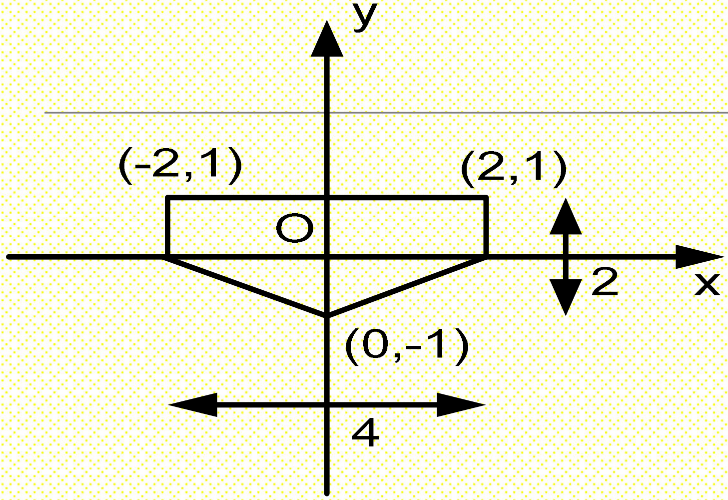
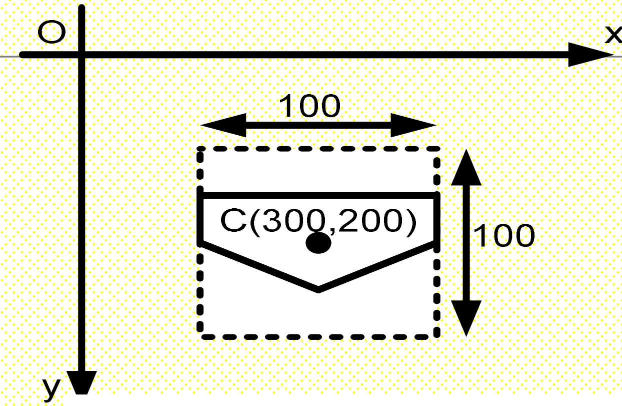
Transformarea de instantiere
CURS 5 TRANSFORMARI -continuare



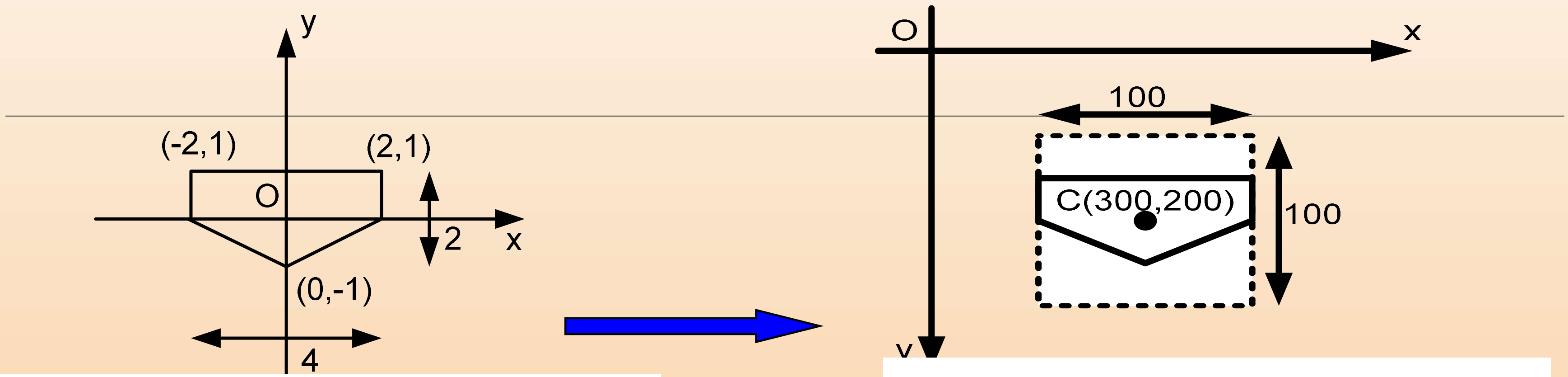


Sistem de coordonate obiect

Sistem de coordonate al suprafetei de afisare

Transformarea de instantiere

Transformarea de instantiere



Sistem de coordonate obiect

Sistem de coordonate al suprafetei de afisare

- Scalare: sx = 100/4 = 25, sy = 100/2 = 50
  - Scalare uniforma: sx = sy = min(sx, sy) = 25
- Oglindire fata de axa x
- Translatie astfel incat O -> C
  - tx = 300 – 0 = 300, ty= 200 – 0 = 200

