Landing trajectory of an Airbus A300 optimized for fuel consumption

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Corresponding video link : `https://youtu.be/wrIkIp4riNM`

Abstract— **The landing of a commercial aircraft is one of the only phases of the flight that is not fully automated. As a result, the optimization of the landing trajectory lies in the hand of the pilots. However, it has been shown that pilots were generally flying their aircraft in a way that was very inefficient for minimizing the fuel consumption. This paper describes and presents the results of a model predictive control which provides the optimal control input for an Airbus A300 aircraft (thrust, elevator, landing gear extension, slats and flaps extension) during the landing phase given an initial state of the aircraft and the wind measured along the descent. Safe operation of the airplane is taken into account at every instant of the landing phase. This 2D model has been developed to prove the feasibility of this problem and the potential improvements in fuel consumption. A video presentation about the findings in this report can be found at: `https://youtu.be/wrIkIp4riNM`**

I. INTRODUCTION

Today, roughly 100,000 planes land every day all over the globe. This operation is the most difficult part of the flight and is in a lot of cases controlled to some extent manually by the pilot but can be easily assisted by an optimization algorithm. Currently, all the commercial flights are not assisted in landing and could be optimized to use less fuel during this phase. This could offer benefits to all the stakeholders, from the airplane company to the pilots, and save a huge amount of energy and carbon emissions every day. In this project, we are designing an optimization algorithm to predict from any original point in the sky, the trajectory that the plane should use to use as little fuel as possible. At every moment, the algorithm will provide the optimal value of each control parameter of the plane to help the pilot optimizing his landing.

II. MODEL DESCRIPTION

A. System dynamics

The different variables describing the longitudinal dynamics (2D) of the plane are:

- V [m/s] : Air speed of the aircraft (different from ground speed in general)
- γ [rad] : Slope angle, angle in the vertical plane between the x axis and the speed vector of the airplane.
- q [rad/s] : Pitch speed, the pitch angle being the angle between the x axis and the aircraft fuselage in the vertical plane.
- α [rad] : Angle of incidence or angle of attack, angle in the vertical plane between the air speed vector and the plane fuselage.
- h [m] : Altitude.
- x [m]: Distance from runway (longitudinal)

The equations describing these variables are

$$\dot{V} = -\frac{\rho V^2 S}{2m} C_x(\alpha, q, \delta m) + \frac{F(\delta x, h)}{m} - g\gamma \tag{1}$$
$$\dot{\gamma} = \frac{\rho V S}{2m} C_z(\alpha, q, \delta m) - \frac{g}{V} \tag{2}$$
$$\dot{q} = \frac{\rho V^2 S l}{2B} C_{m/G}(\alpha, q, \delta m) \tag{3}$$
$$\dot{\alpha} = q - \dot{\gamma} \tag{4}$$
$$\dot{h} = V\gamma \tag{5}$$
$$\dot{x} = V\cos\gamma \tag{6}$$
$$\tag{7}$$

where the expressions for the aerodynamics coefficients and thrust are given by,

$$C_z(\alpha, q, \delta m) = C_{z\alpha}(\alpha - \alpha_0) + C_{z\delta m}\delta m \tag{8}$$
$$C_x(\alpha, q, \delta) = C_{x0} + k_i(C_{z\alpha}(\alpha - \alpha_0) \tag{9}$$
$$+ C_{z\delta m}\delta m)^2 \tag{10}$$
$$C_{m/G}(\alpha, q, \delta m) = C_{m0} + C_{m\alpha}(\alpha - \alpha_0) \tag{11}$$
$$+ C_{mq}\frac{ql}{V} + C_{m\delta m}\delta m \tag{12}$$
$$F(\delta x, h) = 2F_{max}\delta x\frac{\rho(h)}{\rho_0} \tag{13}$$

where $\rho(h)$ is defined as,

$$\rho(h) = \rho_0(1 - 22.6 \cdot 10^{-6} h)^{4.26} \tag{14}$$

From the given differential equations, we define our state vector as

$$z = \begin{bmatrix} V \ \gamma \ q \ \alpha \ h \ x \end{bmatrix}^T \tag{15}$$

Our control input vector is

$$u = \begin{bmatrix} \delta_x \ \delta_m \end{bmatrix} \tag{16}$$

where δ_x is the gas power or the throttle. The throttle is varying from 0 to 1, where 0 is no applied thrust and 1 is maximum thrust. δ_m is the pitch control (angle of the elevator). In addition to these two main control tools, the plane also has the possibility to extend the landing gears as well as the high lift surfaces (the flaps and the slats) which allows the plane to fly at lower speeds and with a higher angle of attack.

