Hash Tables

Data Structures for *Map ADT*

	Find	Insert	Delete
Unsorted Array	$\theta(n)$	$\theta(n)$	$\theta(n)$
Unsorted Linked List	$\theta(n)$	$\theta(n)$	$\theta(n)$
Sorted Array	$\theta(\log(n))$	$\theta(n)$	$\theta(n)$
Sorted Linked List	$\theta(n)$	$\theta(n)$	$\theta(n)$
Binary Search Tree	$\theta(h), \theta(n)$	$\theta(h), \theta(n)$	$\theta(h), \theta(n)$
AVL Tree	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$

Data Structures for *Map ADT*

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Unsorted Array	$\theta(n)$	$\theta(n)$	$\theta(n)$
Unsorted Linked List	$\theta(n)$	$\theta(n)$	$\theta(n)$
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Binary Search Tree	$\theta(h), \theta(n)$	$\theta(h), \theta(n)$	$\theta(h), \theta(n)$
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Bucket Array			

Data Structures for *Map ADT*

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Unsorted Array	$\theta(n)$	$\theta(n)$	$\theta(n)$
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Binary Search Tree	$\theta(h), \theta(n)$	$\theta(h), \theta(n)$	$\theta(h), \theta(n)$
AVL Tree	$\theta(\log(n))$	$\theta(\log(n))$	$\theta(\log(n))$
Bucket Array	$\theta(1)$	$\theta(1)$	$\theta(1)$

Data Structures for *Map ADT*

	Find	Insert	Delete	Space
Unsorted Array	$\theta(n)$	$\theta(n)$	$\theta(n)$	
Unsorted Linked List	$\theta(n)$	$\theta(n)$	$\theta(n)$	
Sorted Array	$\theta(\log(n))$	$\theta(n)$	$\theta(n)$	
Sorted Linked List	$\theta(n)$	$\theta(n)$	$\theta(n)$	
Binary Search Tree	$\theta(h), \theta(n)$	$\theta(h), \theta(n)$	$\theta(h), \theta(n)$	
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Bucket Array	$\theta(1)$	$\theta(1)$	$\theta(1)$	Could be large relative to n

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Binary Search Tree	$\theta(h), \theta(n)$	$\theta(h), \theta(n)$	$\theta(h), \theta(n)$	$\theta(\mathrm{n})$
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Bucket Array	$\theta(1)$	$\theta(1)$	$\theta(1)$	Could be large relative to n
Hash Table				

Data Structures for *Map ADT*

	Find	Insert	Delete	Space
Unsorted Array	$\theta(n)$	$\theta(n)$	$\theta(n)$	$\theta(n)$
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Sorted Array	$\theta(\log(n))$	$\theta(n)$	$\theta(n)$	$\theta(n)$
Sorted Linked List	$\theta(n)$	$\theta(n)$	$\theta(n)$	$\theta(n)$
Binary Search Tree	$\theta(h), \theta(n)$	$\theta(h), \theta(n)$	$\theta(h), \theta(n)$	$\theta(\mathrm{n})$
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Bucket Array	$\theta(1)$	$\theta(1)$	$\theta(1)$	Could be large relative to n
Hash Table				$\theta(\mathrm{n})$

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	Find	Insert	Delete	Space
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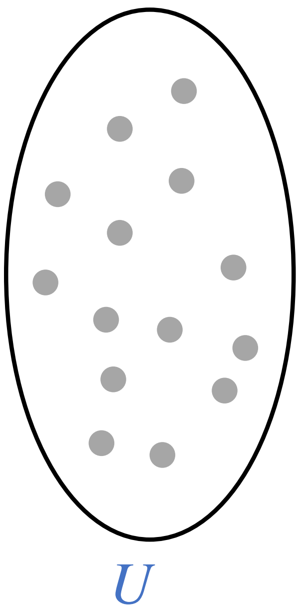
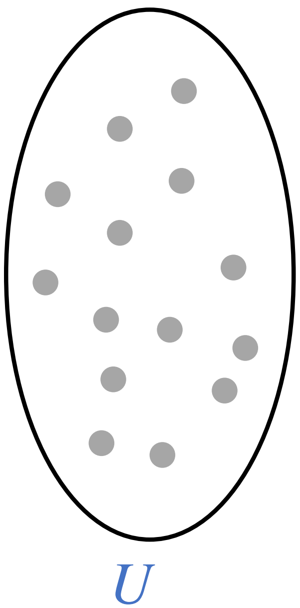
Bucket Array	$\theta(1)$	$\theta(1)$	$\theta(1)$	Could be large relative to n
Hash Table	$\theta(1)$	$\theta(1)$	$\theta(1)$	$\theta(\mathrm{n})$

Hash Tables

Hash Tables

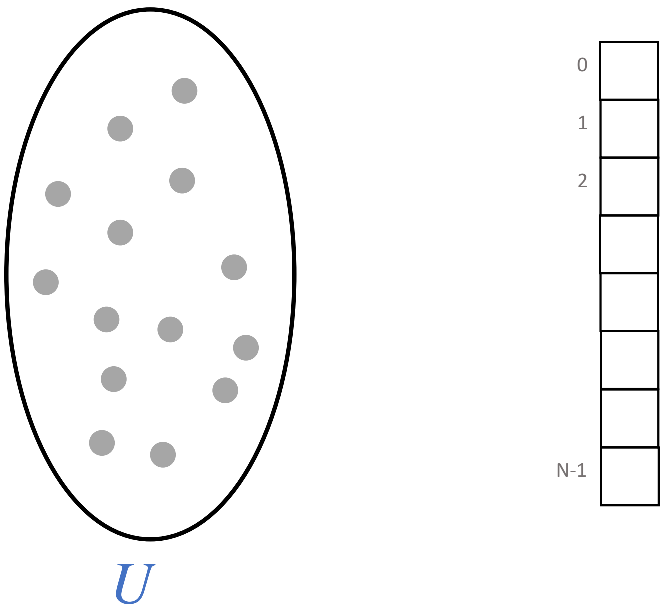


Hash Tables



- U
  Universe from which the keys will be taken

Hash Tables



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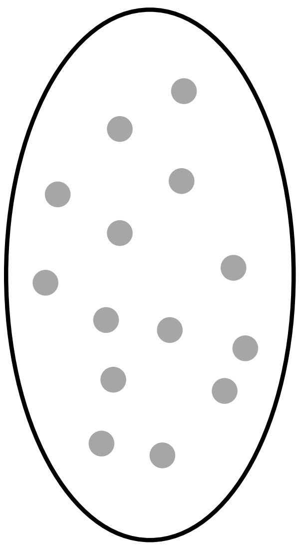

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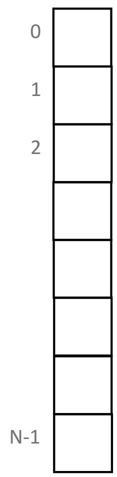
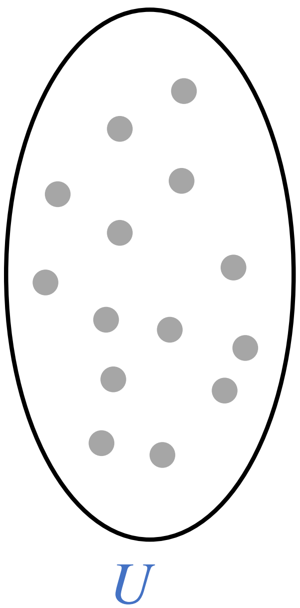
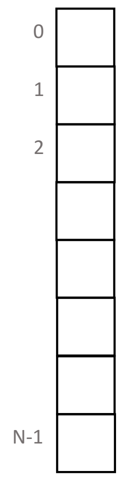


- U
  Universe from which the keys will be taken

- T = [None]*N

Hash Tables

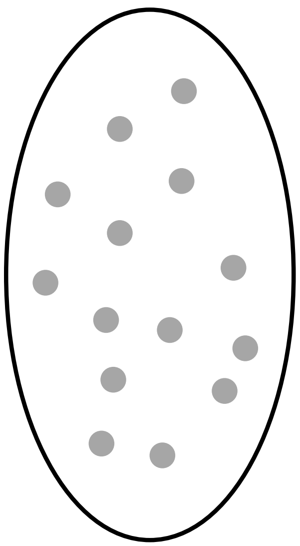




- U
  Universe from which the keys will be taken

- $T = [None]*N$
  Hash table of N slots, where the entries will be stored

Hash Tables



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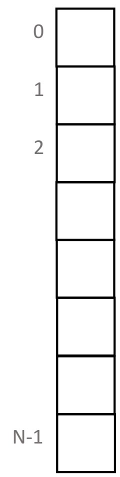

$T\ [0\ldots(N\text{-}1)]$

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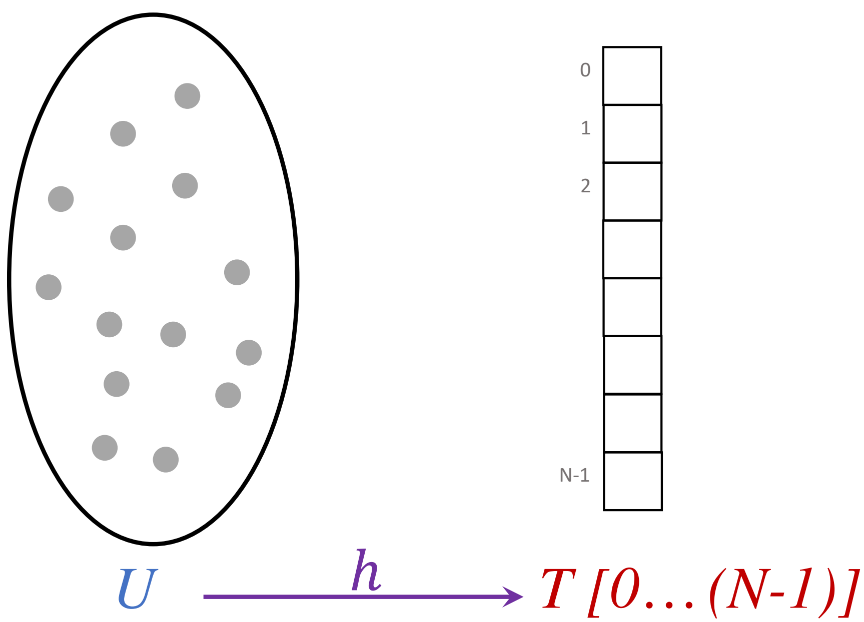

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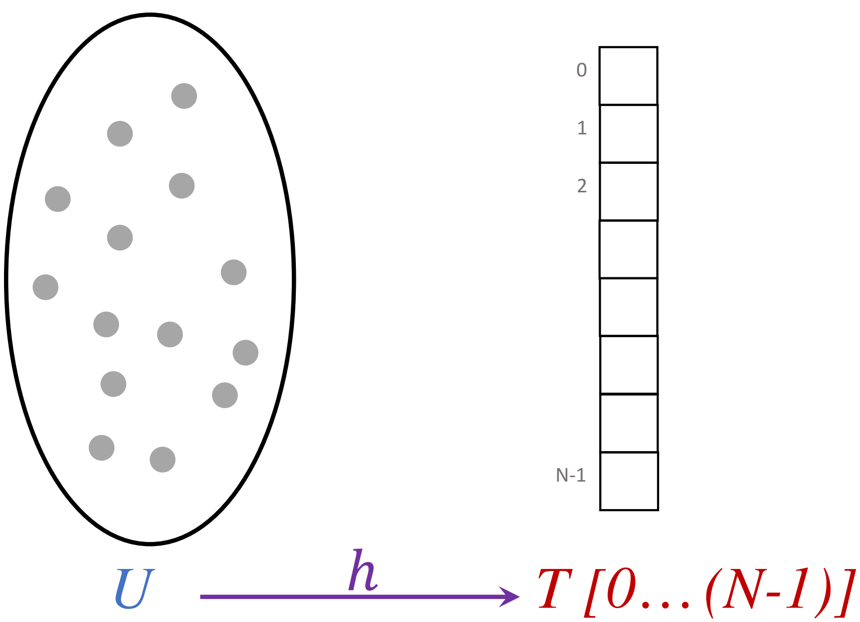

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- $h: U \rightarrow \{0, 1, \ \ldots, (N-1)\}$

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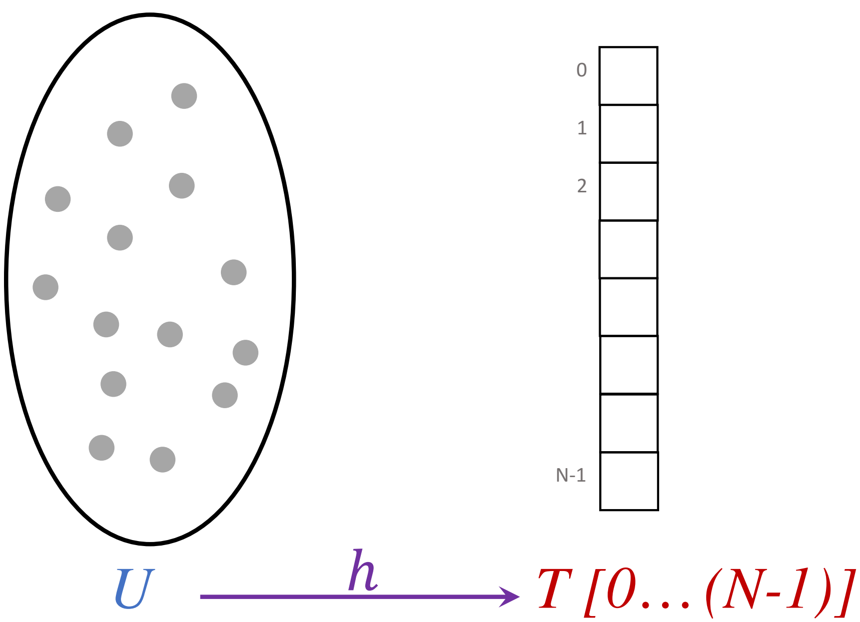