Transformarea de instantiere

(x, y) – in sistemul de coordonate obiect

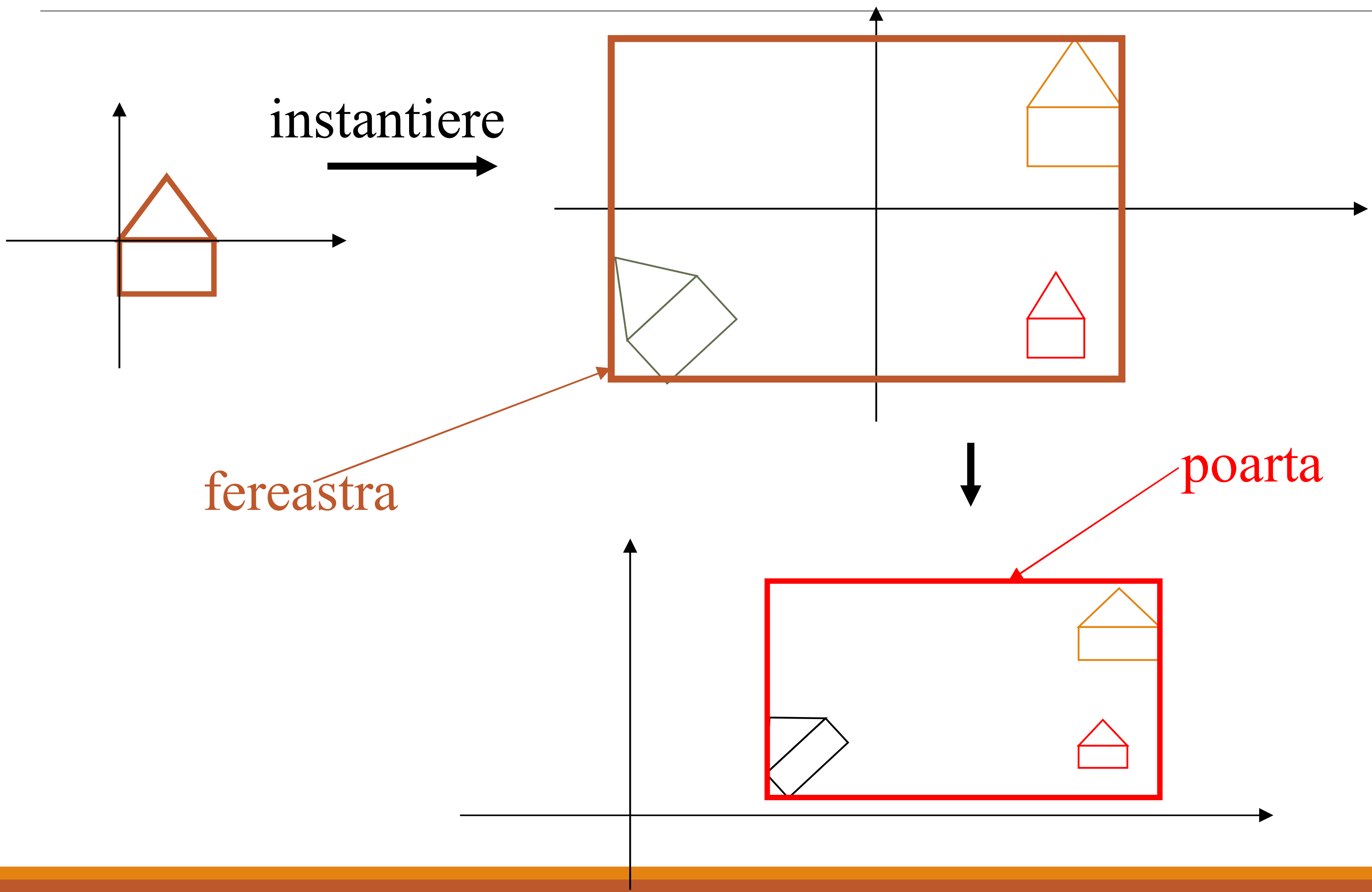
transformarea de instantiere

$$x' = x * 25 + 300$$

$$Y' = -y * 25 + 200$$

(x’, y’) – in sistemul de coordonate asociat suprafetei de afisare

Transformarea fereastra-poarta



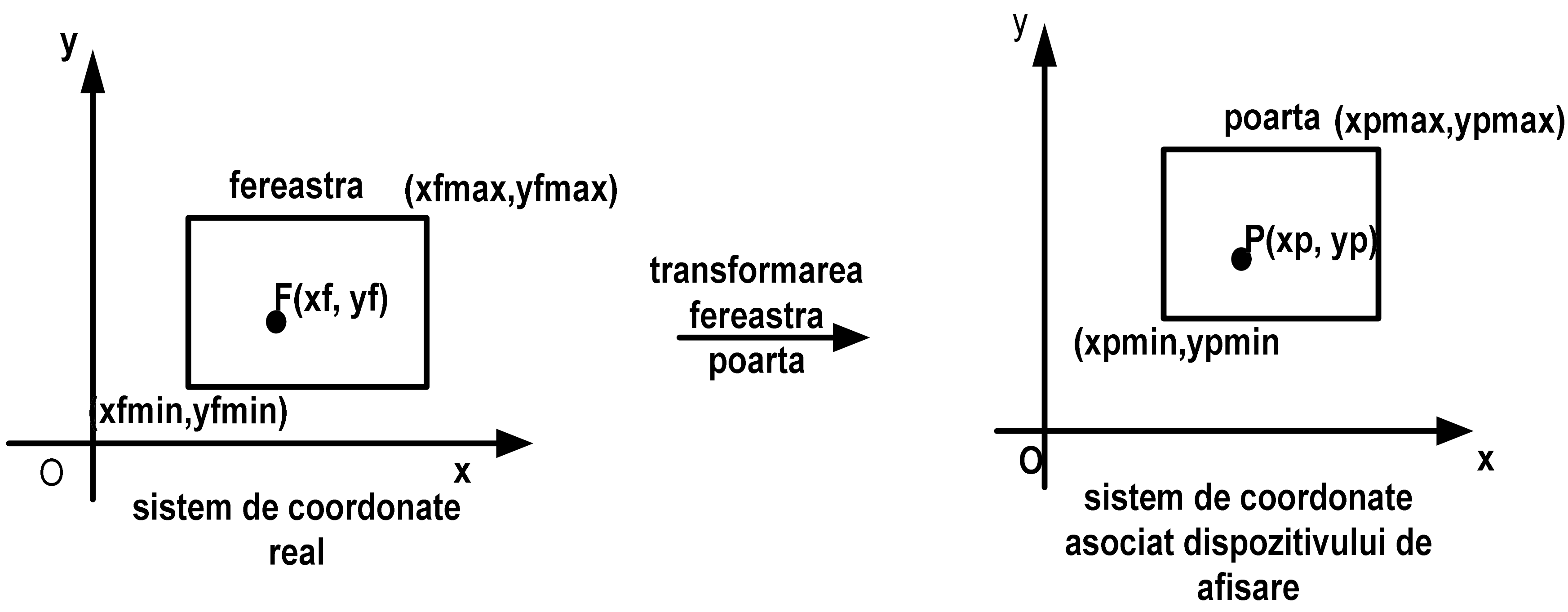
Transformarea fereastra-poarta

Face trecerea din sistem de coordonate 2D real in sistemul de coordonate asociat dispozitivului de afisare

Fereastra – zona dreptunghiulara cu laturile paralele cu axele sistemului de coordonate real ce incadreaza imaginea reprezentata in spatiul 2D real

Poarta – zona dreptunghiulara cu laturile paralele cu axele sistemului de coordonate dispozitiv (in care se va face afisarea)

Transformarea fereastra-poarta



Transformarea fereastra-poarta

Pozitia relativa a punctului F in fereastra trebuie sa fie aceeasi cu pozitia relativa a punctului P in poarta

$$\frac{xf - xf\min}{xf\max - xf\min} = \frac{xp - xp\min}{xp\max - xp\min}$$

$$\frac{yf - yf\min}{yf\max - yf\min} = \frac{yp - yp\min}{yp\max - yp\min}$$

$$sx = \frac{xp\max - xp\min}{xf\max - xf\min}$$

$$sy = \frac{yp\max - yp\min}{yf\max - yf\min}$$

$$tx = xp\min - sx * xf\min$$

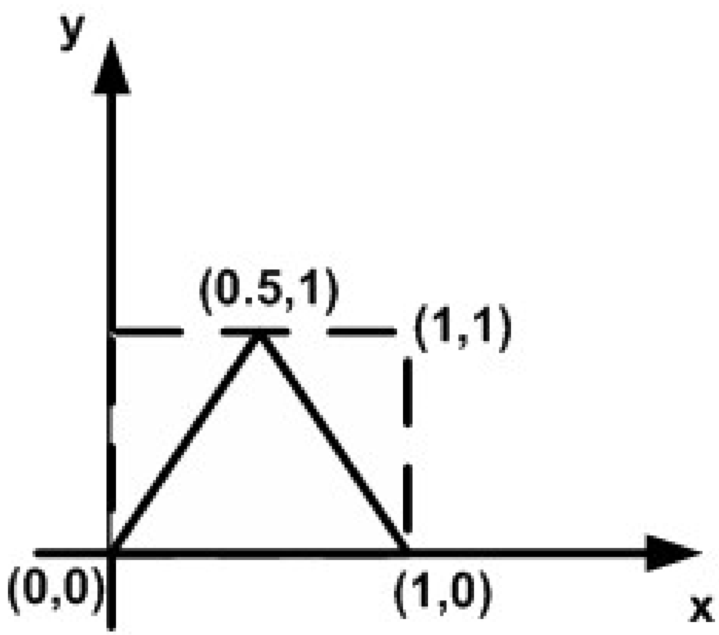
$$ty = yp\min - sy * yf\min$$

$$xp = xf * sx + tx$$

$$yp = yf * sy + ty$$

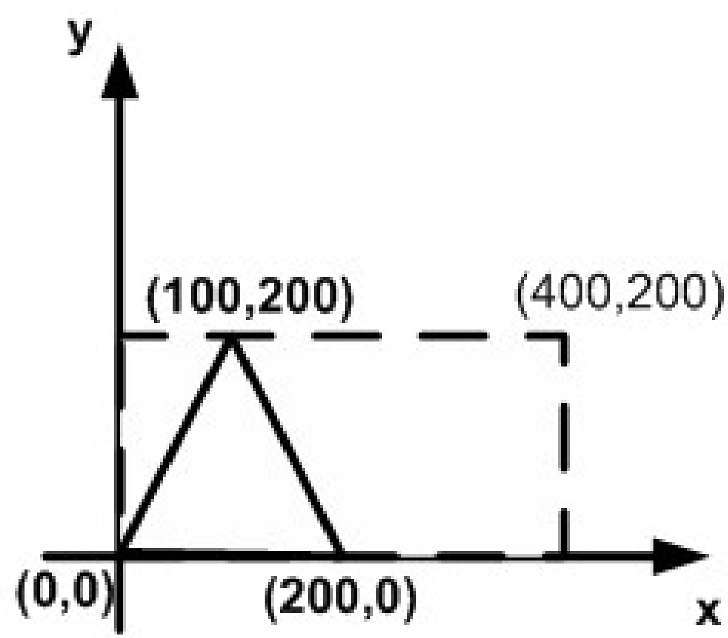
Transformarea fereastra-poarta

Poarta (0, 0) – (400, 200)