B. Cost function

The aim of this project is to optimize the landing trajectory with regards to fuel consumption. This consumption is actually directly proportional to the throttle. Historically for a commercial aircraft, increasing the throttle was simply increasing the kerosene mass flow to the engines. Even if engines work in a slightly different way nowadays we will keep this assumption for this project. As a result the cost constructed to minimize the fuel consumption is as follows,

$$J = \sum_{k=0}^{N} u_k' R u_k \tag{17}$$

where R is given by,

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{18}$$

In this way, the landing trajectory can be optimized with regards to the fuel consumption i.e the thrust only. Note that optimizing the landing trajectory with regard to the fuel consumption only is an arbitrary choice, and other parameters could have been added, as the minimization of the inputs given to the elevator to simplify the control of the airplane, or the minimization of the acceleration and deceleration phases to maximize passenger comfort.

C. Model constraints

In order to simulate a stable flight, several constraints must be fulfilled. These constraints are of different natures: some of them come from physical constraints such as keeping the plane airborne, other are not as critical but are here to ensure the comfort inside the aircraft and finally there are also many constraints linked to the regulations in place around the world concerning commercial aviation.

State and input constraints
For a safe flight, the general recommendations states that the air speed must be at least 23% higher than the stall speed V_{Stall}, so we choose

$$1.3\, V_{\text{Stall}}(k) \leq V(k) \leq 250\,\mathrm{m\,s^{-1}} \quad \text{for } k \in [0, N] \tag{19}$$

Note that the stalling speed varies with time as the slats and the flaps are progressively extended during the landing phase. For the variables γ, q we set the following constraints:

$$-10° \leq \gamma(k) \leq -10° \quad \text{for } k \in [0, N] \tag{20}$$

$$-5\,°\,\mathrm{s}^{-1} \leq q(k) \leq -5\,°\,\mathrm{s}^{-1} \quad \text{for } k \in [0, N] \tag{21}$$

The lower bound constraint for α is determined by the angle at which the lift generated by the wings is equal to zero. This angle is called angle of zero-lift, noted $\alpha_0(k)$, and varies with time as the slats and flaps are progressively extended. The upper bound is limited by the maximum angle of incidence $\alpha_{max}(k)$ which corresponds to the maximum angle of incidence the wing can take before stalling. It also increases as the slats and flaps are extended.

$$\alpha_0(k) \leq \alpha(k) \leq \alpha_{max}(k) \quad \text{for } k \in [0, N] \tag{22}$$

The altitude h and the longitudinal position (distance to the airport) should be between the initial value and 0 as we want the aircraft to get closer to the runway:

$$0\,\mathrm{m} \leq h(k) \leq h_0 \quad \text{for } k \in [0, N] \tag{23}$$

$$0\,\mathrm{m} \leq x(k) \leq x_0 \quad \text{for } k \in [0, N] \tag{24}$$

Because δ_x is defined as the ratio of the actual delivered power to the maximum power of the engines, $\delta_x \in [0, 1]$. For the angle of the elevator, we set the constraints to

$$-30° \leq \delta_m(k) \leq 30° \quad \text{for } k \in [0, N] \tag{25}$$

Final Constraints
To enable the solver of the optimization problem to find a feasible solution, we don't define equality terminal constraints, but allowed a convex terminal set, which should be reached when landing:

$$0.95(1.3V_{\text{Stall}} + \Delta_{\text{s}}) \leq V(N) \leq 1.05(1.3V_{\text{Stall}} + \Delta_{\text{s}}) \tag{26}$$