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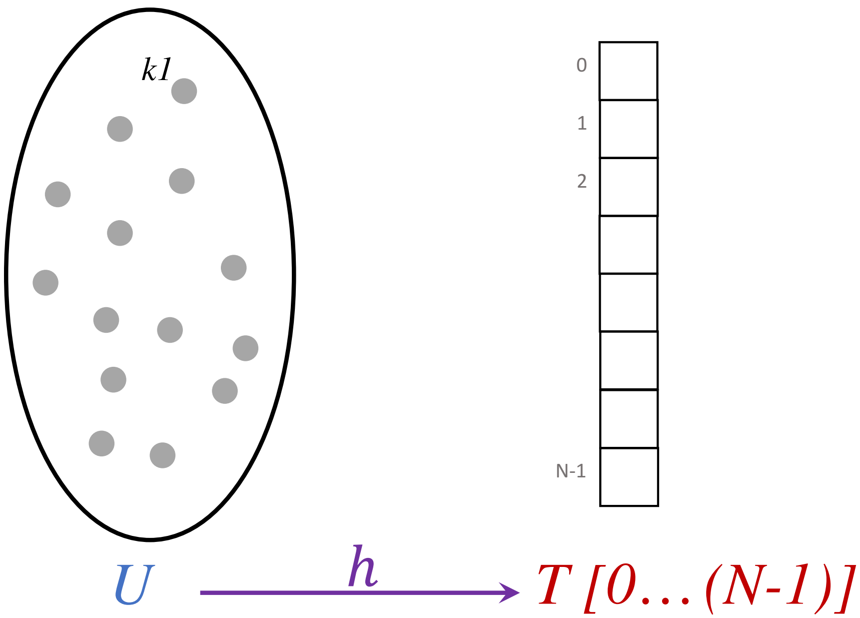
  Hash function that maps keys form the universe to slots in the table

Hash Tables



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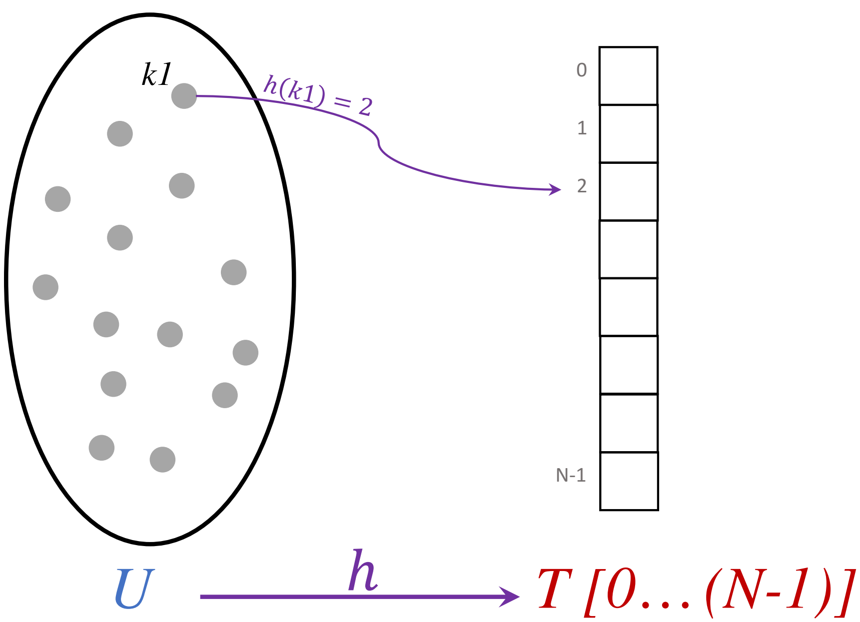

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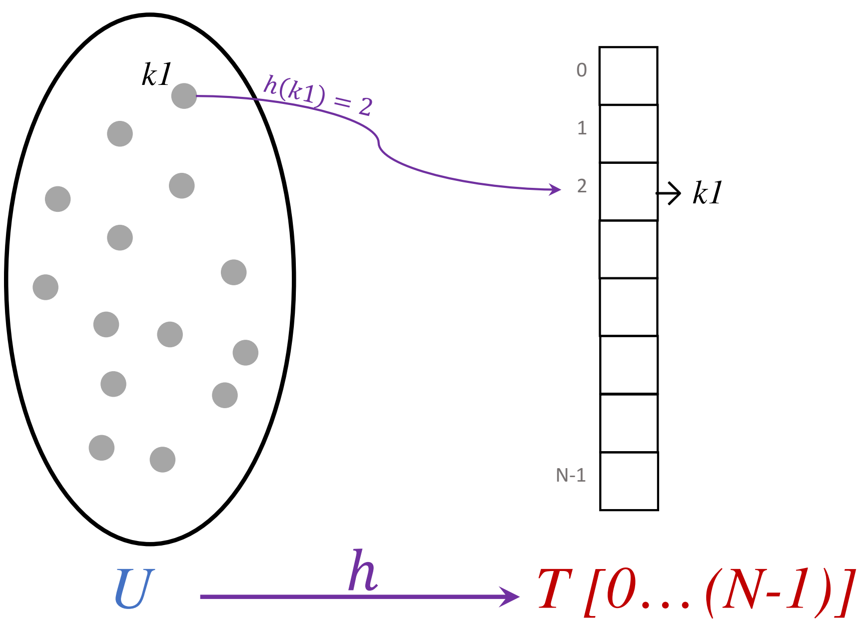

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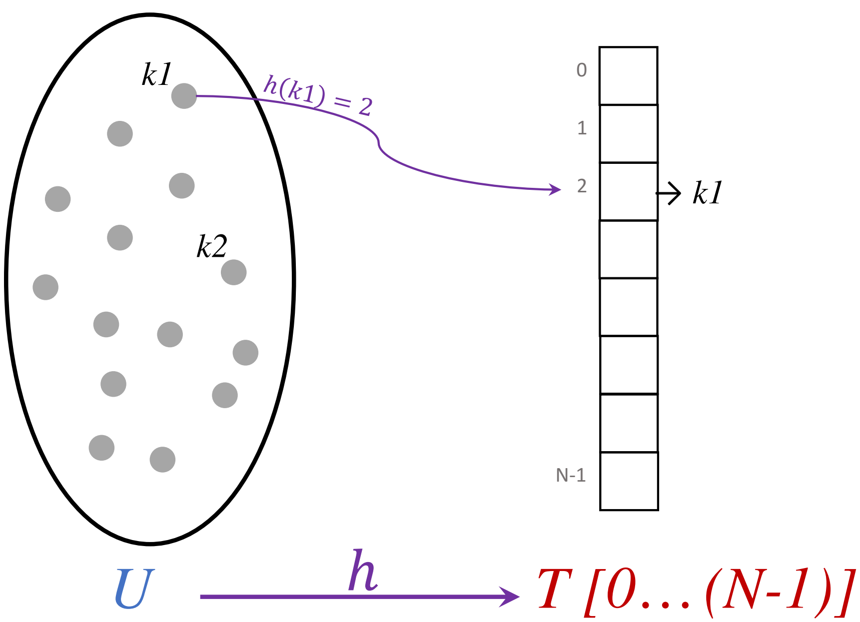

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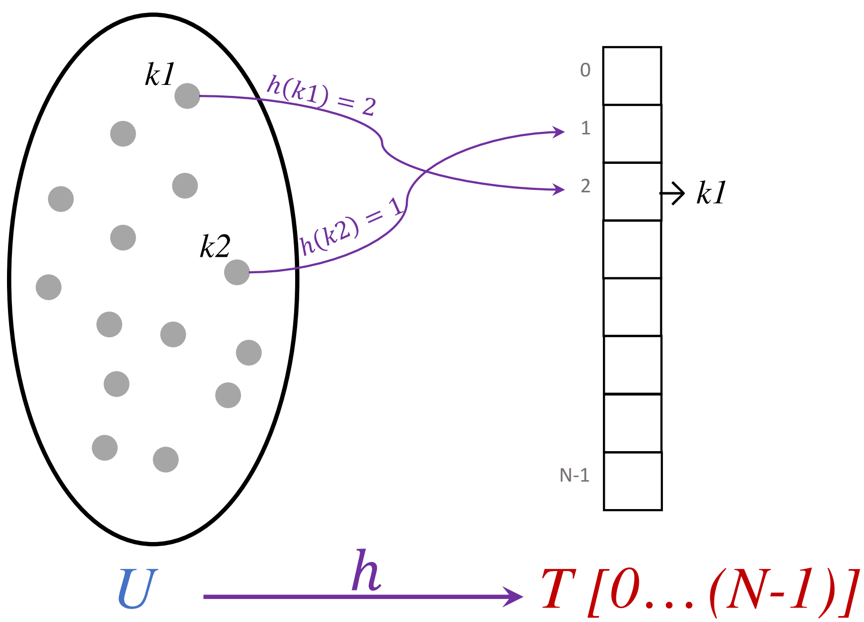

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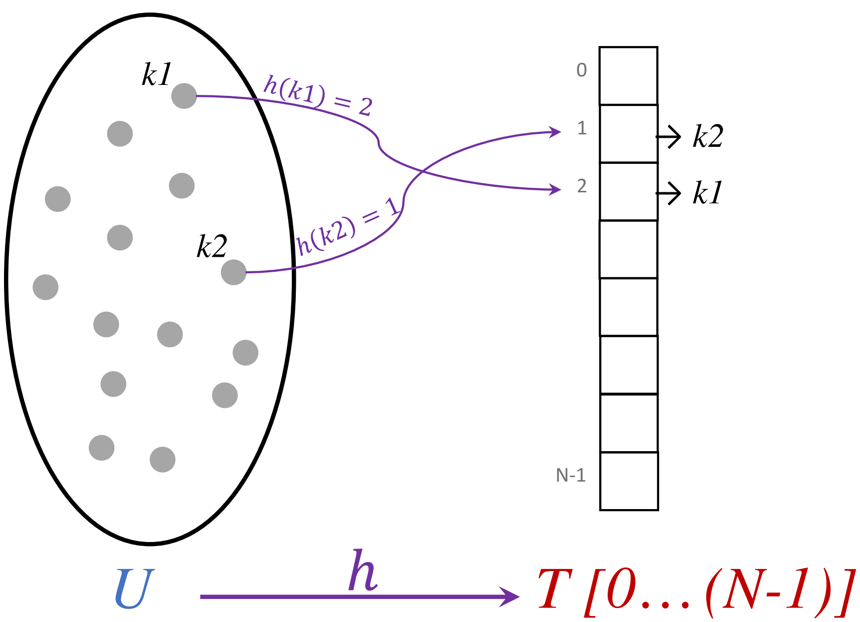

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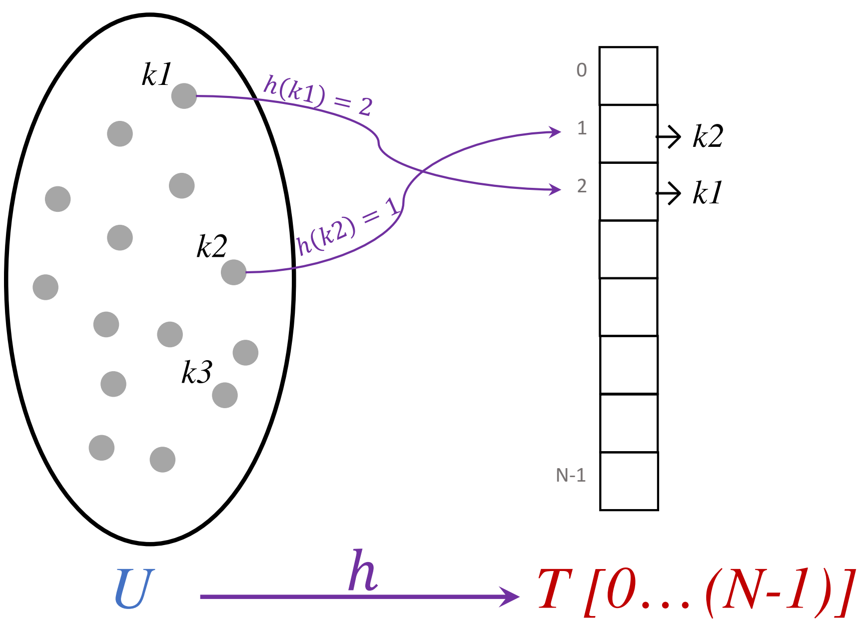

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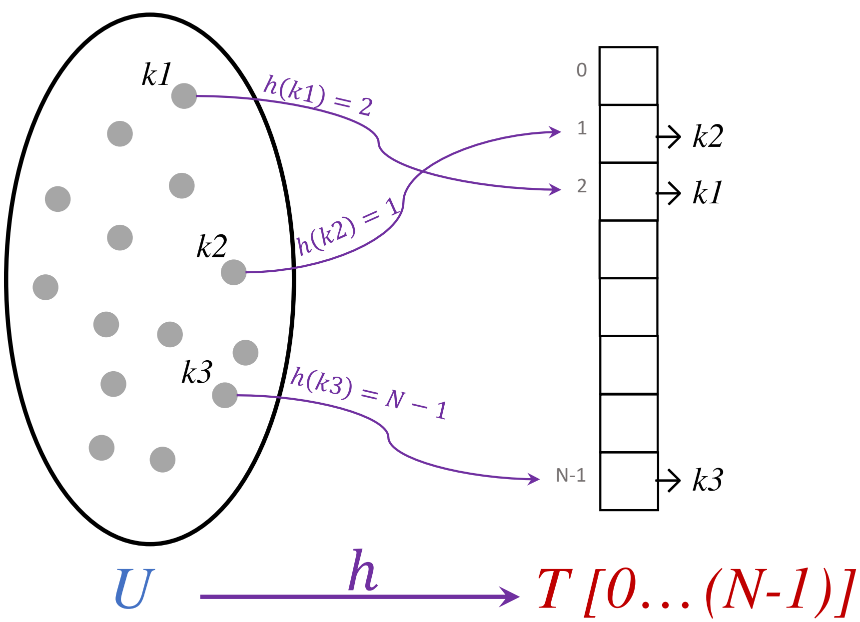

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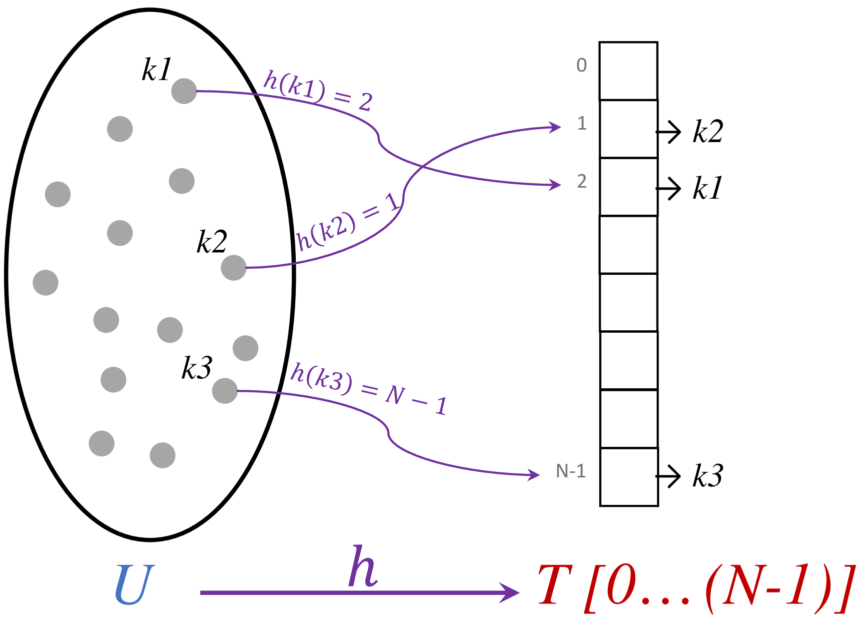