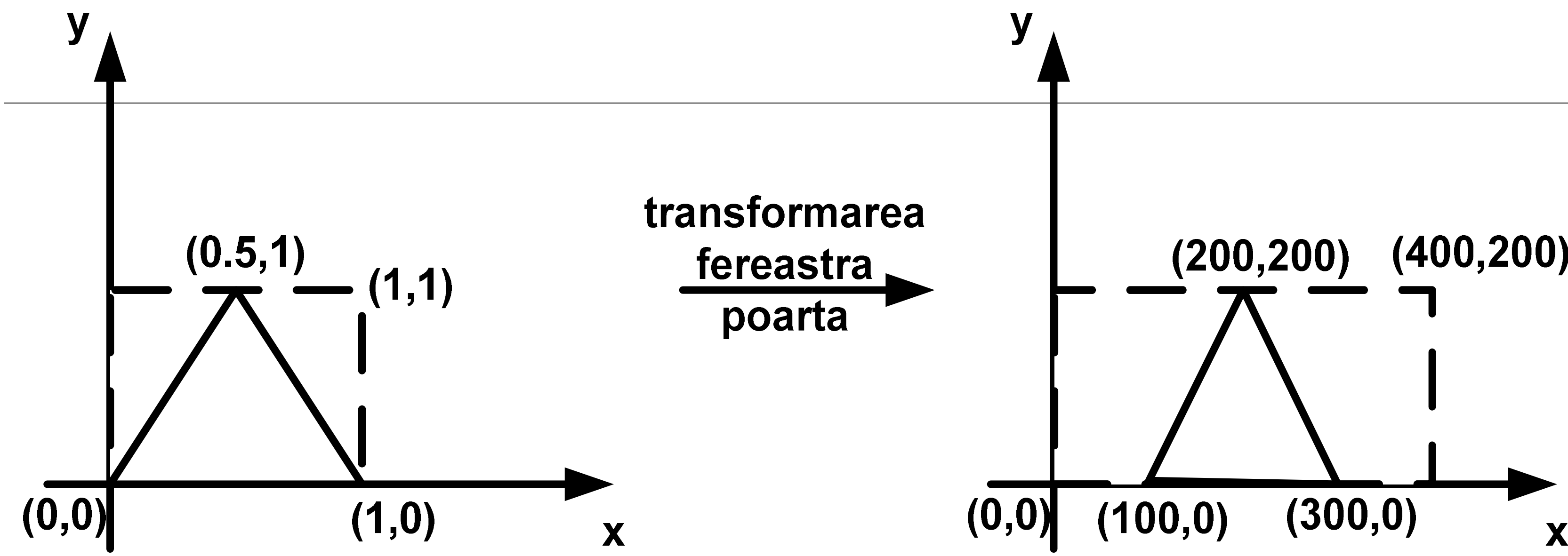


$$sx = \frac{xp\max - xp\min}{xf\max - xf\min} = 400, \quad tx = xp\min - sx * xf\min = 0$$

$$sy = \frac{yp\max - yp\min}{vf\max - vf\min} = 200, \quad ty = yp\min - sy * yf\min = 0$$



Transformarea fereastra-poarta



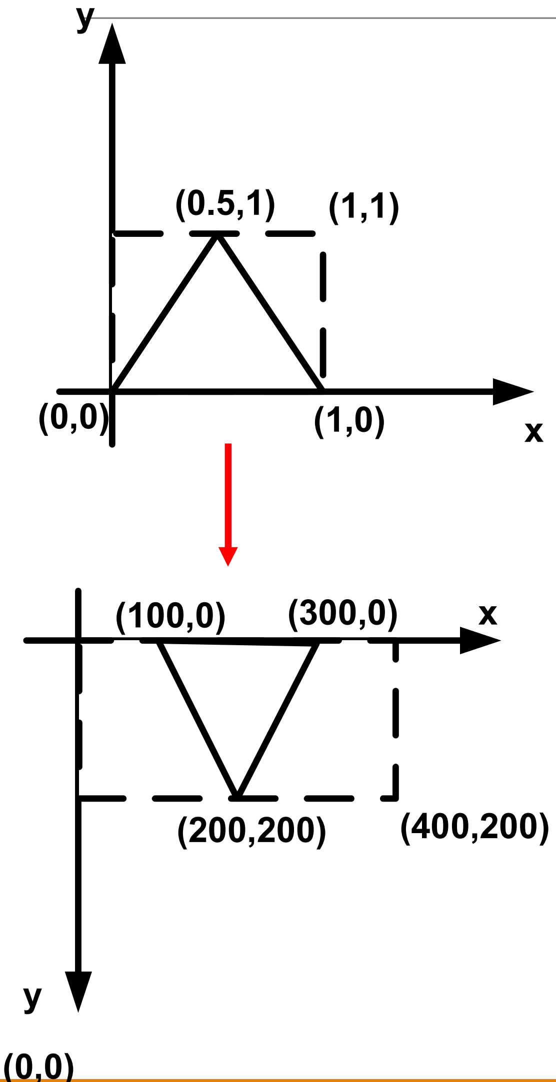
centrarea in poarta de afisare: translatie suplimentara

$$tx' = \frac{xp\max - xp\min - sx * (xf\max - xf\min)}{2}$$

$$ty' = \frac{yp\max - yp\min - sy * (yf\max - yf\min)}{2}$$

Transformarea fereastra-poarta

- Afisare pe ecran – poarta (0,0) – (400,200)



$$sx = \frac{xp\max - xp\min}{xf\max - xf\min} = 400, \quad tx = xp\min - sx * xf\min = 0$$

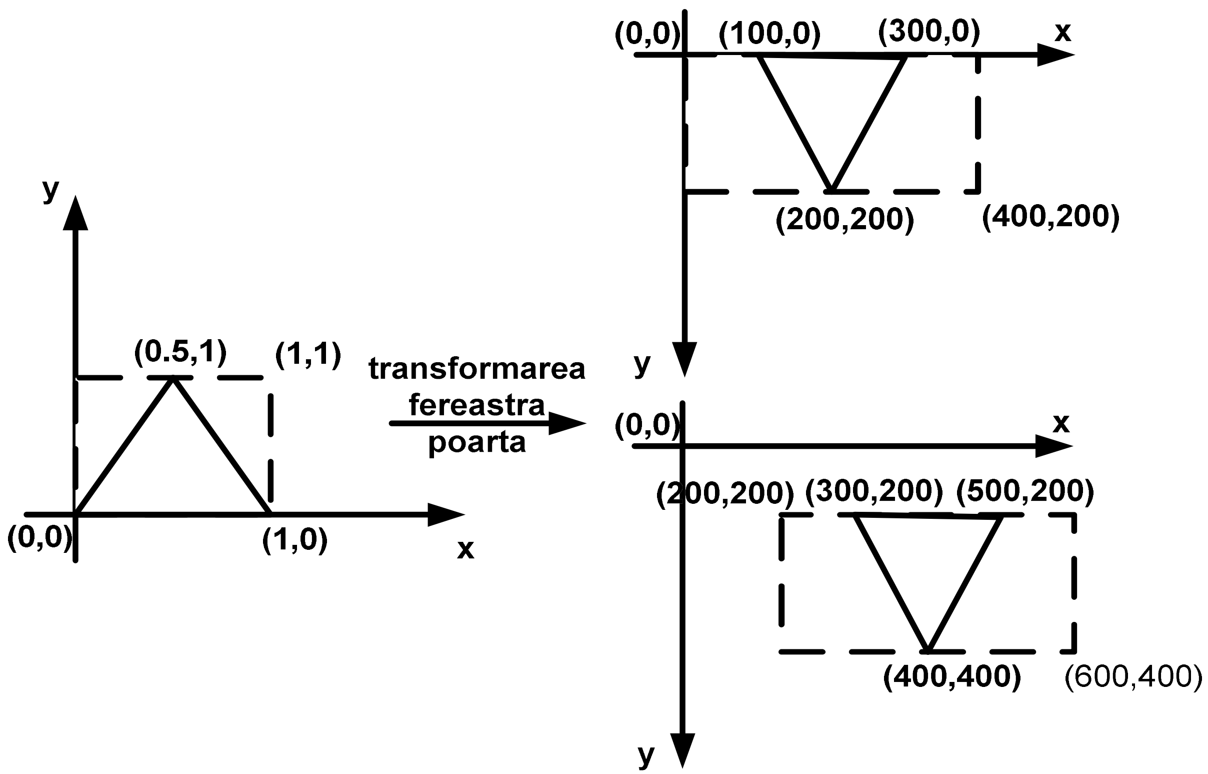
$$sy = \frac{yp\max - yp\min}{yf\max - yf\min} = 200, \quad ty = yp\min - sy * yf\min = 0$$