$$-5° \leq \gamma(N) \leq 5° \tag{27}$$

$$-5\,°\,\mathrm{s}^{-1} \leq q(N) \leq 5\,°\,\mathrm{s}^{-1} \tag{28}$$

$$0.95 \cdot \alpha_L \leq \alpha(N) \leq 1.05 \cdot \alpha_L \tag{29}$$

$$0\,\mathrm{m} \leq h(N) \leq 10\,\mathrm{m} \tag{30}$$

$$0\,\mathrm{m} \leq x(N) \leq 100\,\mathrm{m} \tag{31}$$

$$\tag{32}$$

The landing velocity is - in accordance with the international regulations - equal to 1.3 times the stalling speed at landing, in addition to a safety margin Δ_s which is a function of the wind. This margin is typically half the wind speed in knots added as meters per second. The optimal angle of attack α_L is equal to 0.0788 rad and is a function of the landing speed. The typical slope prior to landing for a commercial jet is around $\gamma = 3.5°$, but this slope has to change a few seconds before that the landing gear touches the ground to make a smoother landing with a slope as close to 0° as possible. Here we added a sufficient margin to ensure the convergence of the optimization problem. The pitch speed constraint has been set more arbitrarily due to a lack of data but while trying to remain realistic.
For the Landing Gear and the Slats/Flaps we set an equality constraint to ensure, that they are fully extended when landing:

$$P_{\text{LG}}(N) = 1 \tag{33}$$

$$P_{\text{SF}}(N) = 1 \tag{34}$$

Slack Constraints
For the landing gear P_{LG} and slats/flaps P_{SF} variables, we define first $P_{\text{LG}}, P_{\text{SF}} \in [0, 1]$. Then, we need to make sure that the model doesn't use them as a way to brake, so we added a constraint specifying that once these controls start

being extended, the aircraft does not abort and complete to full extension of the landing gear and the slats/flaps:

$$P_{\text{LG}}(k+1) \geq P_{\text{LG}}(k) \quad \text{for } k \in [0, N-1] \tag{35}$$

$$P_{\text{SF}}(k+1) \geq P_{\text{SF}}(k) \quad \text{for } k \in [0, N-1] \tag{36}$$

$$\tag{37}$$

Moreover, we also want to ensure that the plane is constantly descending:

$$h(k+1) \leq h(k) \quad \text{for } k \in [0, N-1] \tag{38}$$

At last, we want to limit the acceleration and deceleration of the airplane to ensure passenger comfort:

$$-g \leq \frac{V(k+1) - V(k)}{T_s} \leq g \tag{39}$$

D. Model Predictive Control Implementation

The cost function in the problem given in Equation 17 is quadratic. The constraints on the states and input are linear, but the dynamics of the system are highly non-linear. Thus, this in a non-linear optimization problem. The open-loop and the MPC problem used the non-linear constrained solver IPOPT. The MPC controller was designed to automate the landing process and optimize it for fuel consumption. Wind in the longitudinal direction was also added as a disturbance. This wind was modeled by picking a random number between 0 and 20 m/s which would be the initial wind speed. To imitate wind in real life, the wind would be updated at each time step. Therefore, at each time step, when calling the solver in the MPC, the new wind would be updated to be $\pm$ 5% of the wind speed at the previous time step.