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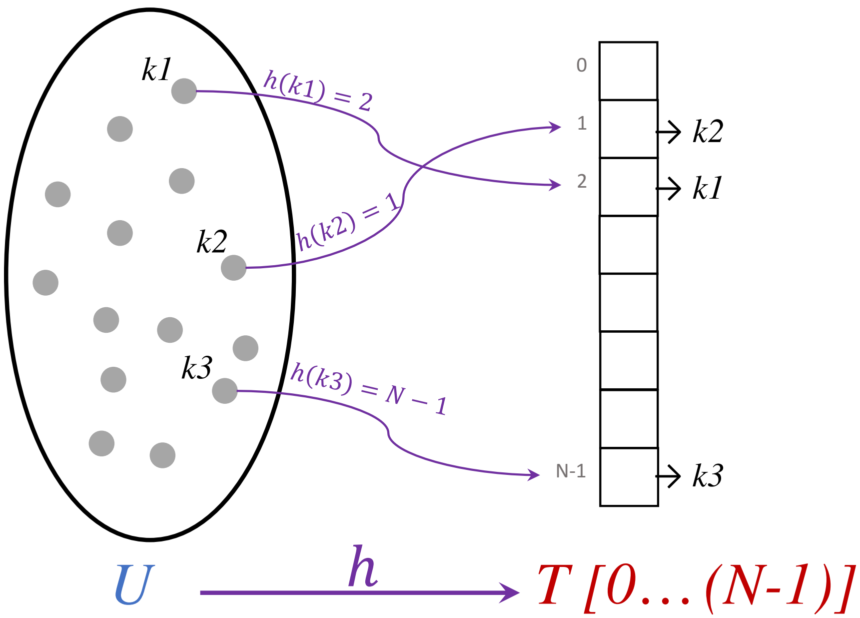
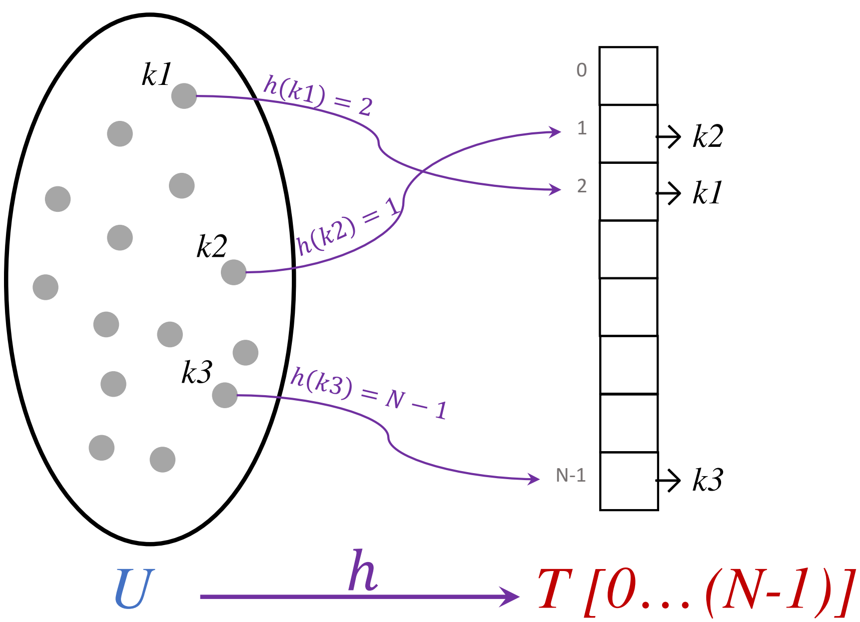


insert(key, value):

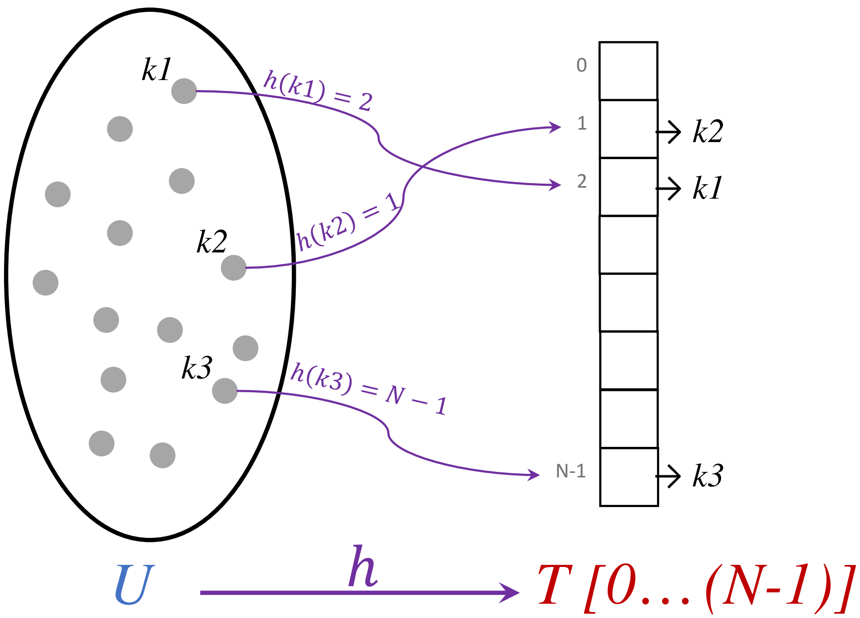
Hash Tables



insert(key, value):

i = h(key)

Hash Tables

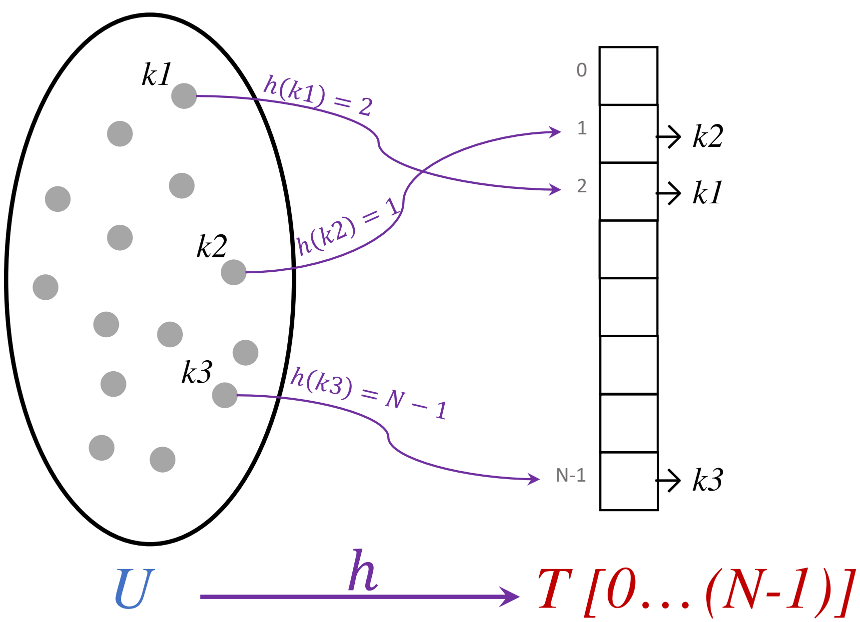


insert(key, value):

i = h(key)

T[i] = value

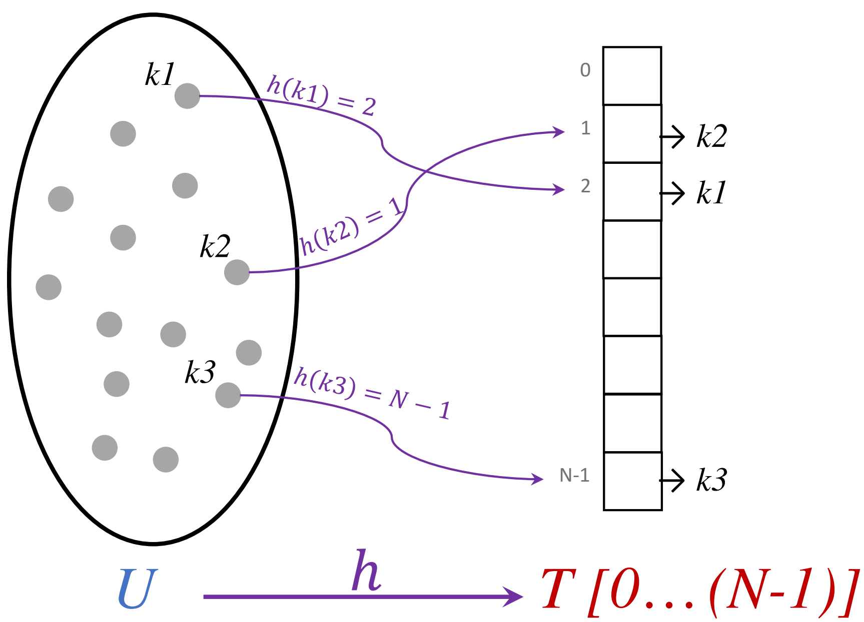
Hash Tables



insert(key, value):
i = h(key)
T[i] = value

find(key):

Hash Tables



insert(key, value):

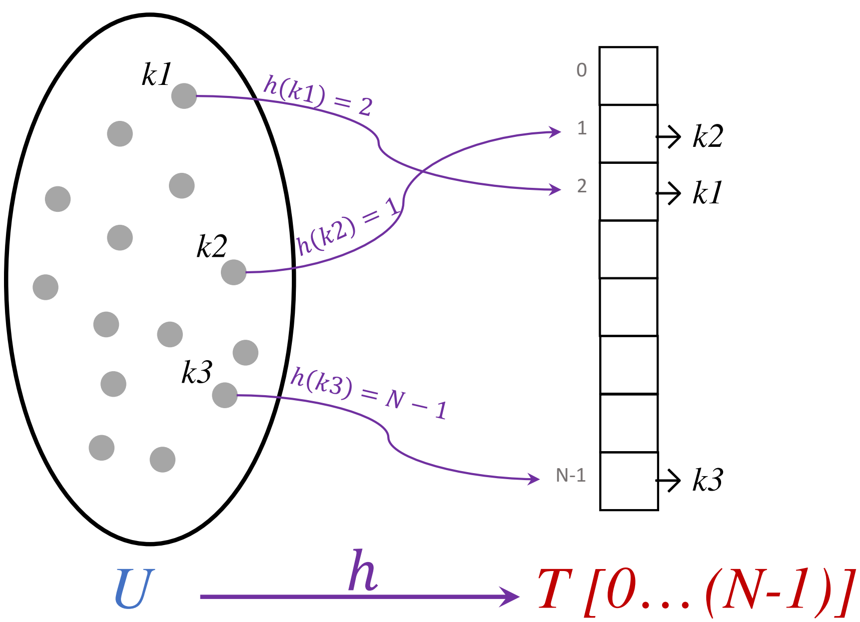
$i = h(key)$

$T[i] = value$

find(key):

$i = h(key)$

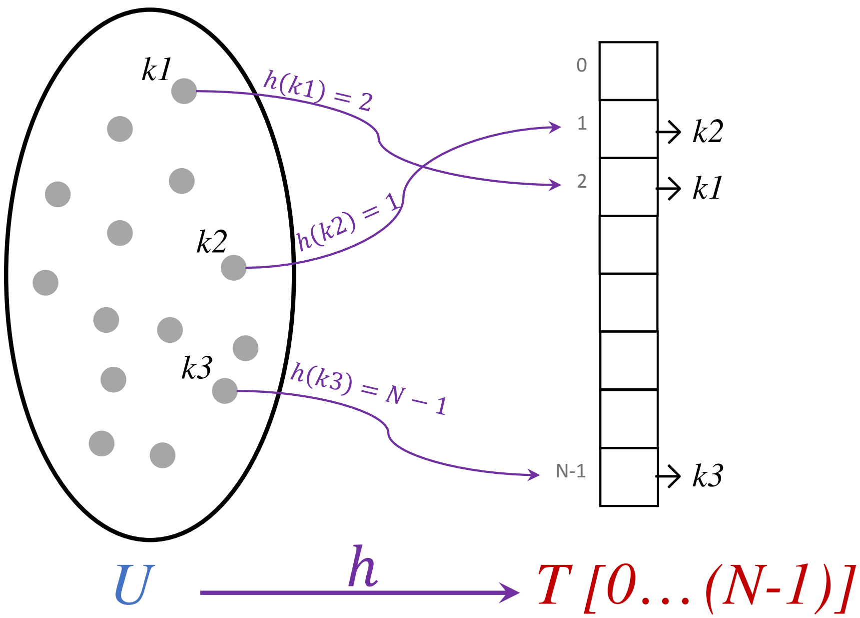
Hash Tables



insert(key, value)
    i = h(key)
    T[i] = value

find(key)
    i = h(key)
    return T[i]

Hash Tables



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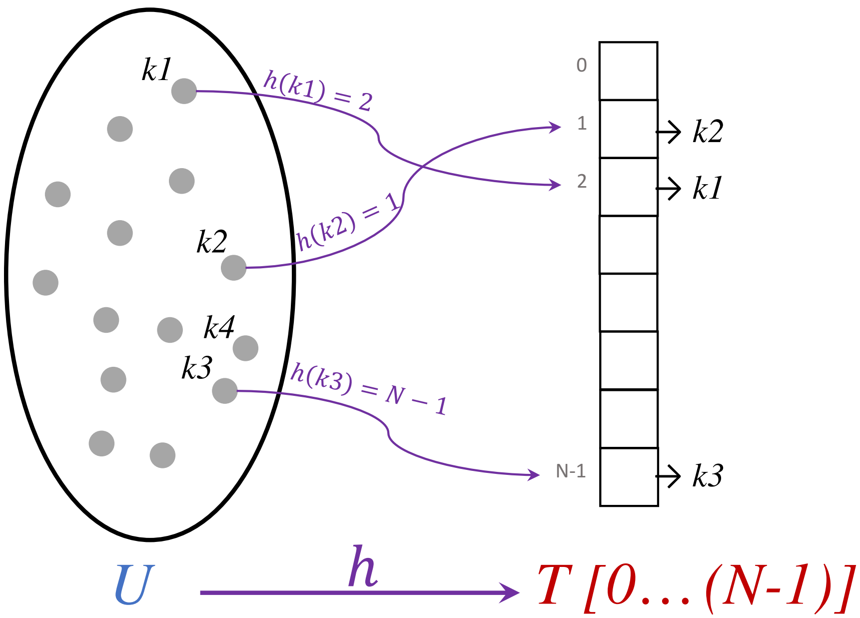

