水坝系统应该与现有的卡里巴水坝具有相同能力，同时现有的水坝要提供给卡里巴湖相同或更高水平的保护和水管管理选择。你的分析必须给出沿赞比西河的新坝的数量和位置。



2017 Problem A: Managing The Zambezi River 管理赞比西河

- In your report for Requirement 2, you should include a strategy for modulating the water flow through your new multiple dam system that provides a reasonable balance between safety and costs. In addition to addressing known or predicted normal water cycles, your strategy should provide guidance to the ZRA managers that explains and justifies the actions that should be taken to properly handle emergency water flow situations (i.e. flooding and/or prolonged low water conditions).
- 在你要求2的报告中，你应该在安全和成本方面为你的新水坝的水流量系统提供合理的策略。除了解决已知或预测的正常水周期，你的战略应该为ZRA的管理者提供指导，解释和证明应该采取的行动，以正确处理紧急水流情况。（即洪水和长期低水位状况）。



2017 Problem A: Managing The Zambezi River 管理赞比西河

- Your strategy should provide specific guidance for extreme water flows ranging from maximum expected discharges to minimum expected discharges. Finally, your recommended strategy should include information addressing any restrictions regarding the locations and lengths of time that different areas of the Zambezi River should be exposed to the most detrimental effects of the extreme conditions.
- 你的策略应为从**最大预期排放到最小预期排放的极端水流**提供具体指导。最后，你的建议战略应包括解决对**赞比西河不同地区中暴露于极端条件最有害影响的位置和时间长度的任何限制**的信息。



2017 Problem A: Managing The Zambezi River 管理赞比西河

- Your MCM submission should consist of three elements: a standard 1 page MCM Summary Sheet, a 1-2 page brief assessment report (Requirement 1), and your main MCM solution (Requirement 2) not to exceed 20 pages for a maximum submission of 23 pages. Note: Any appendices or reference pages you include will not count towards the 23 page limit.
- 你的MCM提交应包含三个要素：1页MCM摘要，1-2页简要评估报告（要求1）和您的主要MCM解决方案（要求2），最多提交20页到23页。注意：您添加的任何附录或参考页都不会计入23页的限制。



2018MCM Multi-hop HF Radio Propagation 多跳高频HF无线电传播



2018 Problem A: Multi-hop HF Radio Propagation 多跳高频HF无线电传播

- **Background:** On high frequencies (HF, defined to be 3 – 30 MHz), radio waves can travel long distances (from one point on the earth's surface to another distant point on the earth's surface) by multiple reflections off the ionosphere and off the earth. For frequencies below the *maximum usable frequency* (MUF), HF radio waves from a ground source reflect off the ionosphere back to the earth, where they may reflect again back to the ionosphere, where they may reflect again back to the earth, and so on, travelling further with each successive hop. Among other factors, the characteristics of the reflecting surface determine the strength of the reflected wave and how far the signal will ultimately travel while maintaining useful signal integrity. Also, the MUF varies with the season, time of day, and solar conditions.
- **背景:** 在高频 (HF, 定义为3 – 30MHz), 无线电波可以通过离开电离层和离开地球的多次反射而行进很长距离 (从地球表面的一个点到地球表面上的另一个远点)。对于低于最大可用频率 (MUF) 的频率, 来自地面源的HF无线电波将电离层反射回地球, 在那里它们可能再次反射回到电离层, 在那里它们可能再次反射回地球, 等等, 随着每个连续的跳跃继续前进。除了其他因素之外, 反射表面的特性决定了反射波的强度以及信号最终行进的程度, 同时保持有用的信号完整性。而且, MUF随季节, 一天中的时间和太阳能条件而变化。



2018 Problem A: Multi-hop HF Radio Propagation 多跳高频HF无线电传播

- Frequencies above the MUF are not reflected/refracted, but pass through the ionosphere into space. In this problem, the focus is particularly on reflections off the ocean surface. It has been found empirically that reflections off a turbulent ocean are attenuated more than reflections off a calm ocean. Ocean turbulence will affect the electromagnetic gradient of seawater, altering the local permittivity and permeability of the ocean, and changing the height and angle of the reflection surface. A turbulent ocean is one in which wave heights, shapes, and frequencies change rapidly, and the direction of wave travel may also change.
- MUF以上的频率不被反射/折射， 而是通过电离层进入太空。
- 在这个问题上，重点是特别关注海洋表面的反射。从经验上发现，在一个平静的海洋上，反射出汹涌的海洋的反射比衰减更多。海洋湍流将影响海水的电磁梯度，改变海洋的局部介电常数和渗透率，改变反射面的高度和角度。汹涌的海洋是波浪高度，形状和频率迅速变化的地方，波浪的行进方向也可能发生变化。