$$tx' = \frac{xp\max - xp\min - sx * (xf\max - xf\min)}{2} = 100$$

$$ty' = \frac{yp\max - yp\min - sy * (yf\max - yf\min)}{2} = 0$$

Transformarea fereastra-poarta

- Afisare pe ecran



$$xp = xf * sx + tx$$

$$yp = yp\min + yp\max - (yf * sy + ty)$$

Transformarea fereastra-poarta

- Matricea transformarii fereastra-poarta

$$xp = xf * sx + tx$$

$$\begin{aligned} yp &= yp\min + yp\max - (yf * sy + ty) \\ &= -yf * sy + yp\min + yp\max - ty \end{aligned}$$

$$M = \begin{bmatrix} sx & 0 & tx \\ 0 & -sy & yp\min + yp\max - ty \\ 0 & 0 & 1 \end{bmatrix}$$

Specificarea transformarii fereastra - poarta

```
float xFm, xFM, yFm, yFM;
int xPm, xPM, yPm, yPM;
float sx, sy, tx, ty;
boolean  tip_tran;  //tipul transformarii: true - scalare uniforma

…

void fereastra(float xFmin, float yFmin, float xFmax, float yFmax)
 {…  }

void poarta(int xPmin, int yPmin, int xPmax, int yPmax)
 {  …}
```

Specificarea transformarii fereastra - poarta

```
void calcul_transformare()
 {
   if(xFM > xFm && yFM > yFm)
   {
      sx = (xPM - xPm) / (xFM - xFm);
      sy = (yPM - yPm) / (yFM - yFm);

      if (tip_tran) //scalare uniforma
               sx = sy = (sx < sy) ? sx : sy;

      tx = xPm - sx * xFm + (xPM - xPm - sx * (xFM - xFm)) / 2;
      ty = yPm - sy * yFm + (yPM - yPm - sy * (yFM - yFm)) / 2;
   }
   else     sx = sy = tx = ty = 0;
 }
```

Specificarea transformarii fereastra - poarta

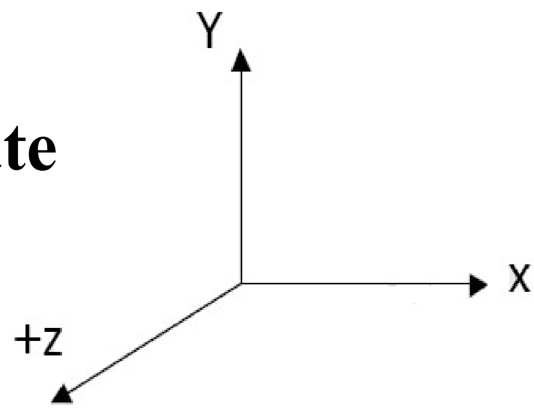
```
MATRICE matrice()
  { return MATRICE(sx, 0.f, 0.f, -sy, tx, -ty+yPM+yPm);}

int xDisp(float xf)
  {
    return (int) (xf * sx + tx);
  }

int yDisp(float yf)
  {
    return (int) (yPM + yPm - (yf * sy + ty));
  }

}
```

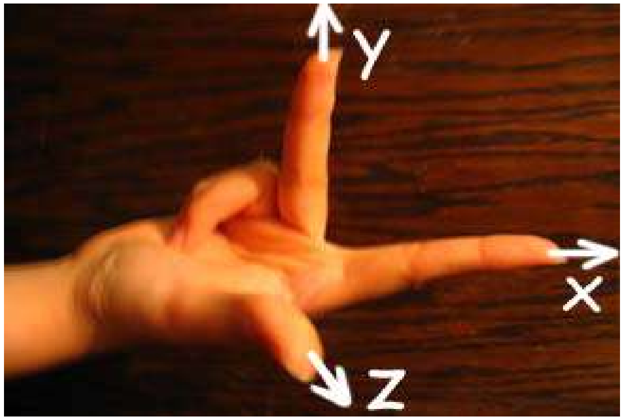
Vizualizarea scenelor 2D

1. Definirea obiectelor componente scenei in spatiul 2D real propriu
2. Plasarea obiectelor in scena – raportarea lor la acelasi sistem de coordonate – sistem 2D de coordonate real (transformarea de instantiere)
3. Aplicarea transformarilor geometrice 2D asupra obiectelor ce compun scena
4. Transformarea fereastra-poarta
5. Scena definita in sistemul de coordonate asociat dispozitivului de afisare (coordonate ecran)