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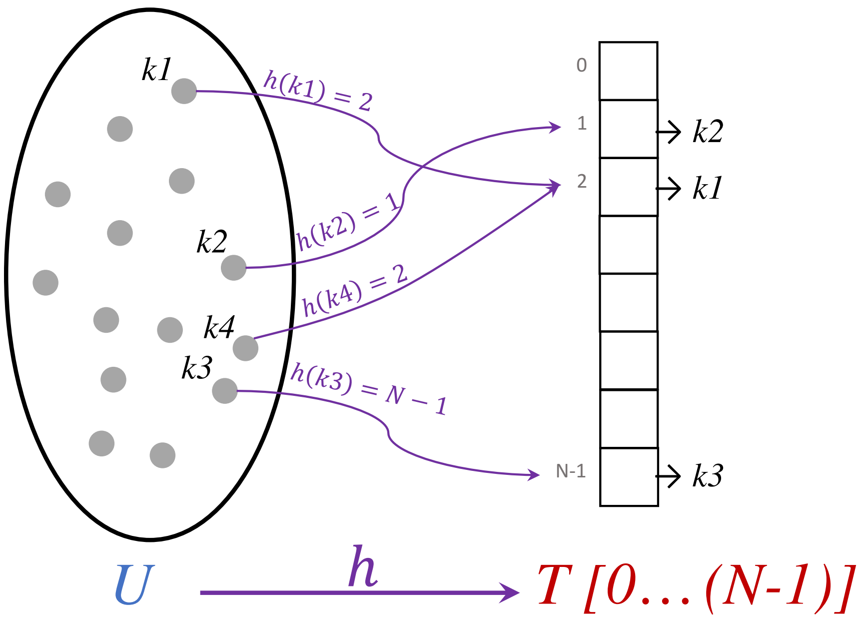

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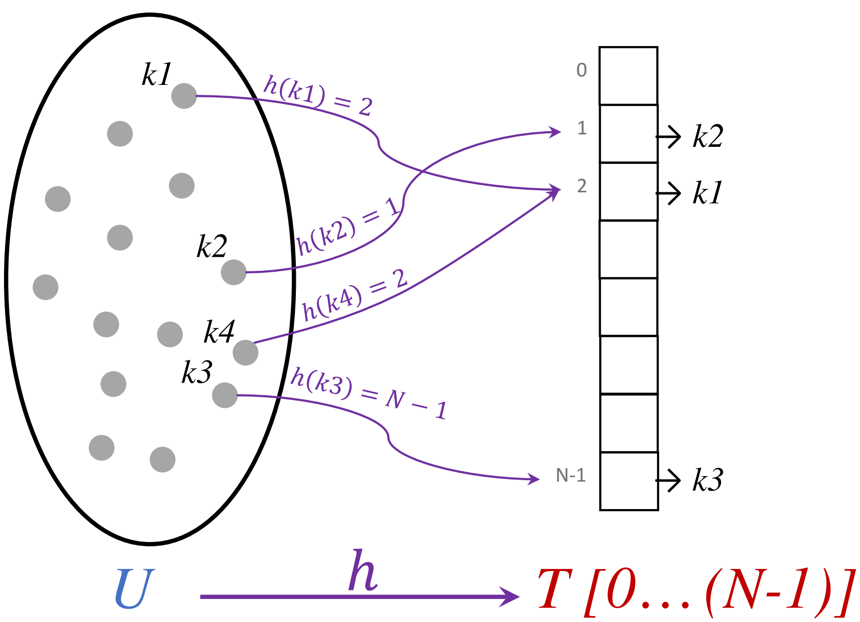
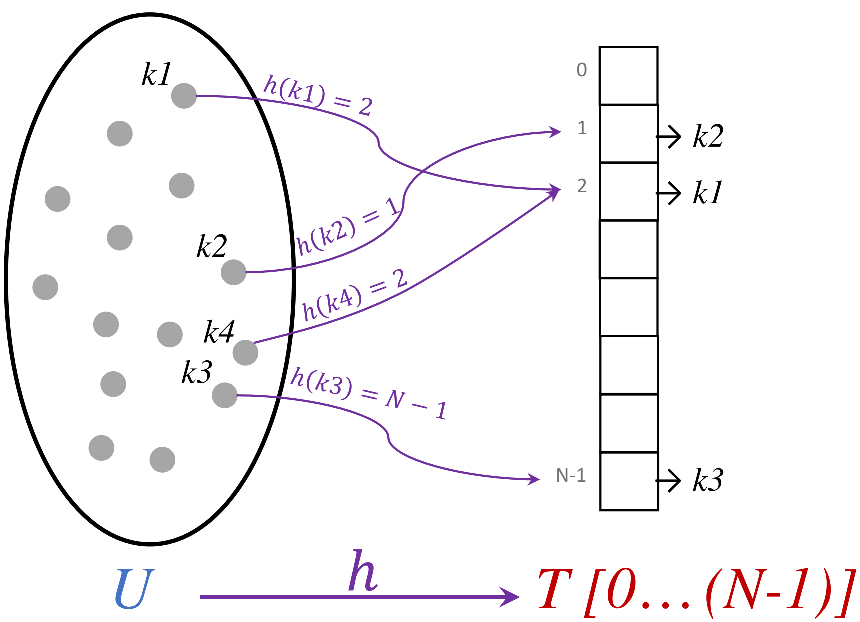


<u>The Collisions Problem</u>:
multiple keys could be mapped to the same slot

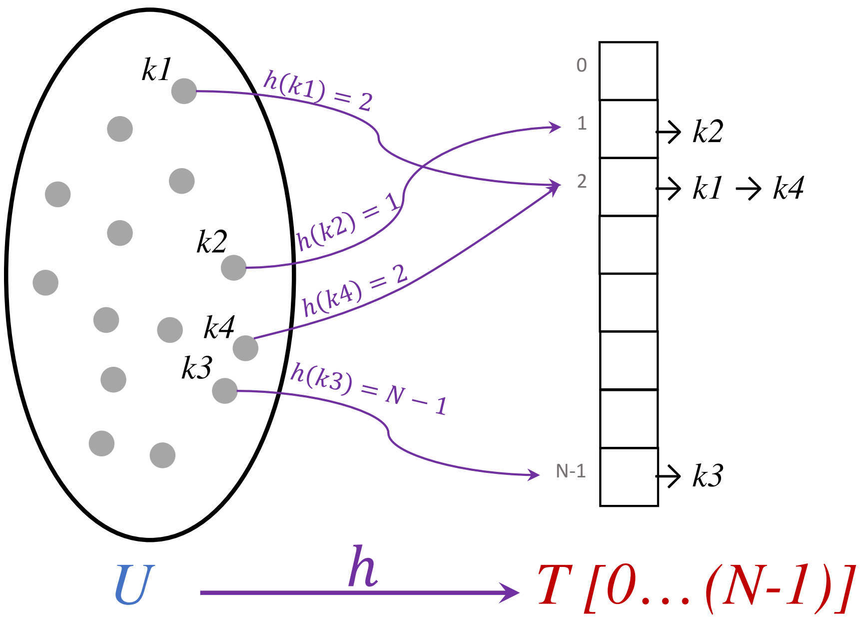
Hash Tables



The Collisions Problem:
multiple keys could be mapped to the same slot

An easy solution:
Chaining – store all entries that are mapped to the same slot, in some secondary collection

Hash Tables



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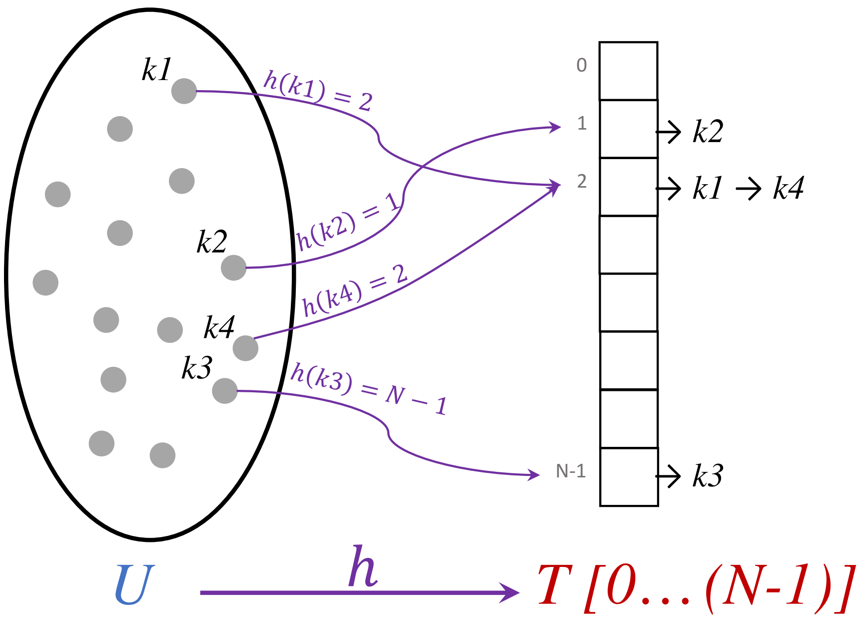

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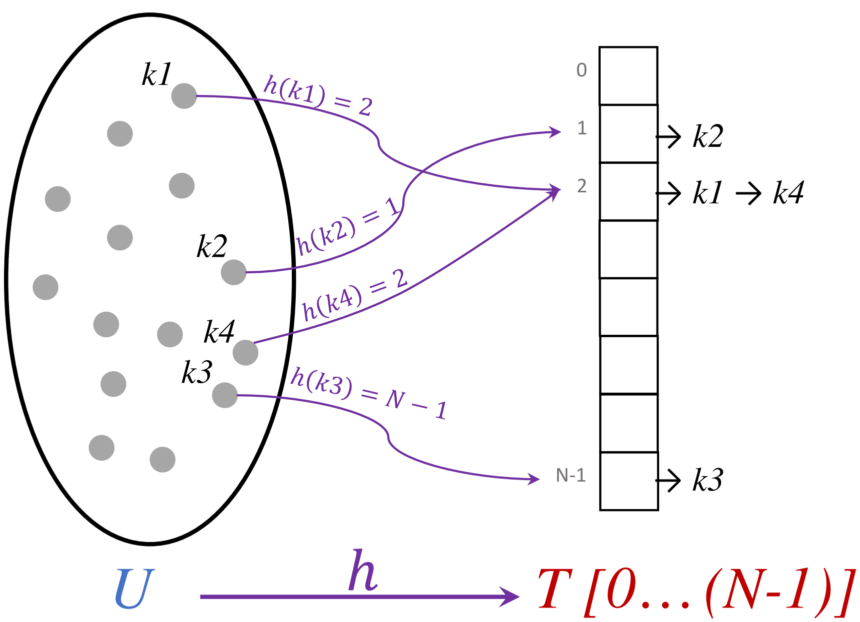
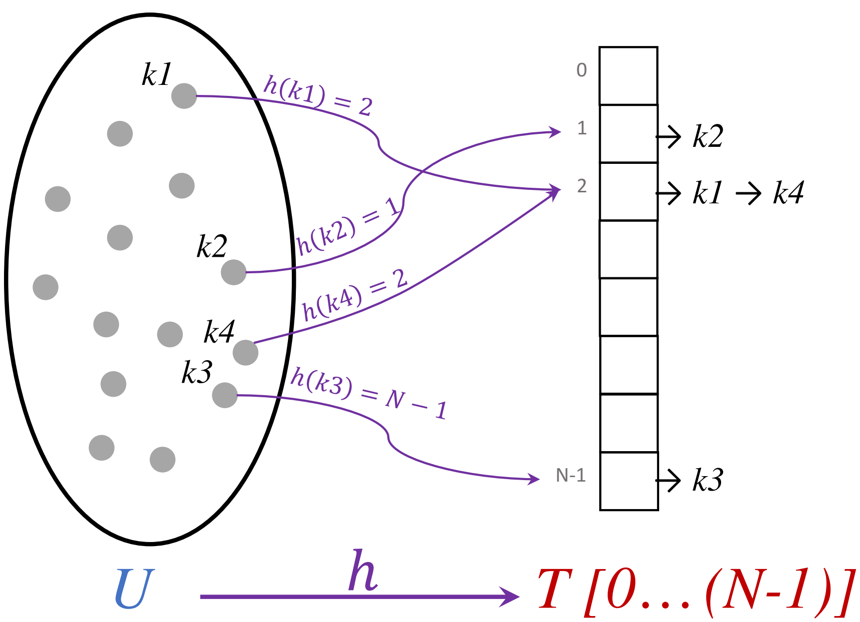


Problem: When using chaining, a lot of keys could end up being stored at the same slot $\Rightarrow$ bad performance

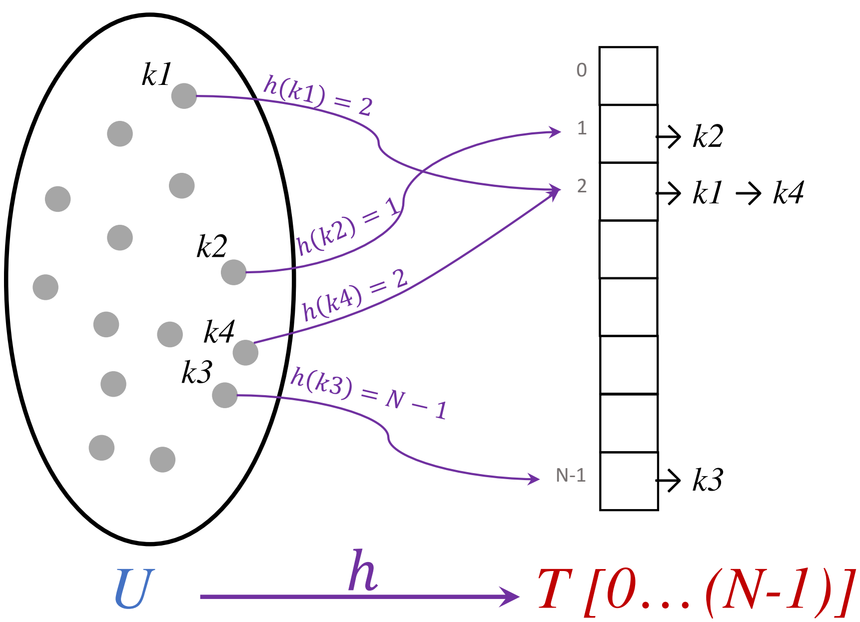
Hash Tables



Problem: When using chaining, a lot of keys could end up being stored at the same slot $\Rightarrow$ bad performance

Solution: Use a "good" hash function (a function that evens out the collisions)

Hash Tables



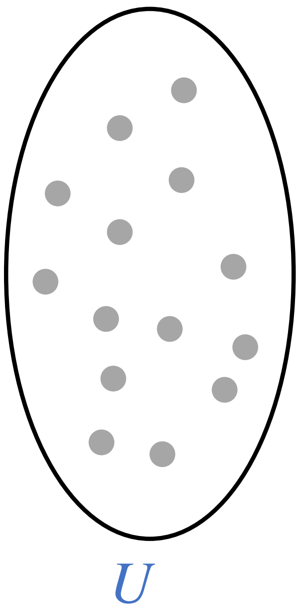
Problem: When using chaining, a lot of keys could end up being stored at the same slot $\Rightarrow$ bad performance