2018 Problem A: Multi-hop HF Radio Propagation 多跳高频HF无线电传播

- **Problem:**
- **Part I:** Develop a mathematical model for this signal reflection off the ocean. For a 100-watt HF constant-carrier signal, below the MUF, from a point source on land, determine the strength of the first reflection off a turbulent ocean and compare it with the strength of a first reflection off a calm ocean. (Note that this means that there has been one reflection of this signal off the ionosphere.) If additional reflections (2 through n) take place off calm oceans, what is the maximum number of hops the signal can take before its strength falls below a usable signal-to-noise ratio (SNR) threshold of 10 dB?
- **第一部分：**为海洋信号反射建立一个数学模型。对于一个100瓦的HF恒定载波信号，低于MUF，从陆地上的一个点源，确定从一个湍流海洋的第一反射的强度，并将其与平静海洋的第一反射的强度进行比较。（注意，这意味着这个信号离开电离层有一个反射。）如果在平静的海洋附近发生附加的反射（2到 n ），在信号强度降至可用信噪比（SNR）阈值10dB之前，信号能跳的最大跳数是多少？



2018 Problem A: Multi-hop HF Radio Propagation多跳HF无线电传播

- **Part II:** How do your findings from **Part I** compare with HF reflections off mountainous or rugged terrain versus smooth terrain?
- **Part III:** A ship travelling across the ocean will use HF for communications and to receive weather and traffic reports. How does your model change to accommodate a shipboard receiver moving on a turbulent ocean? How long can the ship remain in communication using the same multi-hop path?
- **Part IV:** Prepare a short (1 to 2 pages) synopsis of your results suitable for publication as a short note in *IEEE Communications Magazine*.
- **第二部分：** 第一部分的研究结果与高山或崎岖地形与光滑地形的HF反射比较如何？
- **第三部分：** 穿越海洋的船舶将使用HF进行通信，并接收天气和交通报告。你的模型如何改变，以适应在动荡的海洋上移动的船上接收器？船舶能够使用相同的多跳路径保持多久？
- **第四部分：** 在IEEE通信杂志上准备一份简短的（1至2页）简短摘要。

2018 Problem A: Multi-hop HF Radio Propagation多跳HF无线电传播

- Your submission should consist of:
 - One-page Summary Sheet,
 - Two-page synopsis,
 - Your solution of no more than 20 pages, for a maximum of 23 pages with your summary and synopsis.
- Note: Reference list and any appendices do not count toward the 23-page limit and should appear after your completed solution.
- 您的提交应该包括：
 - 一页总结表，
 - 两页简介，
 - 您的解决方案不超过20页，最多23页的摘要和摘要。
 - 注意：参考列表和任何附录不计入23页限制，应在完成解决方案后出现。



2019MCM Game of Ecology: 生态游戏



2019 MCM Problem A: Game of Ecology 生态游戏

- In the fictional television series *Game of Thrones*, based on the series of epic fantasy novels *A Song of Ice and Fire*[1], three dragons are raised by Daenerys Targaryen, the “Mother of Dragons.” When hatched, the dragons are small, roughly 10 kg, and after a year grow to roughly 30-40 kg. They continue to grow throughout their life depending on the conditions and amount of food available to them.
- 在电视剧《权力的游戏》中，根据史诗奇幻小说《冰与火之歌》改编而成，三条龙由“龙之母”丹妮莉丝·坦格利安抚养长大。“刚孵化时，这些龙很小，大约10公斤重，一年后会长到大约30-40公斤。它们在一生中不断生长，这取决于它们所处的环境和所能获得的食物量。