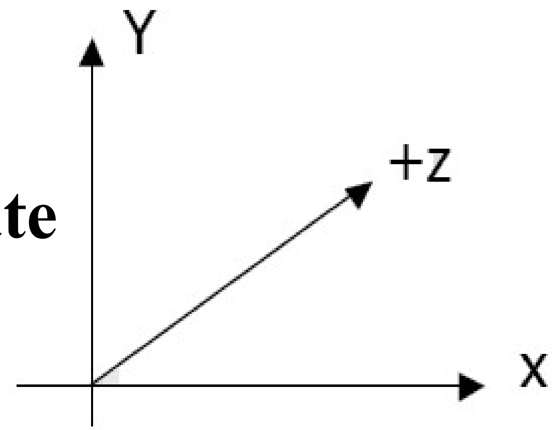
Sisteme de coordonate 3D

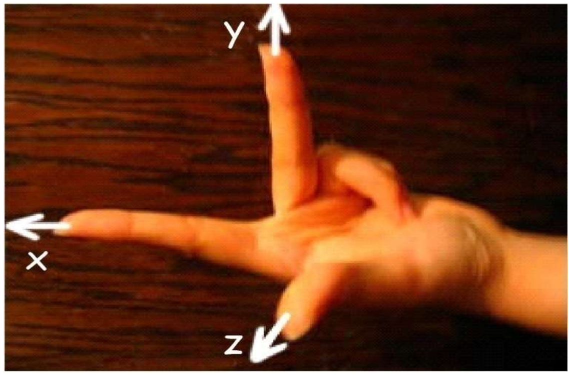
Sistem de coordonate dreapta





Sistem de coordonate stanga





Transformari geometrice 3D

Translatia

Scalarea

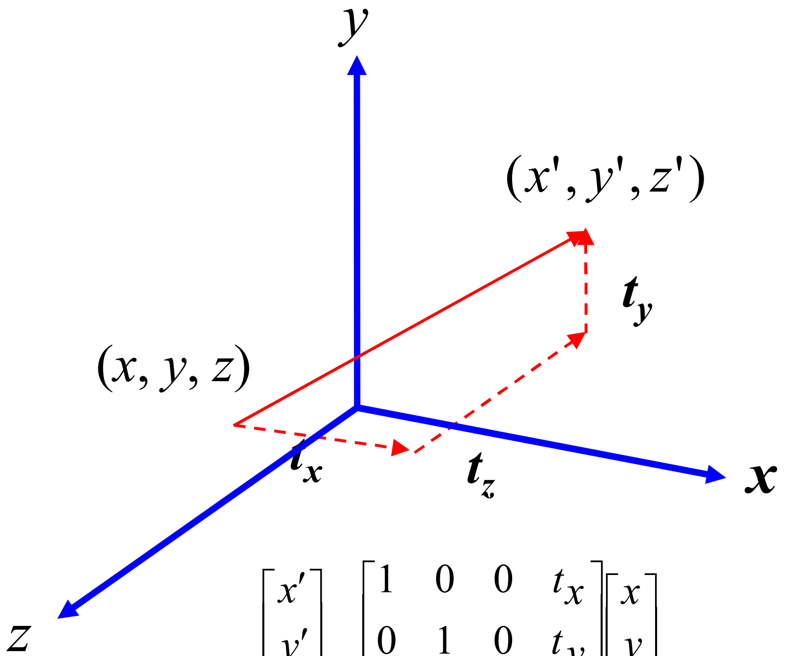
Rotatia:

- in jurul unei axe a sistemului de coordonate
- in jurul unei drepte oarecare

Oglindirea

Forfecarea

Translatia



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = x + t_x$$

$$y' = y + t_y$$

$$z' = z + t_z$$

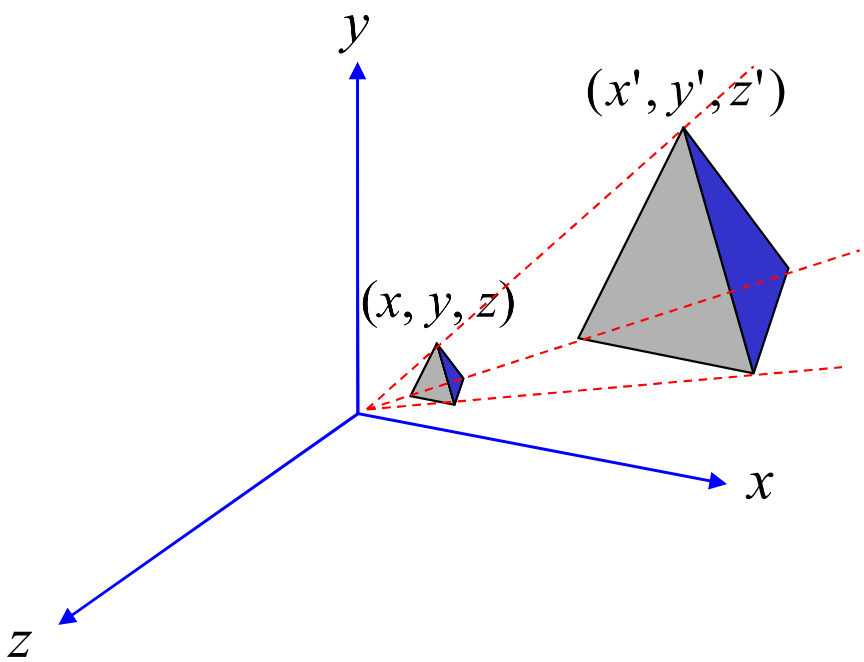
$$\uparrow$$

$T(t_x, t_y, t_z)$

Scalarea fata de origine

$$S_x = S_y = S_z \quad \Rightarrow \quad \text{scalare uniforma}$$

altfel, scalare neuniforma



$$x' = x \cdot s_x$$

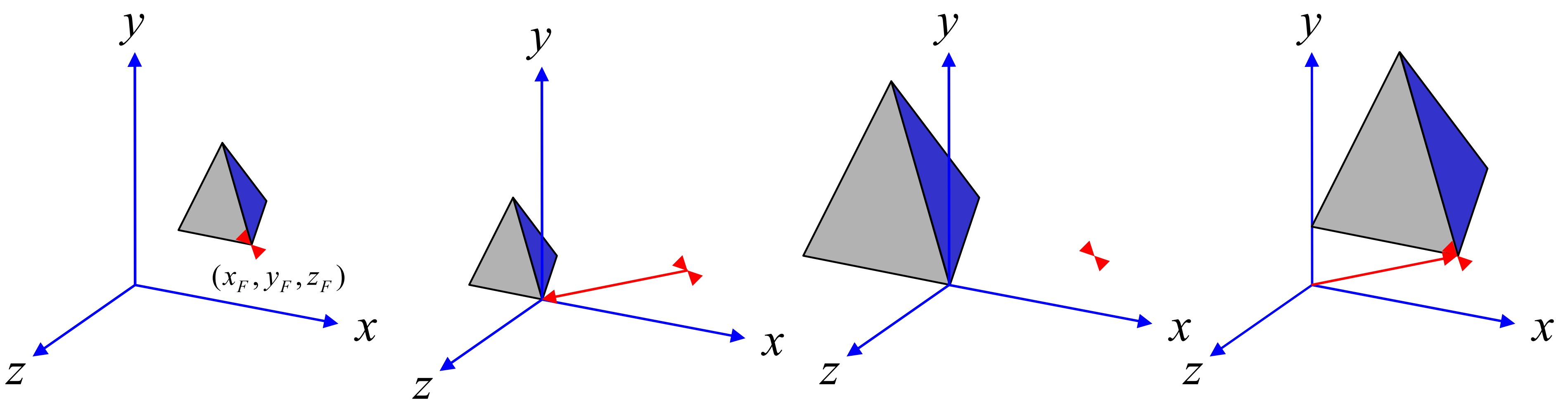
$$y' = y \cdot s_y$$

$$z' = z \cdot s_z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$S(s_x, s_y, s_z)$

Scalarea fata de un punct fix



$$P' = T(x_F, y_F, z_F)\, S(s_x, s_y, s_z)\, T(-x_F, -y_F, -z_F)\, P$$

$$T = \begin{bmatrix} 1 & 0 & 0 & -x_F \\ 0 & 1 & 0 & -y_F \\ 0 & 0 & 1 & -z_F \\ 0 & 0 & 0 & 1 \end{bmatrix}, S = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T^{-1} = \begin{bmatrix} 1 & 0 & 0 & x_F \\ 0 & 1 & 0 & y_F \\ 0 & 0 & 1 & z_F \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Rotatia

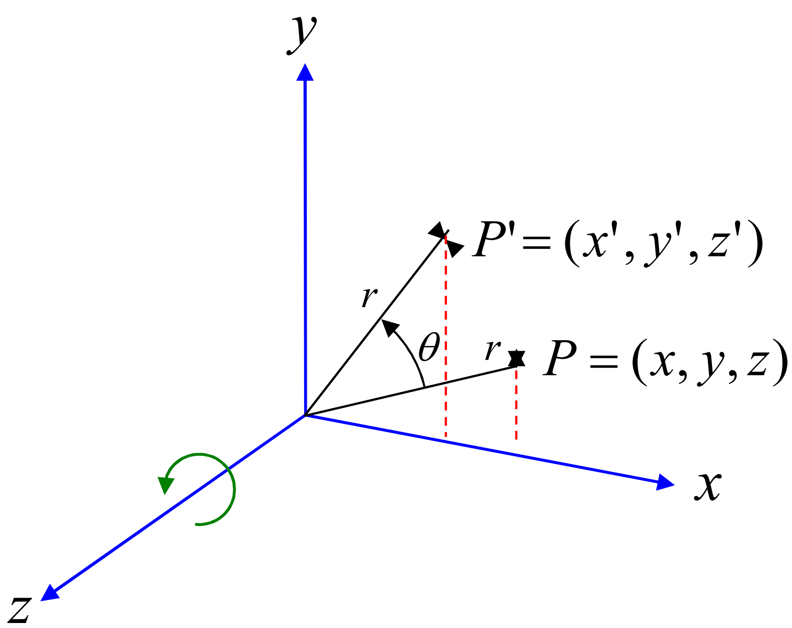
Rotatia($\boldsymbol{\theta}$)