Solution: Use a "good" hash function (a function that evens out the collisions)

Uniform Hashing Function:
A function that when given a randomly chosen key, it will be equally likely mapped to any of the N slots of T, independently of where any other key has hashed to

Hash Functions

Hash Functions



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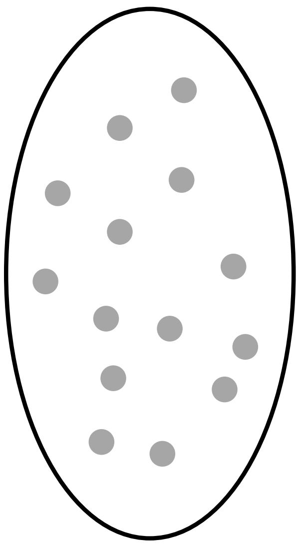

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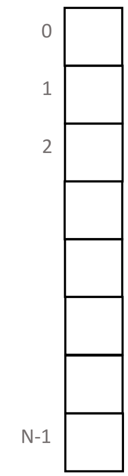
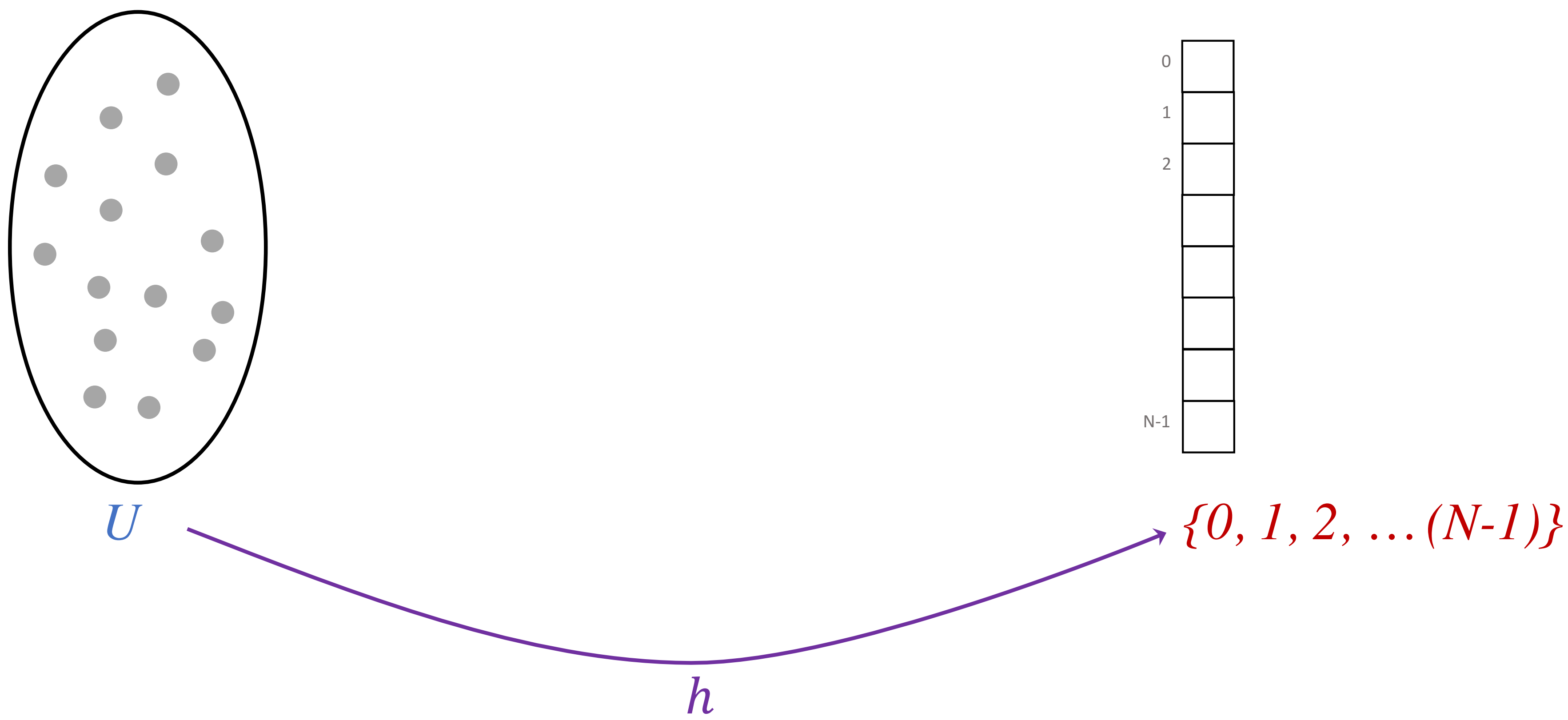


$\{0, 1, 2, \ldots (N\text{-}1)\}$

Hash Functions



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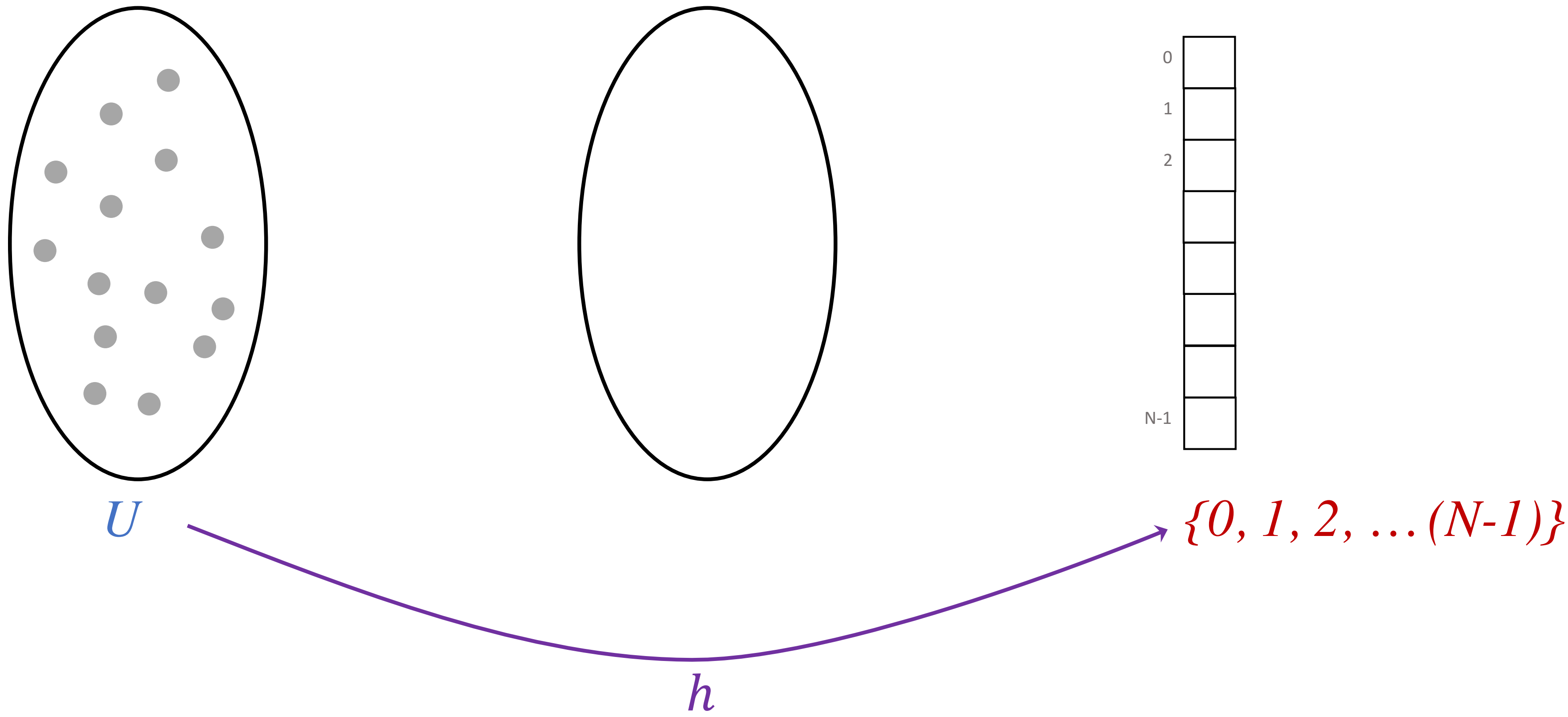

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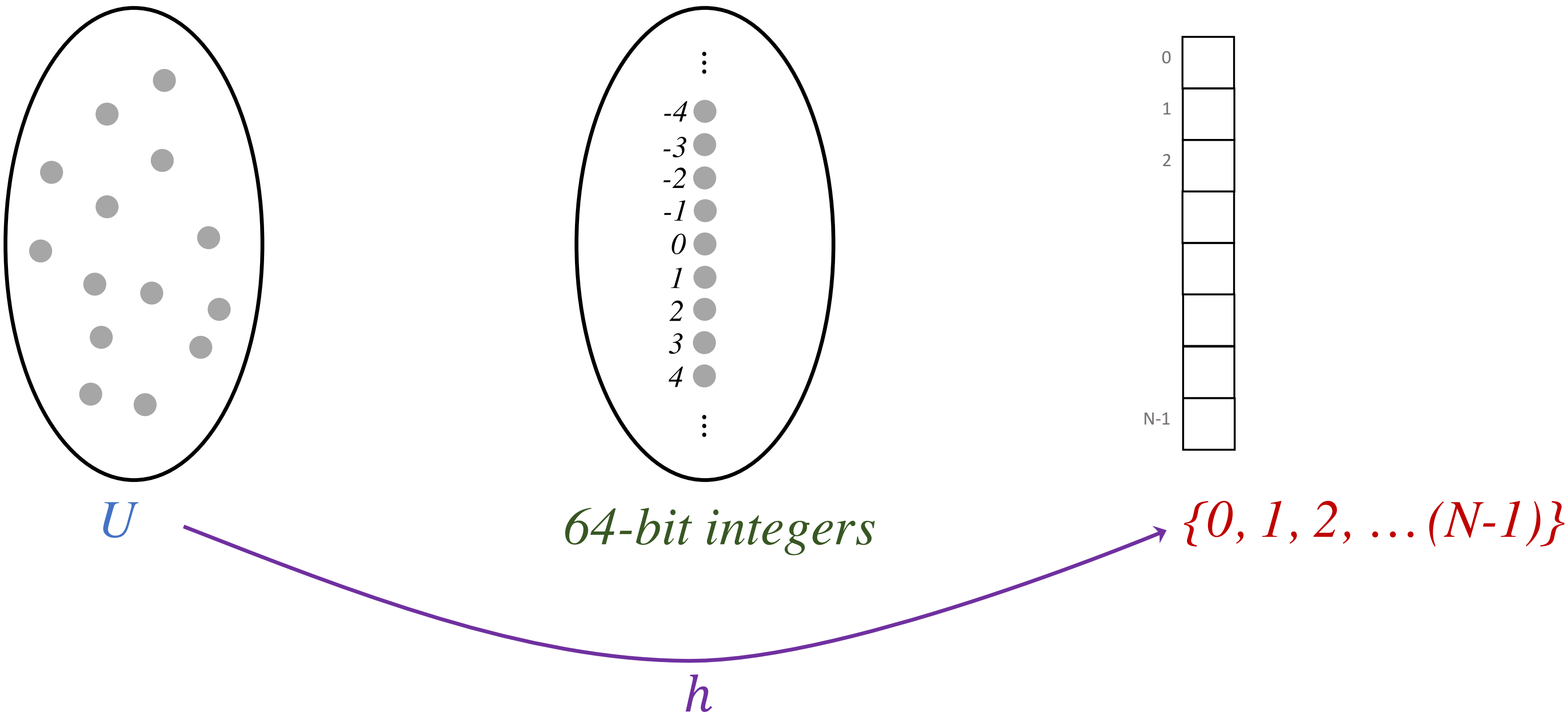

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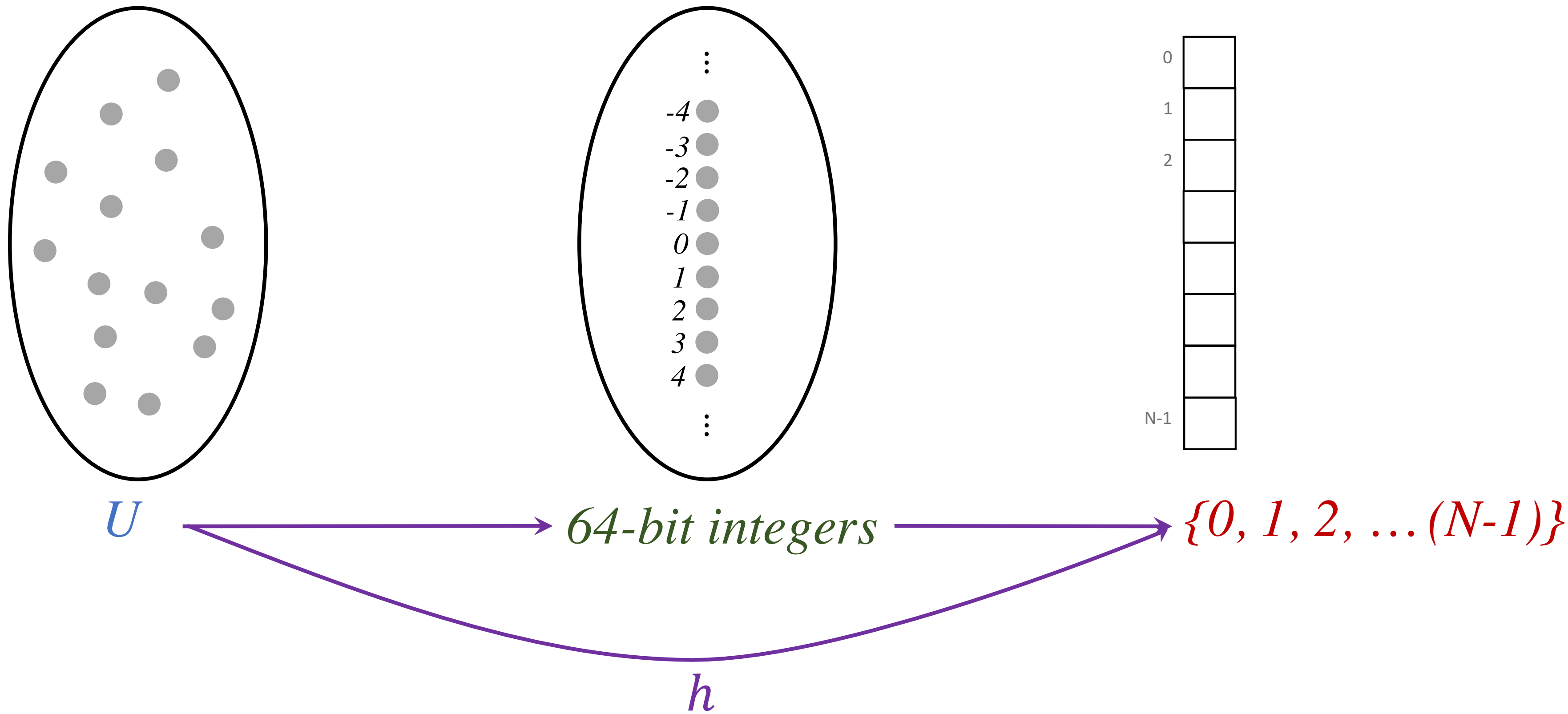