2019 MCM Problem A: Game of Ecology 生态游戏

- For the purposes of this problem, consider these three fictional dragons are living today. Assume that the basic biology of dragons described above is accurate. You will need to make some additional assumptions about dragons that might include, for example, that dragons are able to fly great distances, breath fire, and resist tremendous trauma. As you address the problem requirements, it should be clear how your assumptions are related to the physical constraints of the functions, size, diet, changes, or other characteristics associated with the animals.
- 对于这个问题的目的，考虑这三个虚构的龙是生活在今天。假设上面描述的龙的基本生物学是正确的。你需要对龙做一些额外的假设，比如，龙能飞很远的距离，能喷火，能抵抗巨大的创伤。当您处理问题需求时，应该清楚您的假设与动物的功能、大小、饮食、变化或其他相关特征的物理约束之间的关系。



2019 MCM Problem A: Game of Ecology 生态游戏

- Your team is assigned to analyze dragon characteristics, behavior, habits, diet, and interaction with their environment. To do so, you will have to consider many questions. At a minimum, address the following: What is the ecological impact and requirements of the dragons? What are the energy expenditures of the dragons, and what are their caloric intake requirements? How much area is required to support the three dragons? How large a community is necessary to support a dragon for varying levels of assistance that can be provided to the dragons? Be clear about what factors you are considering when addressing these questions.
- 您的团队被分配来分析龙的特征，行为，习惯，饮食以及与环境的互动。要做到这一点，你必须考虑许多问题。至少，解决以下问题：龙的生态影响和需求是什么？龙的能量消耗是多少？它们的卡路里摄取量需要多少？这三条龙需要多少活动面积？一个社区需要多大的规模来支持龙以提供不同程度的帮助？清楚你在考虑哪些因素。



2019 MCM Problem A: Game of Ecology 生态游戏

- As with other animals that migrate, dragons might travel to different regions of the world with very different climates. How important are the climate conditions to your analysis? For example, would moving a dragon between an arid region, a warm temperate region, and an arctic region make a big difference in the resources required to maintain and grow a dragon?
- 与其他迁徙的动物一样，龙可能会以不同的气候前往世界不同的地区。气候条件对您的分析有多重要？例如，在干旱地区，暖温带地区和北极地区之间移动龙会对维持和种植龙所需的资源产生很大影响吗？



2019 MCM Problem A: Game of Ecology 生态游戏

- Once your dragon analysis is complete, draft a two-page letter to the author of *A Song of Ice and Fire*, George R.R. Martin, to provide guidance about how to maintain the realistic ecological underpinning of the story, especially with respect to the movement of dragons from arid regions to temperate regions and to arctic regions.
- 一旦你的龙分析完成，请写一封两页的信给“冰与火之歌”的作者George RR Martin，以提供关于如何保持故事的现实生态基础的指导，特别是关于从干旱地区到温带地区和北极地区的龙。



2019 MCM Problem A: Game of Ecology 生态游戏

- While your dragon analysis does not directly apply to a real physical situation, the mathematical modeling itself makes use of many realistic features used in modeling a situation. Aside from the modeling activities themselves, describe and discuss a situation outside of the realm of fictional dragons that your modeling efforts might help inform and provide insight?
- 虽然您的龙分析并不直接适用于真实的物理情况，但数学建模本身会利用许多用于建模情境的实际特征。除了建模活动本身之外，您的建模工作可能有助于为描述和讨论虚构龙领域之外的情况提供哪些信息和见解？



2019 MCM Problem A: Game of Ecology 生态游戏

- Your submission should consist of:
 - One-page Summary Sheet,
 - Two-page letter,
 - Your solution of no more than 20 pages, for a maximum of 23 pages with your summary and letter.
- Note: Reference list and any appendices do not count toward the 23-page limit and should appear after your completed solution.
- 您的提交应包括：
 - 一页摘要表，
 - 两页的信，
 - 您的解决方案不超过20页，最多23页，包含您的摘要和信函。
 - 注意：参考列表和任何附录不计入23页的限制，应在完成的解决方案后显示。



2019 MCM Problem A: Game of Ecology 生态游戏

- laws. Please be careful in how you use and cite the sources for your ideas and the materials used in your report.

Reference

- 1. Penguin Random House (2018). *A Song of Ice and Fire Series*. Retrieved from <https://www.penguinrandomhouse.com/series/SOO/a-song-of-ice-and-fire/>.
- 注意：您不应使用未经授权的图像和使用受版权法限制的材料。请注意您如何使用和引用您的想法 和报告中使用的材料的来源。

参考

- 1. 企鹅兰登书屋（2018年）。冰与火之歌系列。从 <https://www.penguinrandomhouse.com/series/SOO/a-song-of-ice-and-fire/>. 获得