III. RESULTS AND DISCUSSION

A. Open loop

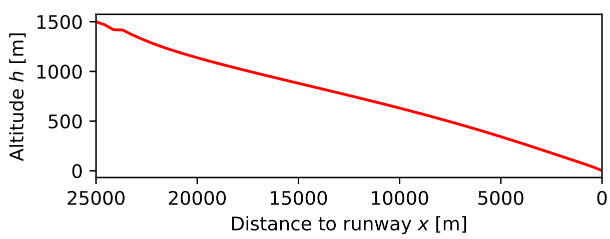

Fig. 1: Optimal Trajectory for the Open-Loop Solution

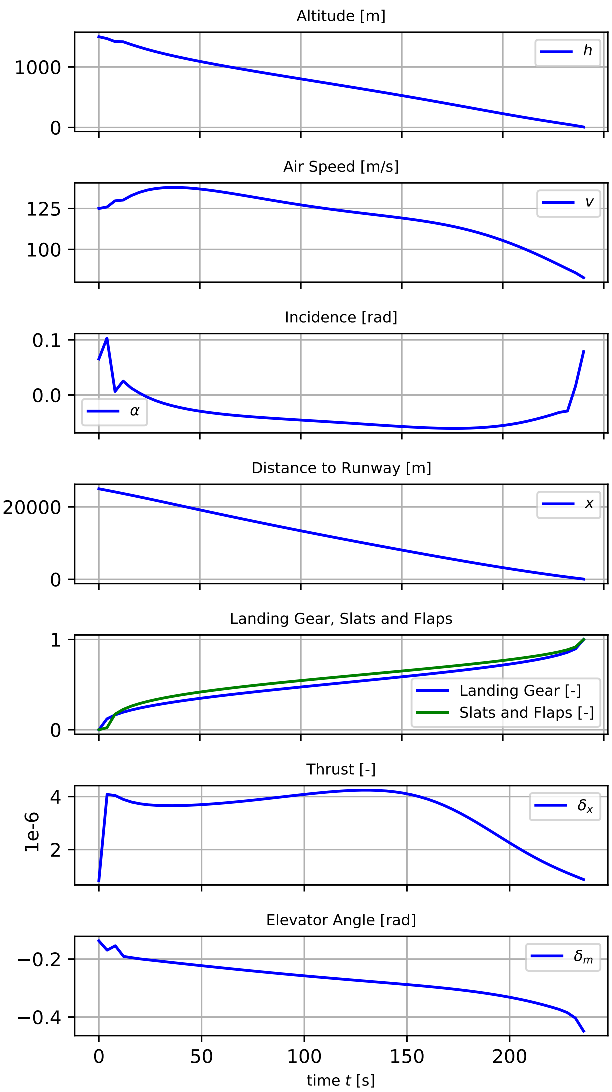

Fig. 2: Evolution of states and control inputs of the open-loop solution

For our first example, the aircraft starts its approach at a distance $x = 25km$ from the runway at an altitude of $h = 1500m$ and an air speed of $v = 100m/s$. The optimal trajectory of the open-loop solution is given in Figure 1. This is the trajectory from the given initial point that is the most optimal with regards to fuel consumption. The evolution of the different states and control inputs are given in Figure 2. In the subplot showing the thrust applied over time, it has the order of magnitude of 10^{-6}. In other words, the plane is almost purely gliding through the sky to reach the runway and as such the amount of thrust actually needed to land is virtually equal to zero. We can observe that during the first fifty seconds the plane accelerates even though there is no significant thrust applied. This is simply the potential energy being converted to kinetic energy. However, as the plane gets closer to the ground, the landing gear, the flaps and the slats are extended and the plane starts to slow down because its aerodynamic performances are degraded. The optimizer is able to remain in control of the aircraft by adjusting the elevator angle to adapt the angle of incidence and generate the adequate lift to continue the descent following a slow and steady slope. Eventually, the aircraft reaches the ground using virtually no fuel but thanks to a precise control of the elevator, of the landing gear and of the high lift surfaces.