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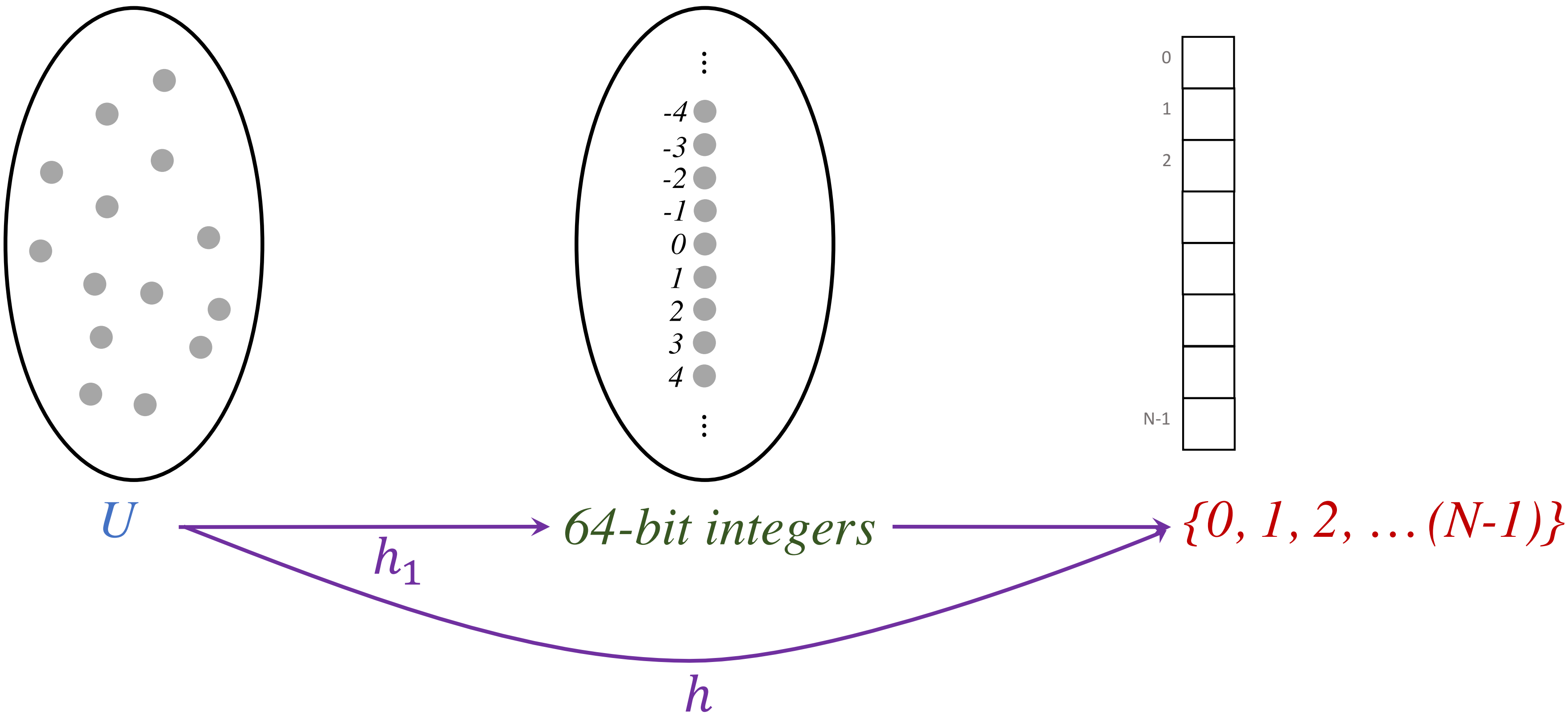

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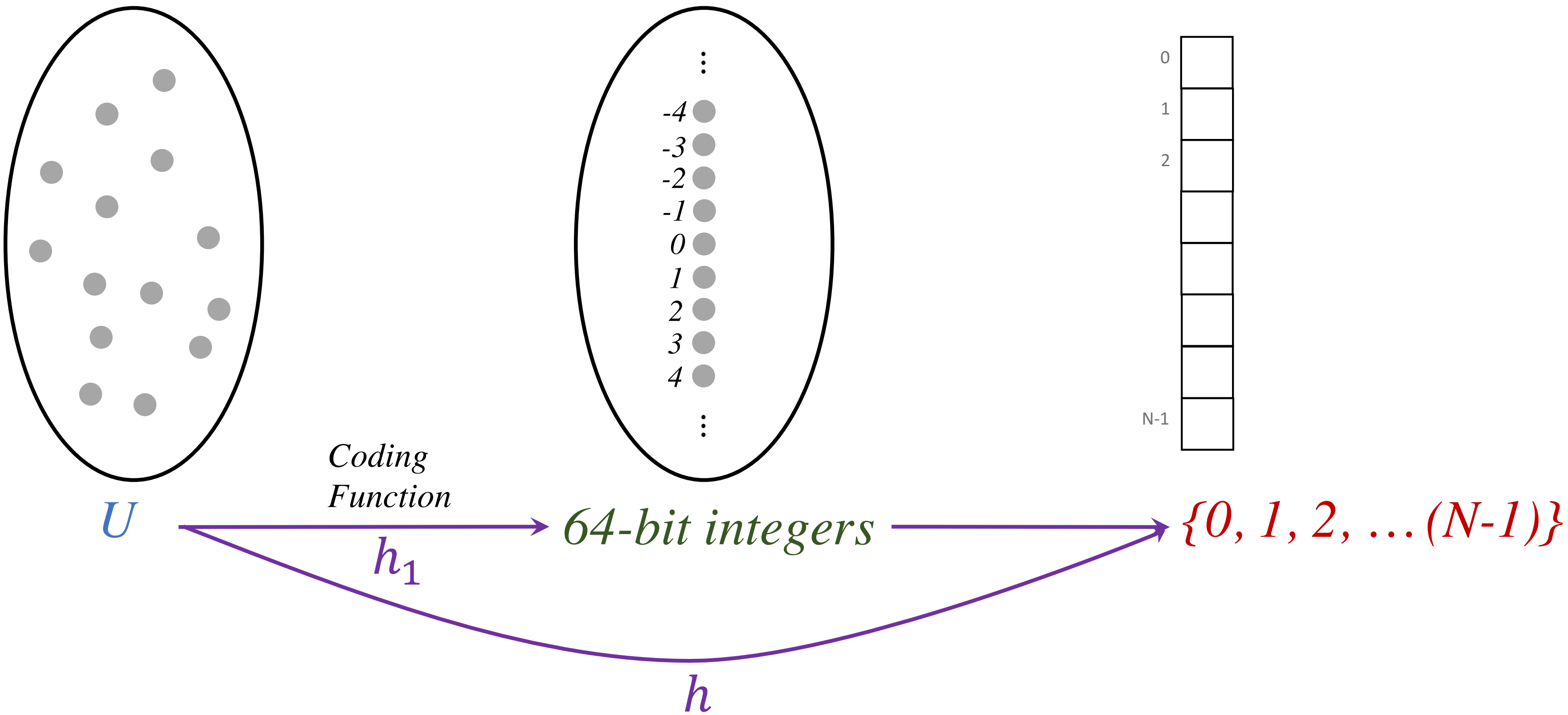

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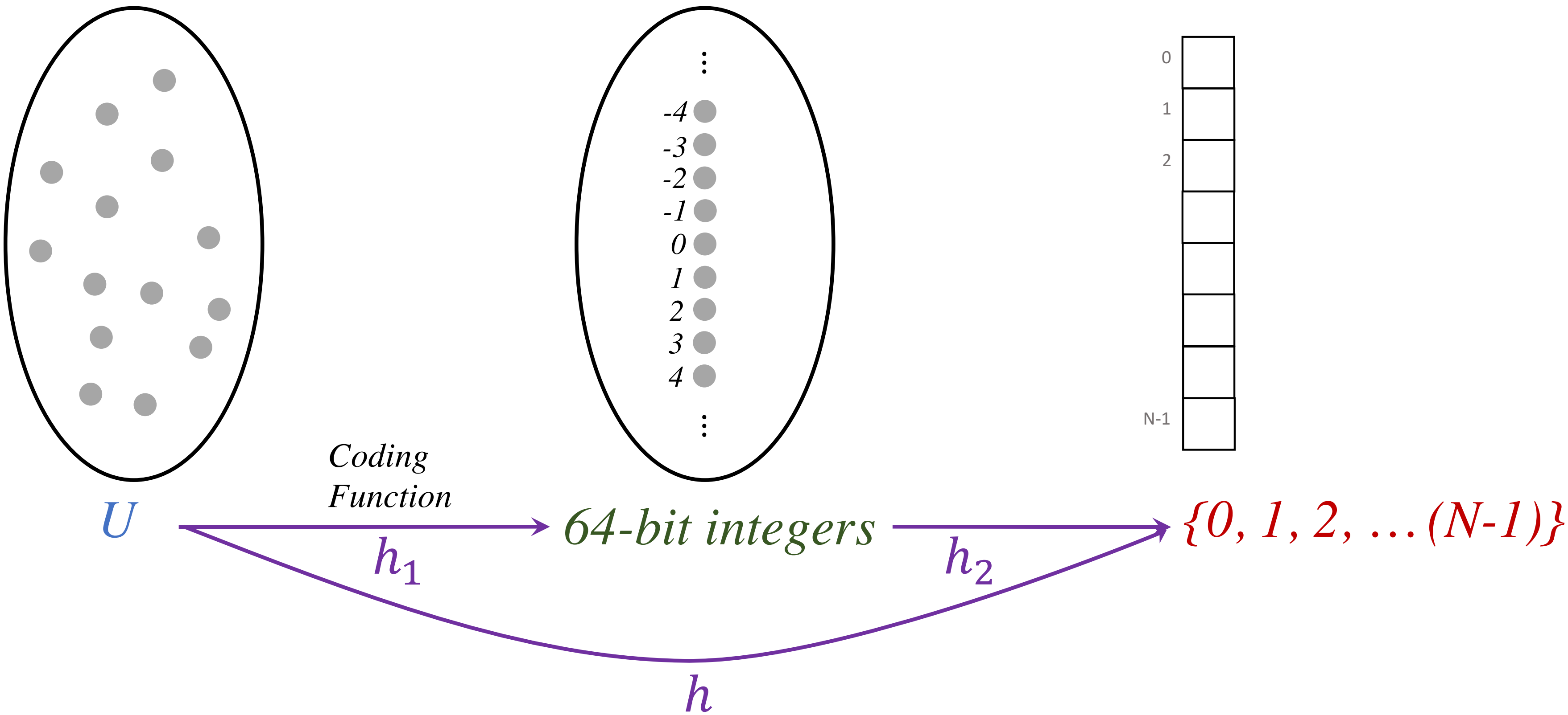

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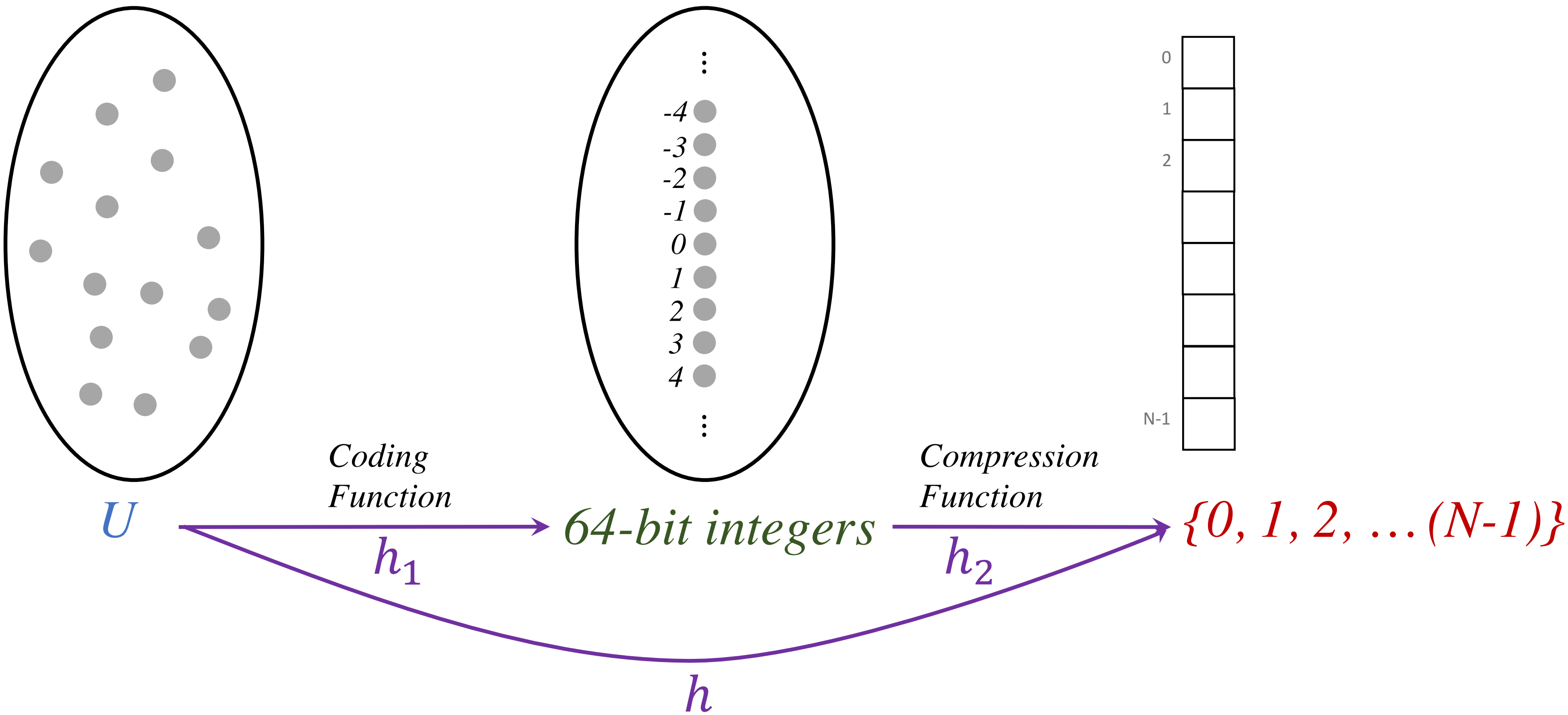