(y, y, z) $\longrightarrow$ (x', y', z')

Rotatia in jurul

- axei ox
- axei oy
- axei oz
- unei drepte oarecare

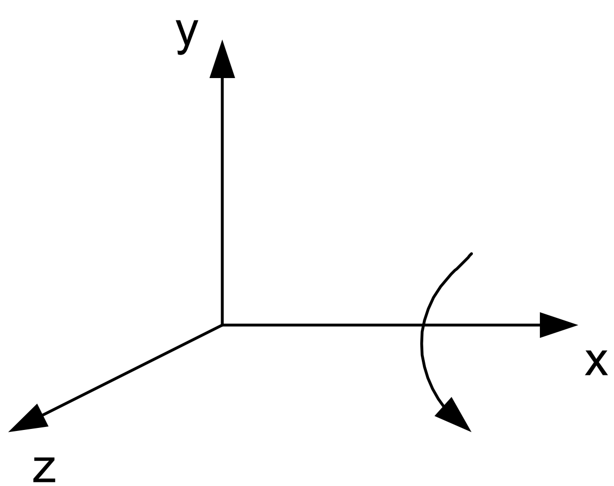
Rotatia pozitiva in jurul axei oz



$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_z(\theta)} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

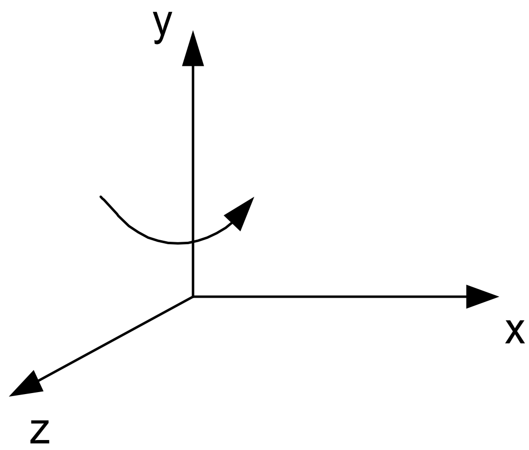
Rotatia pozitiva in jurul axei ox



$$x' = x$$
$$y' = y\cos\theta - z\sin\theta$$
$$z' = y\sin\theta + z\cos\theta$$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

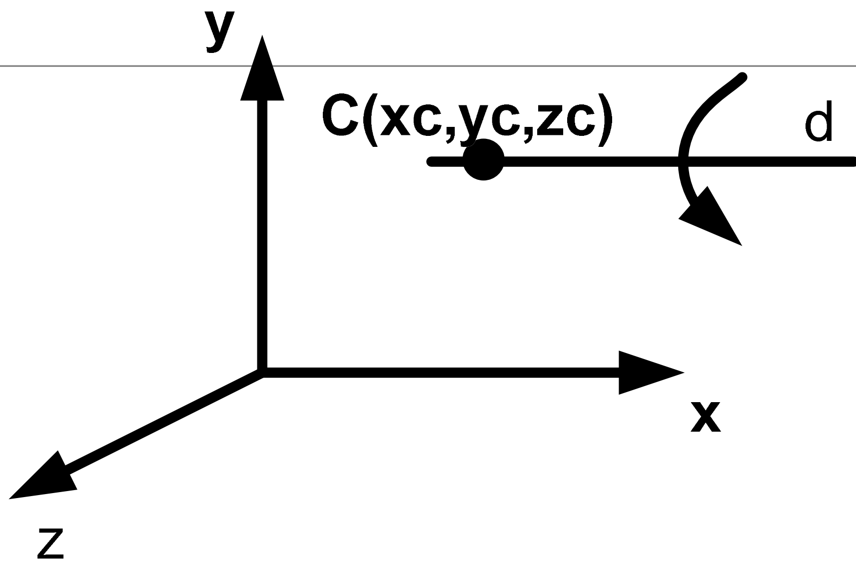
Rotatia pozitiva in jurul axei oy



$$x' = x\cos\theta + z\sin\theta$$
$$y' = y$$
$$z' = -x\sin\theta + z\cos\theta$$

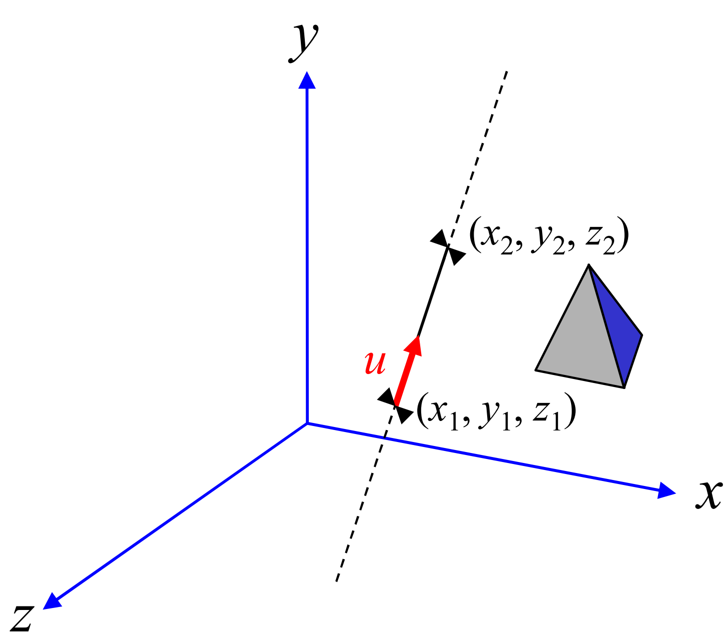
$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotatia in jurul unei drepte paralele cu ox



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = T(xc, yc, zc)\, R_x(u)\, T(-xc, -yc, -zc) \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotatia in jurul unei drepte oarecare



1. Translatie astfel incat dreapta sa treaca prin origine
2. Aliniere dreapta cu una din axe
3. Rotatia propriu-zisa.
4. Transformarea inversa de la punctul 2
5. Transformarea inversa de la punctul 1

Rotatia in jurul unei drepte oarecare

$$V = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

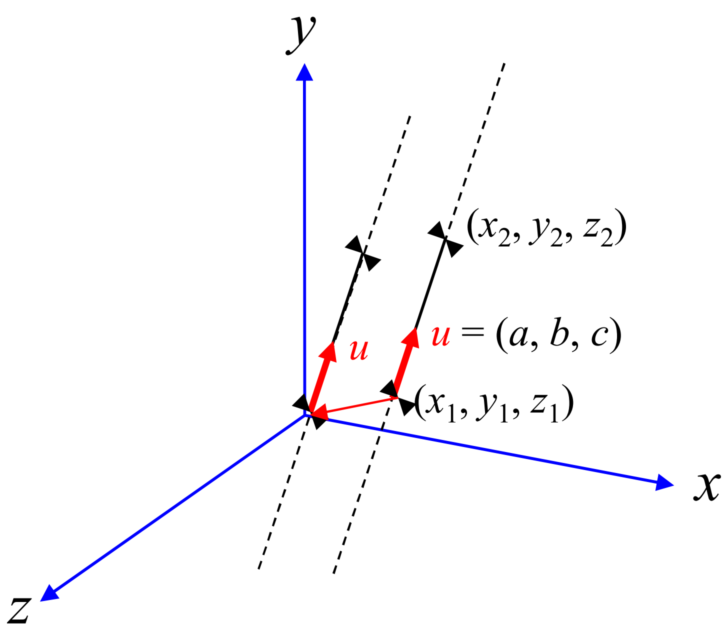
$$\text{Normalizare vector V}: u = \frac{V}{|V|} = (a, b, c)\,,\ a^2 + b^2 + c^2 = 1$$

$$\Delta x \equiv x_2 - x_1,\ \ \Delta y \equiv y_2 - y_1,\ \ \Delta z \equiv z_2 - z_1$$

$$|V| = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2},\ \ a = \frac{\Delta x}{|V|},\ \ b = \frac{\Delta y}{|V|},\ \ c = \frac{\Delta z}{|V|}$$