B. MPC

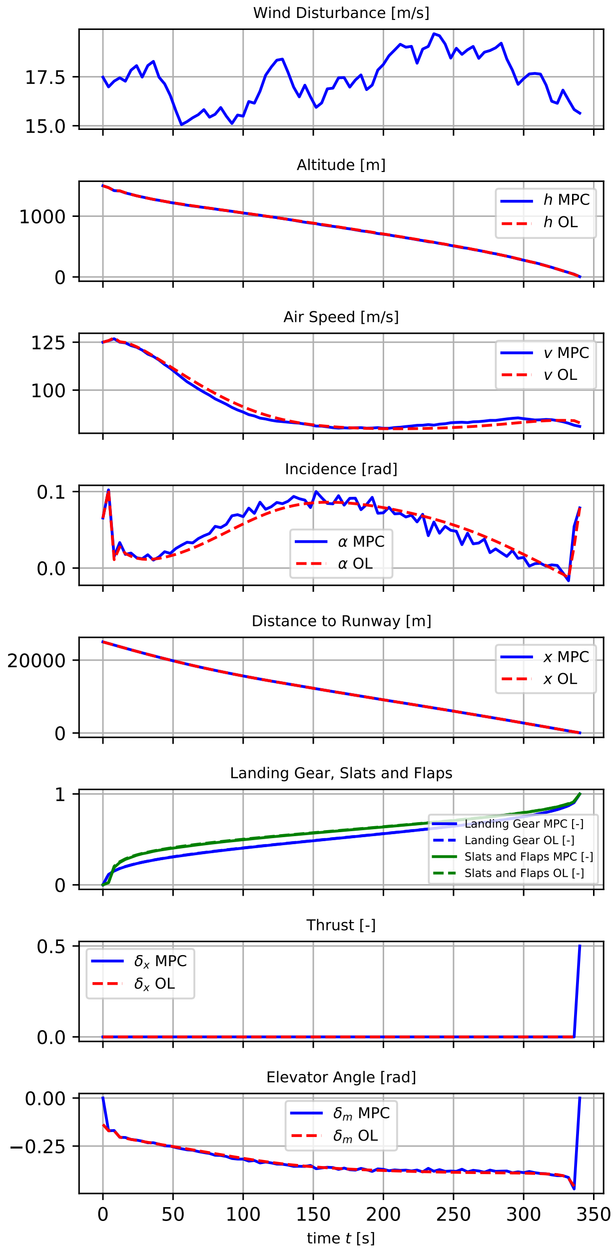

Fig. 3: Evolution of states and control inputs of the Model-Predictive Controller (MPC) compared to the open-loop (OL) solution

Achieving the optimal open-loop trajectory manually is difficult for pilots. In order to ensure the required accuracy over the controls of the aircraft, an MPC controller was implemented to automate the process. This MPC is computing the new optimized control sequence every 4 seconds using the new wind speed measured at the time of the simulation. Note that more powerful calculators could be used to further reduce the sampling time. The optimal trajectory of the open-loop solution is given in Figure 4. In this figure, the overall trajectory obtained the MPC controller is almost identical to the open-loop trajectory which has been computed at the first time step. That is because the variations of the wind remain relatively small in comparison to the air speed of the aircraft. However, looking at the evolution of the different states given in Figure 3 we can observe some differences between the two approaches. The main significant difference visible is the angle of attack of the wings which is much smoother for the open-loop as this approach assumes constant wind (the one measured when the optimizer solves the problem) through the entire landing process. We can also observe a sudden peak of thrust at the very end of the simulation when using the MPC. This is because the wind is dropping at the exact instant when the plane has to land, which means that the air speed decreases. To avoid a dangerously slow speed, the aircraft has to generate thrust to keep a relatively constant speed and operate a smooth landing. The optimizer also indicates that a quick adjustment of the elevator angle has to be made for the same reason of stability. This gust does not exist in the open-loop where the wind is assumed to be constant and the plane does not operate a last second adjustment before landing in this approach as we can see in the plots.

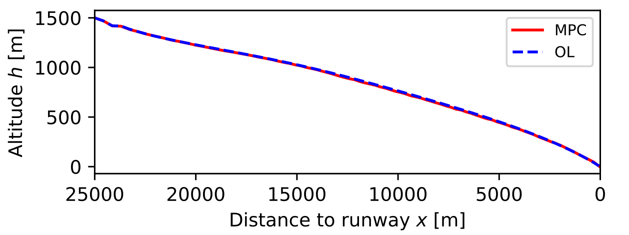

Fig. 4: Optimal Trajectory for Model Predictive Controller (MPC) and Open Loop Solution (OL)

IV. CONCLUSION

In this project, we designed a MPC controller to reduce the fuel consumption of an Airbus A300 during the landing phase. Our controller is working with a simplified 2D model of an Airbus A300 and is able to control a plane with an initial direction parallel to the runway. We have successfully optimized the landing phase of an aircraft flying at an altitude of 1500m and located 25km away from the runway. Thus, in addition to offering a safe landing, this study could allow pilots to save the equivalent amount of fuel that they would have used without this optimization. According to [7]: at least 118L of kerosene, responsible for about 295kg of CO2. For an airport such as SFO and its 230 000 annual flights this economy would add up to around 27 million liters of kerosene or 68 thousand tons of CO2 that would not be emitted in the atmosphere (equivalent to the annual emissions of 4200 Americans).

Now that this controller is working and able to handle wind perturbations, a future work would be to extend the dimensions of the model and design an MPC controlling a 3D trajectory. The concept would be extremely similar but a series of states would be added and inputs controls corresponding to the lateral dynamics of the plane would also be required. This can be done using our algorithm structure but will demand more computation time.
Commercial aviation is responsible for 2 to 3 percents of global CO2 emissions every year. Therefore, this kind of optimization could be implemented in every flying plane and this change could save tons of CO2 rejections and be a small step to reduce plane's carbon footprint.

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