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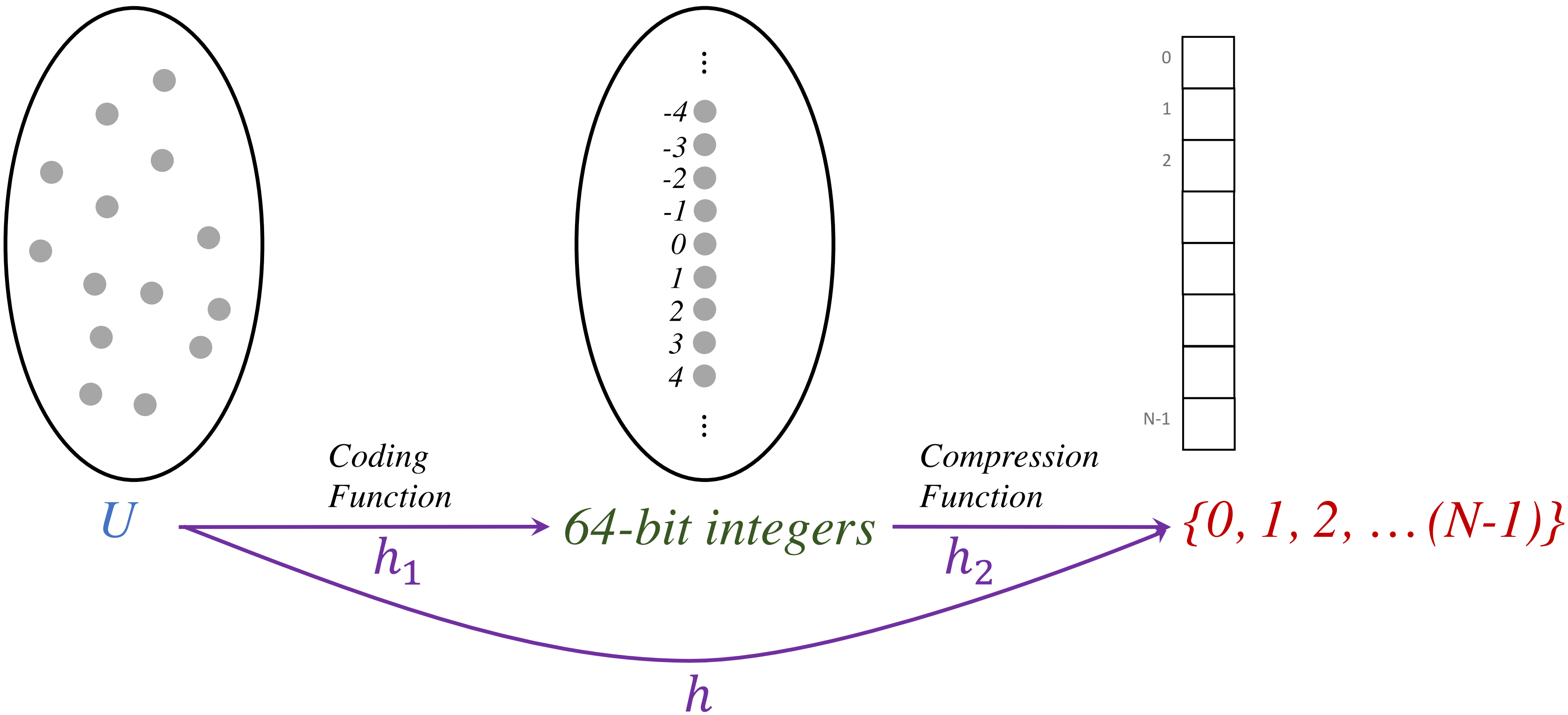
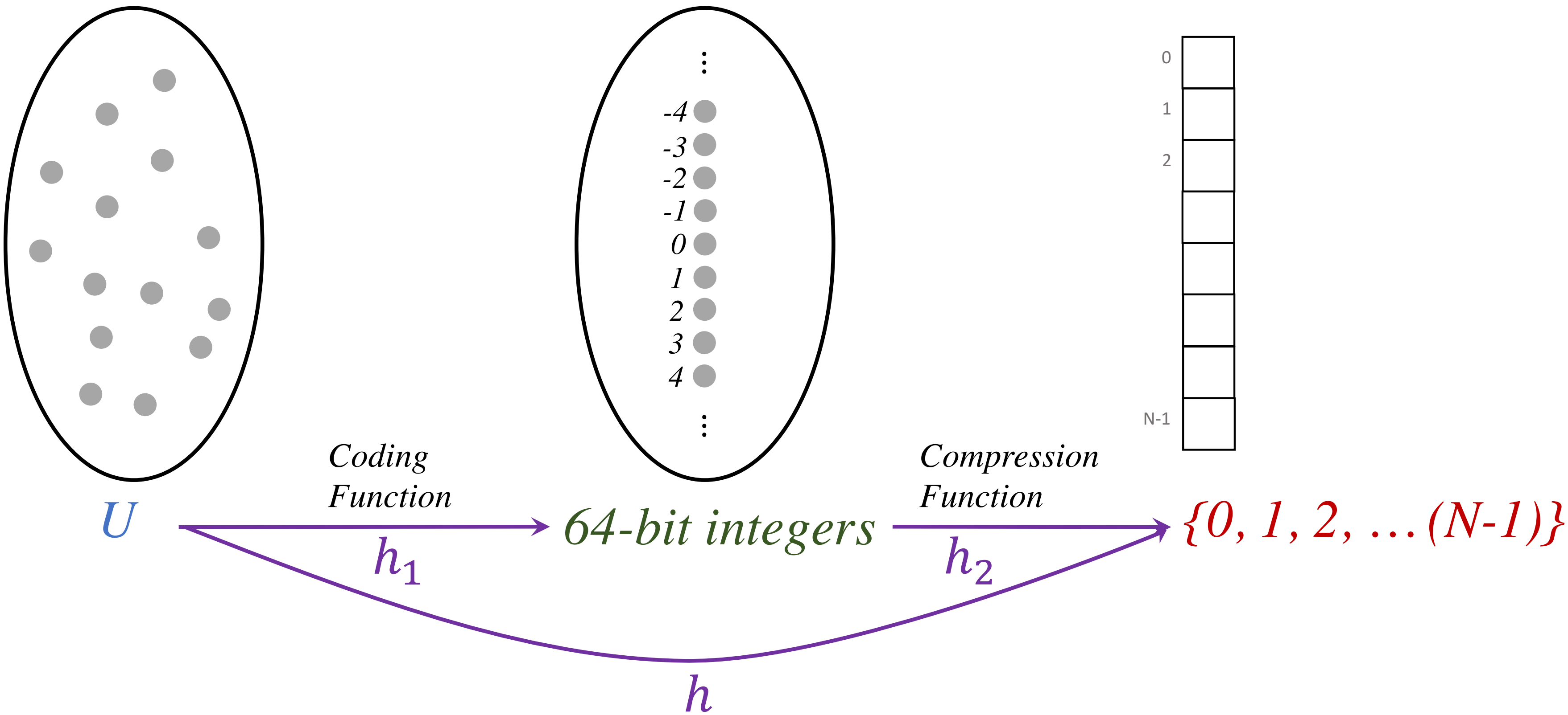


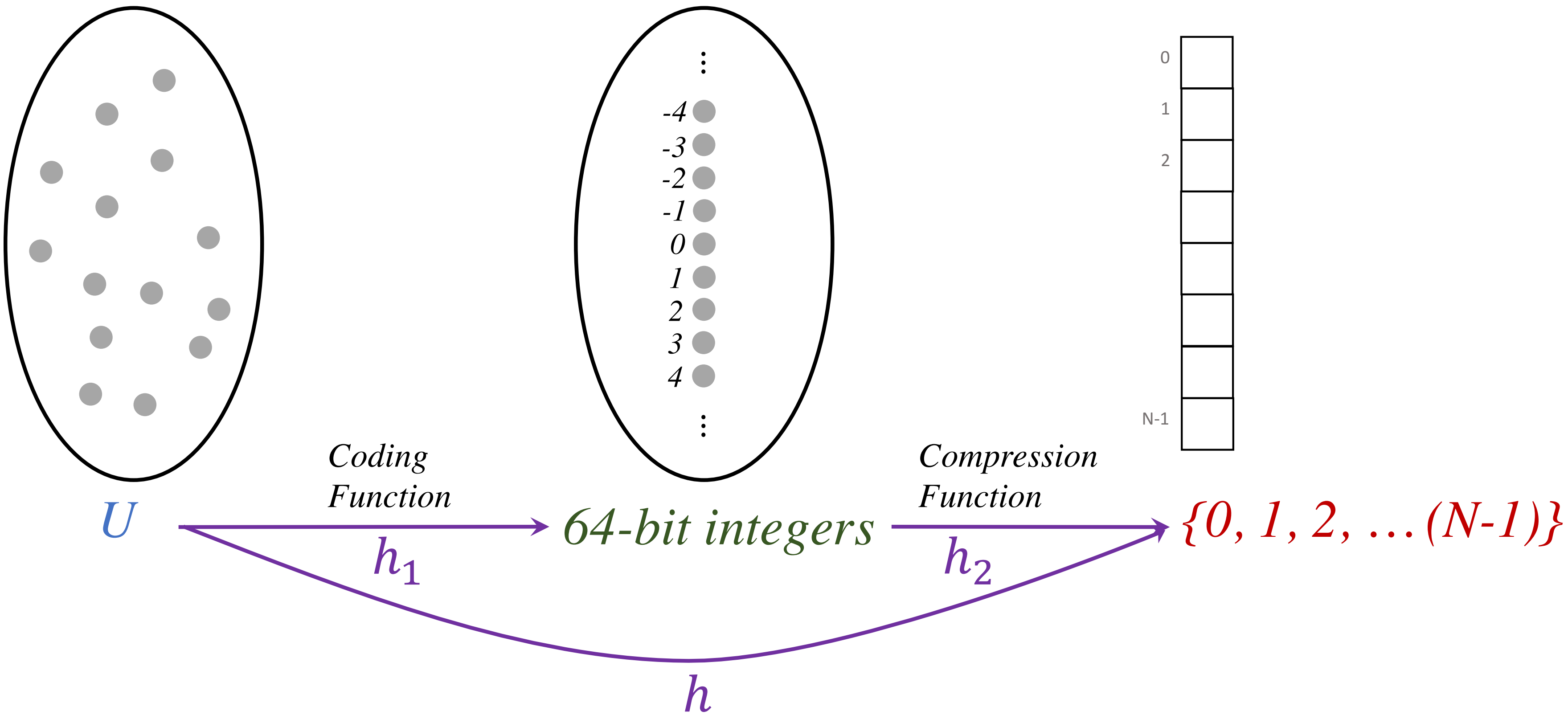
(k)