Rotatia in jurul unei drepte oarecare

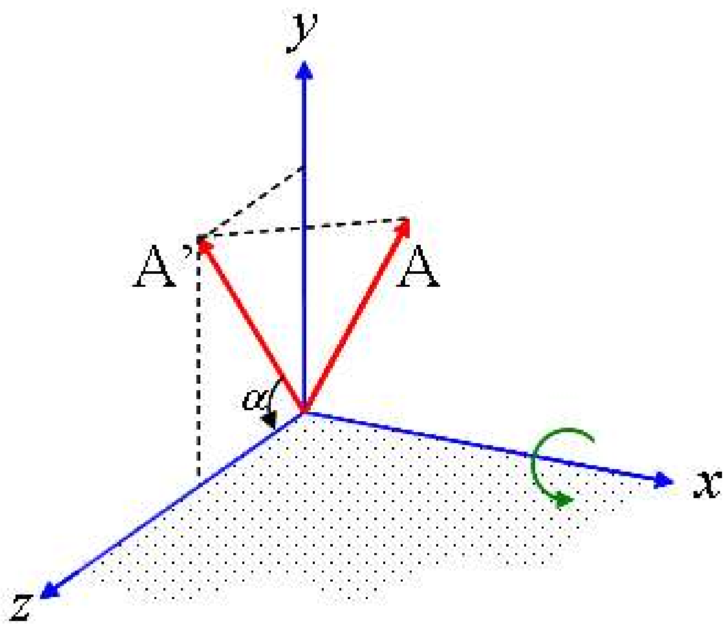
1. Translatie

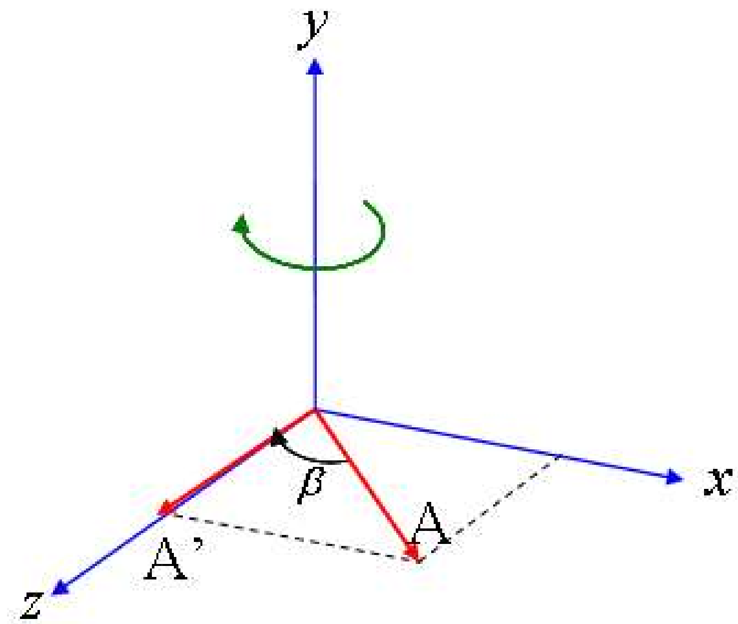


$$T = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotatia in jurul unei drepte oarecare

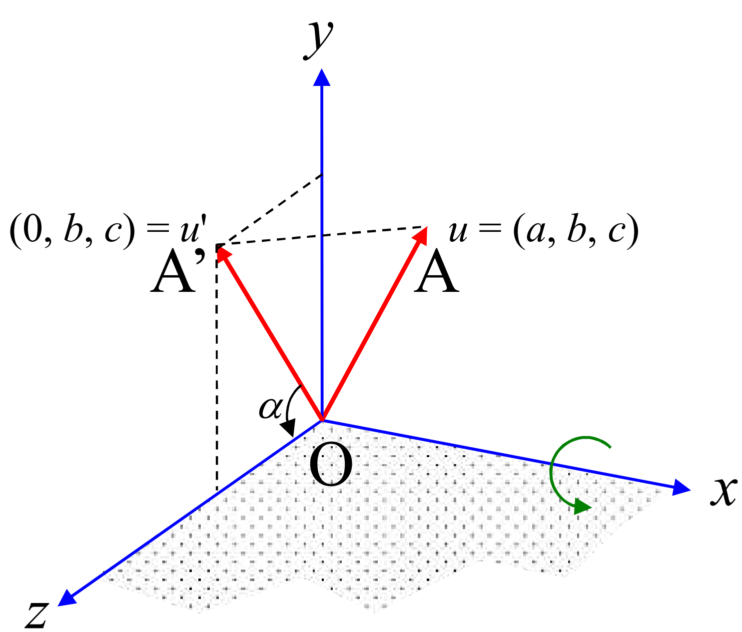
2. Aliniere *u* cu axa z





Rotatia in jurul unei drepte oarecare

2. Aliniere u cu axa z



$R_x(\alpha)$

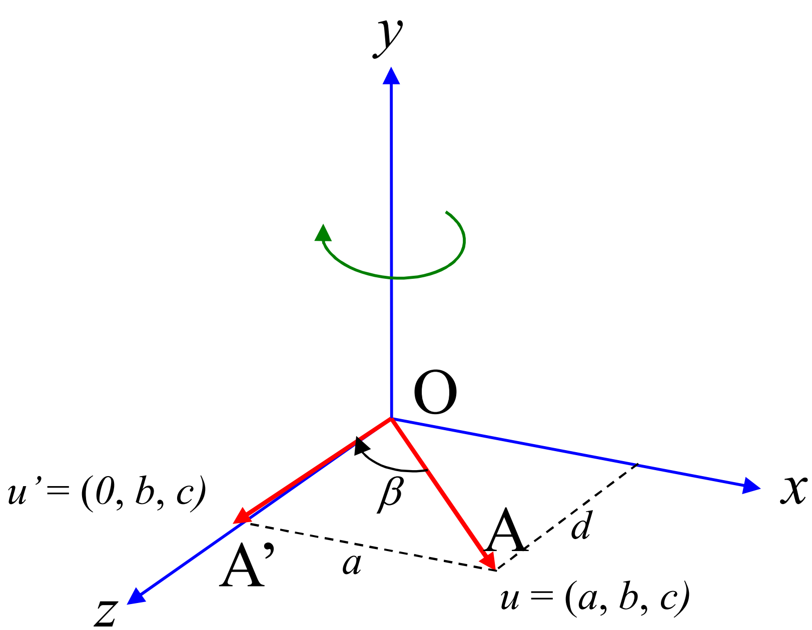
$$\sin\alpha = \frac{b}{\sqrt{b^2 + c^2}}$$

$$\cos\alpha = \frac{c}{\sqrt{b^2 + c^2}}$$

$$d = \sqrt{b^2 + c^2}$$

Rotatia in jurul unei drepte oarecare

- 2. Aliniere u cu axa z

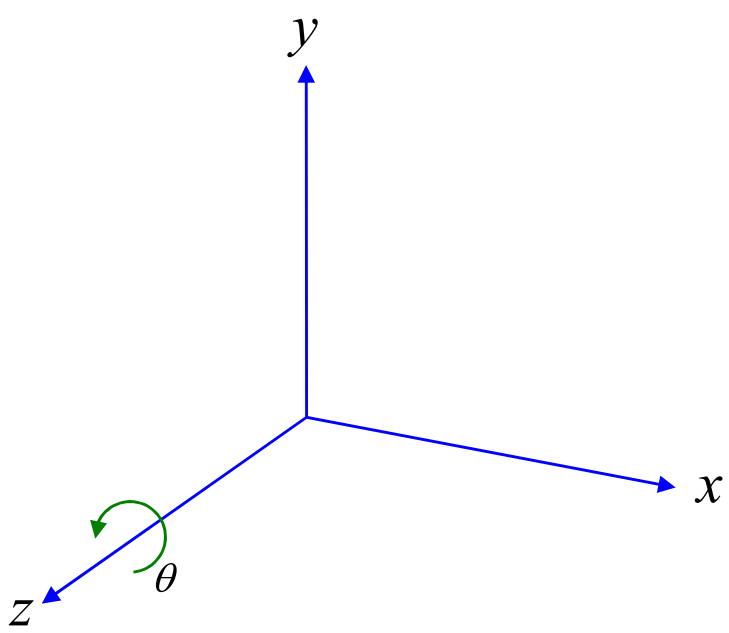


$$R_y(-\beta)$$

$$\sin\beta = a$$
$$\cos\beta = d$$

Rotatia in jurul unei drepte oarecare

3. Rotatia in jurul axei z cu unghiul dat



$$R_z(\theta)=\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotatia in jurul unei drepte oarecare

4. Transformarea inversa de la punctul 2

•Aliniere dreapta cu una din axe: $R_y(-\beta)*R_x(\alpha)$

4. $[A^{-1}] = [\, R_y(-\beta)\, R_x(\alpha)]^{-1} = [R_x(\alpha)^{-1}][R_y(-\beta)^{-1}]$

5. Transformarea inversa de la punctul 1

•Translatie astfel incat dreapta sa treaca prin origine: $T(-x_1,-y_1,-z_1)$

5: $[T^{-1}(-x_1,\, -y_1,\, -z_1)] = T(x_1,\, y_1,\, z_1)$

Rotatia in jurul unei drepte oarecare

1. Translatie astfel incat dreapta sa treaca prin origine
2. Aliniere dreapta cu una din axe
3. Rotatia propriu-zisa.
4. Transformarea inversa de la punctul 2
5. Transformarea inversa de la punctul 1

$$P' = \left[T^{-1}\right] \cdot \left[R_x(\alpha)^{-1}\right] \cdot \left[R_y(-\beta)^{-1}\right] \cdot$$
$$\left[R_z(\theta)\right] \cdot$$
$$\left[R_y(-\beta)\right] \cdot \left[R_x(\alpha)\right] \cdot \left[T\right] \cdot P$$

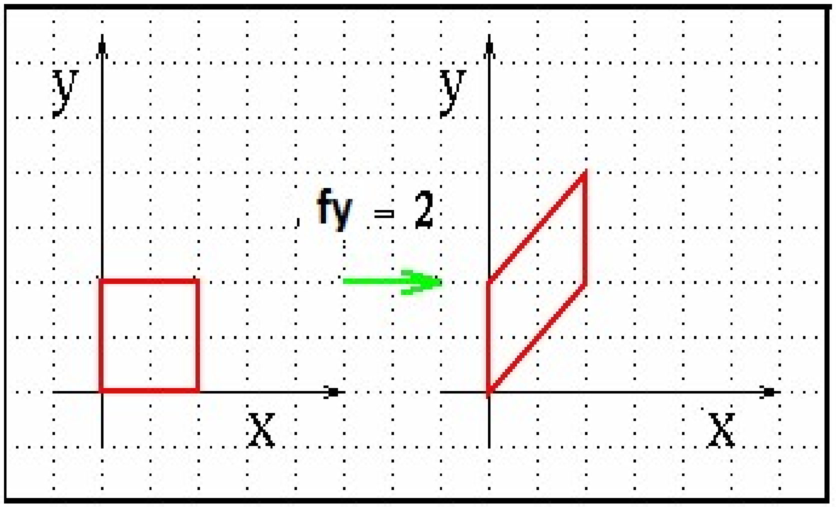
Oglindirea

Oglindirea

- Fata de planul xoy:
  - $x'=x$, $y'=y$, $z'=-z$
- Fata de planul yoz
  - $x'=-x$, $y'=y$, $z'=z$
- Fata de planul xoz
  - $x'=x$, $y'=-y$, $z'=z$
- Fata de un plan oarecare

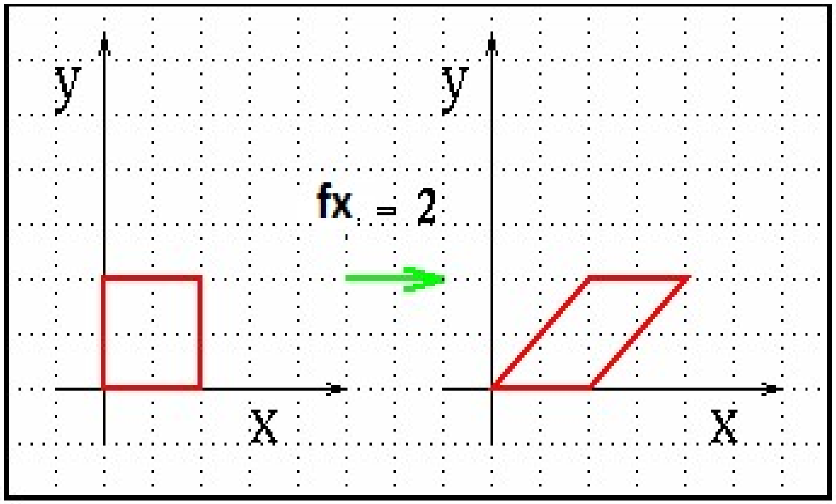
Forfecarea

- Forfecarea 2D



Forfecarea fata de oy

$$x' = x$$
$$y' = y + x * fy$$

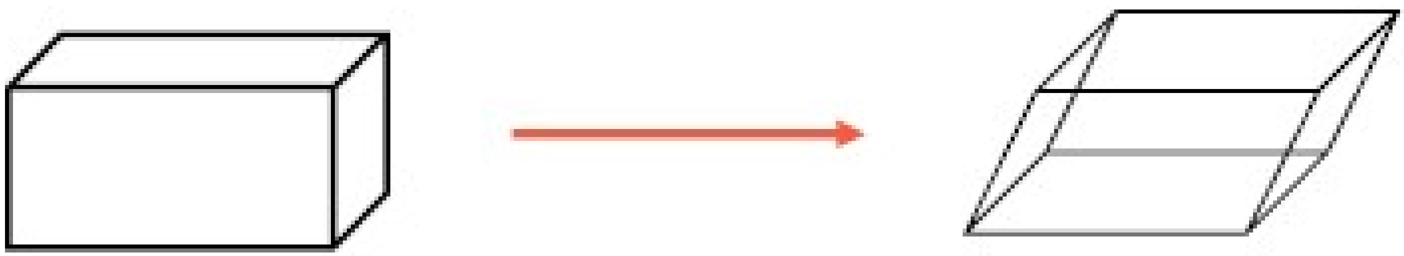


Forfecarea fata de ox

$$x' = x + y * fx$$
$$y' = y$$

Forfecarea

Forfecarea 3D



Forfecarea fata de axa z

x’=x+f*z
y’=y+t*z
z’=z

Proiecții paralele

- OBLICE (proiectori neperpendiculari pe plan)
  - Cavaliere (unghi de 45^0)
  - Cabinet generale (unghi aprox. de 63^0)
- ORTOGRAFICE (proiectori perpendiculari pe plan)
  - Axonometrice
    - $s_x = s_y = s_z$ (Izometrice)
    - $s_x = s_y$
    - $s_x \neq s_y \neq s_z$
  - Vederi (proiecții în planele principale ale sistemelor de coordonate)