Hash Functions



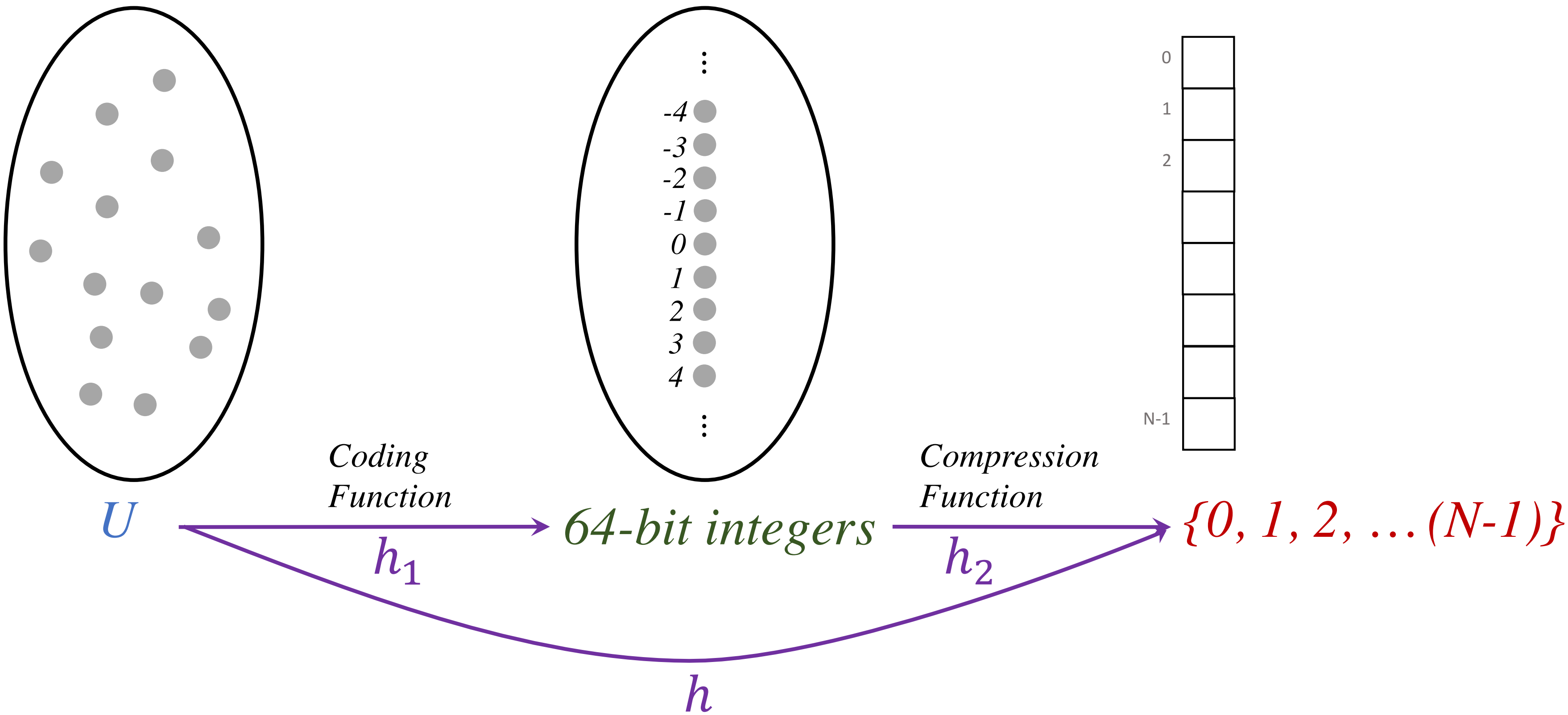
$h_1(k)$

Hash Functions



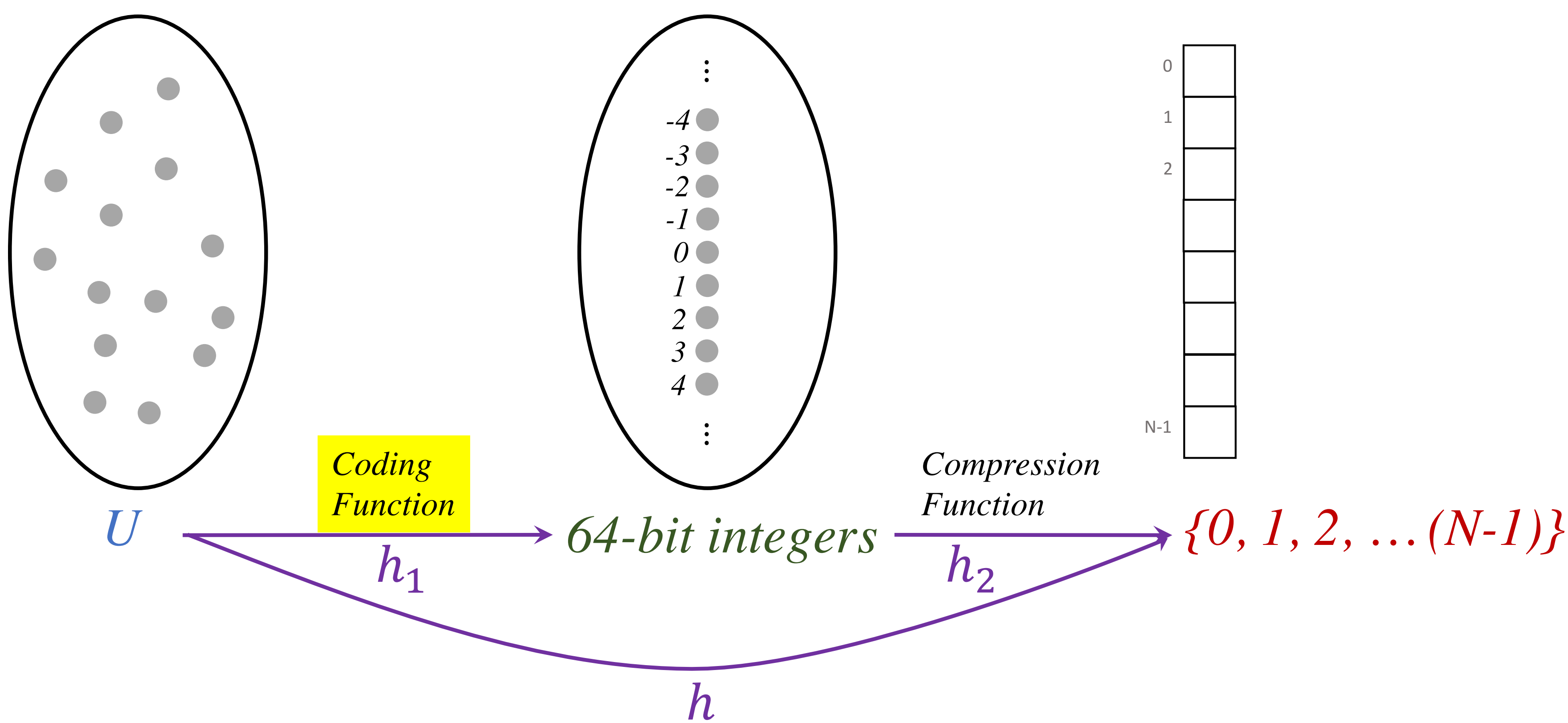
$h_2(h_1(k))$

Hash Functions



$$h(k) = h_2(h_1(k))$$

Hash Functions



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Coding Functions

$h_1 : U \rightarrow (64\text{−}bit\ integers)$

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i. <u>Common Approaches</u>:

Coding Functions

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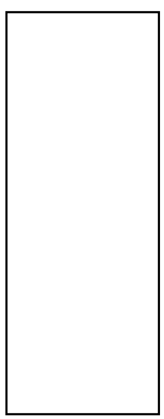

k_1

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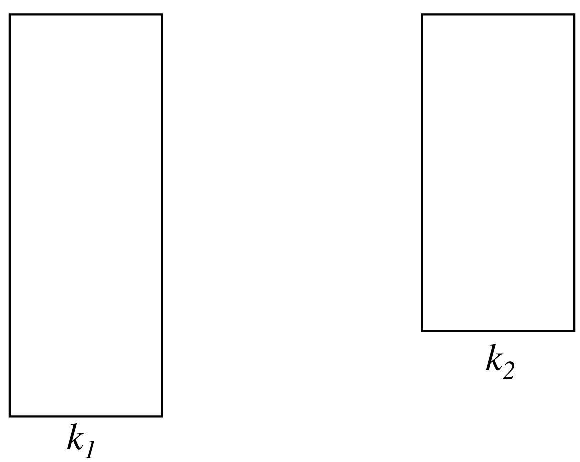

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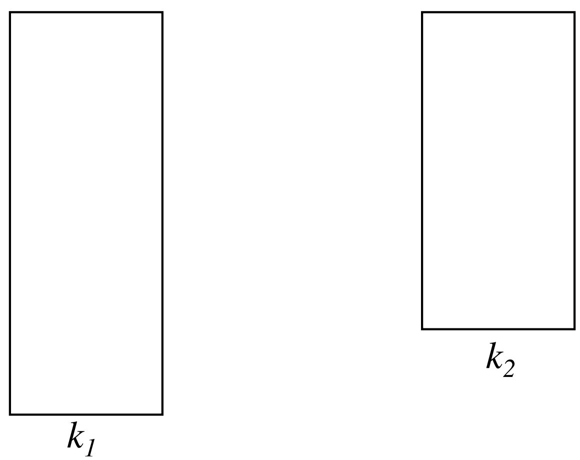

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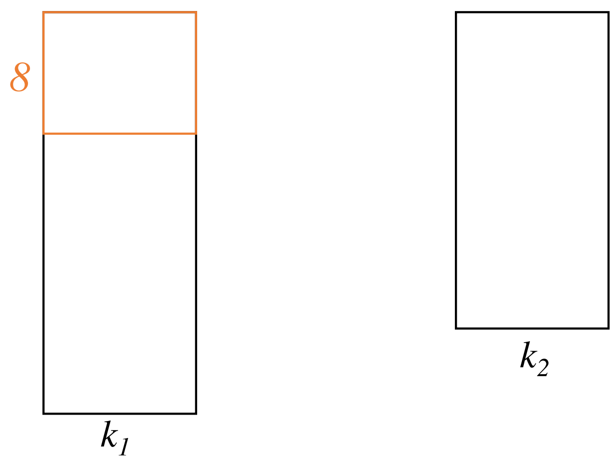

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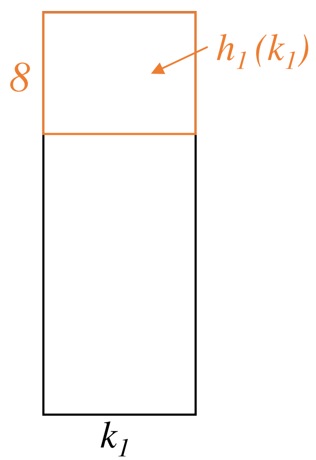

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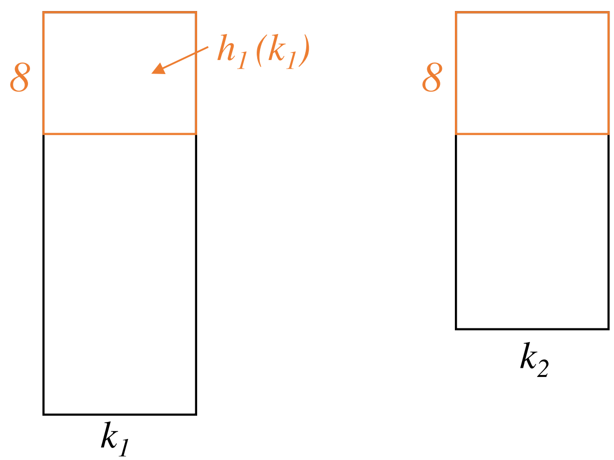

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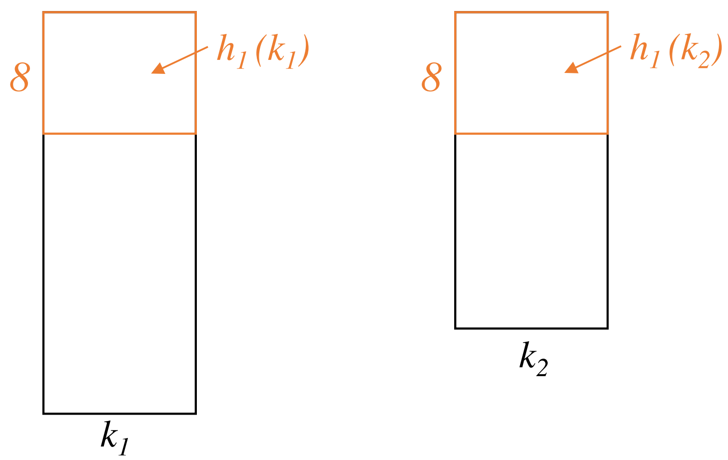

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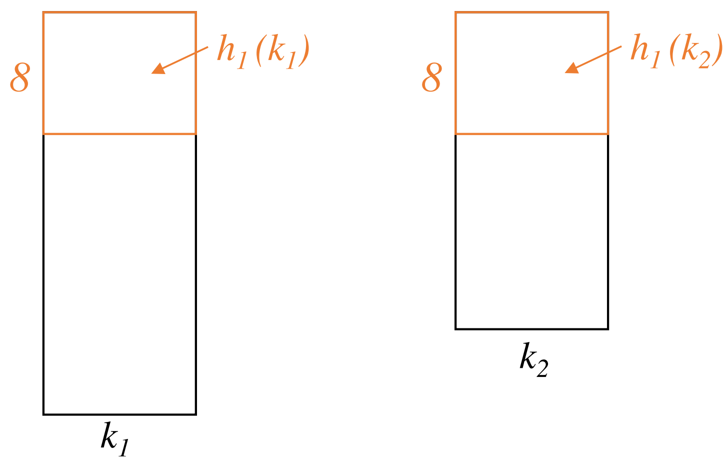

<u>Problem</u>:

This approach ignores part of the data.

In a biased set of keys, these parts could be where the keys differ

Coding Functions

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Coding Functions

$h_1 : U \rightarrow (64\text{-}bit\ integers)$

i. <u>Common Approaches</u>:
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   - Component Sum

Coding Functions

$h_1: U \rightarrow (64\text{-}bit\ integers)$

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  Break key to its components: $key = (k_0, k_1, k_2, \ldots, k_{m-1})$.

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  The coding function h_1 would add all the components of key.

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"stop"

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all coded to the same value

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Problem:

This approach doesn't take the positions of the components into account

Coding Functions

$h_1 : U \rightarrow (64\text{–}bit\ integers)$

i. Common Approaches:
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Coding Functions

$h_1 : U \rightarrow (64\text{−}bit\ integers)$

i. <u>Common Approaches</u>:
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  Let z be an integer ≥ 2

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  We define: $h_1(key) =$

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  Fun Fact:

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  Fun Fact: If we take $z = 33$, When coding 50,000 English words, have at most 6 collisions

Coding Functions

$h_1 : U \rightarrow (64\text{−}bit\ integers)$

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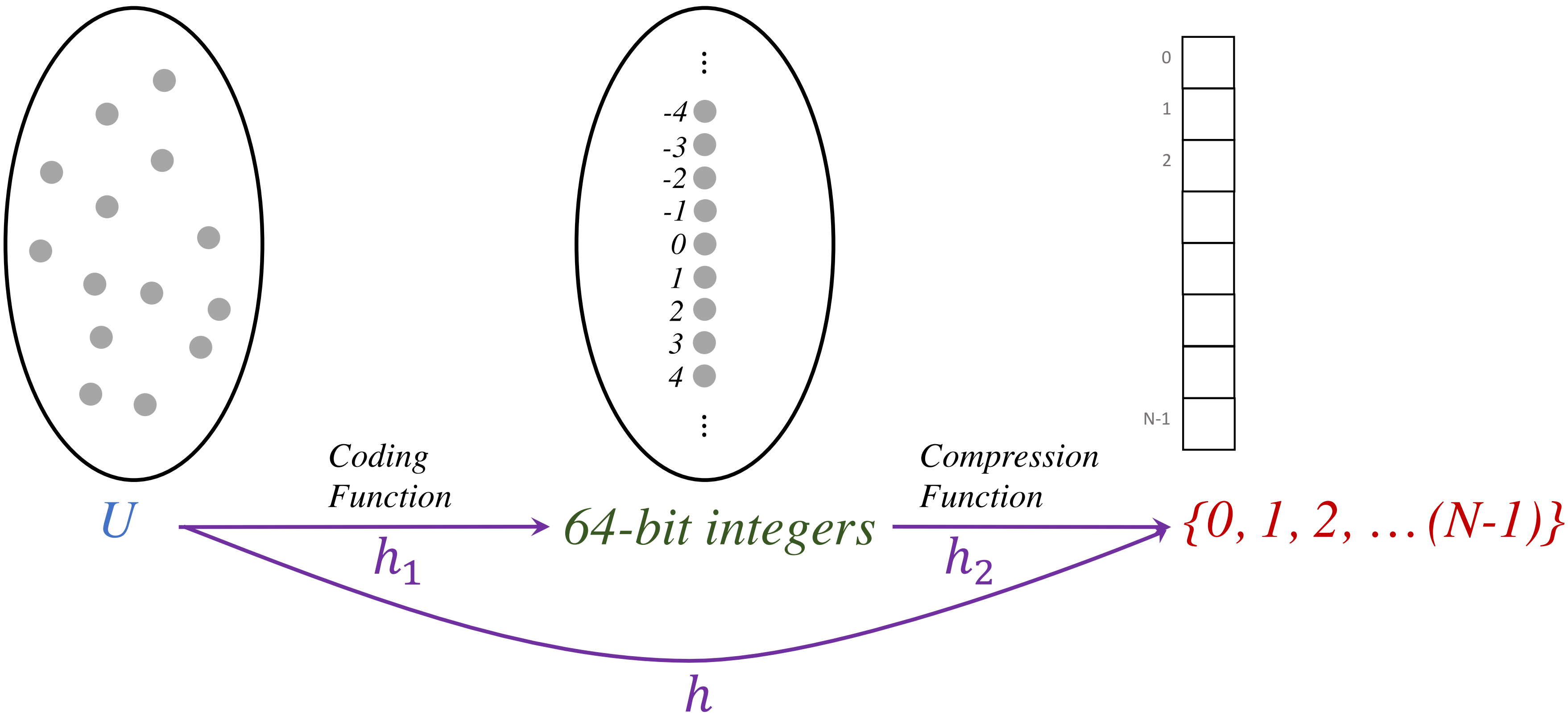
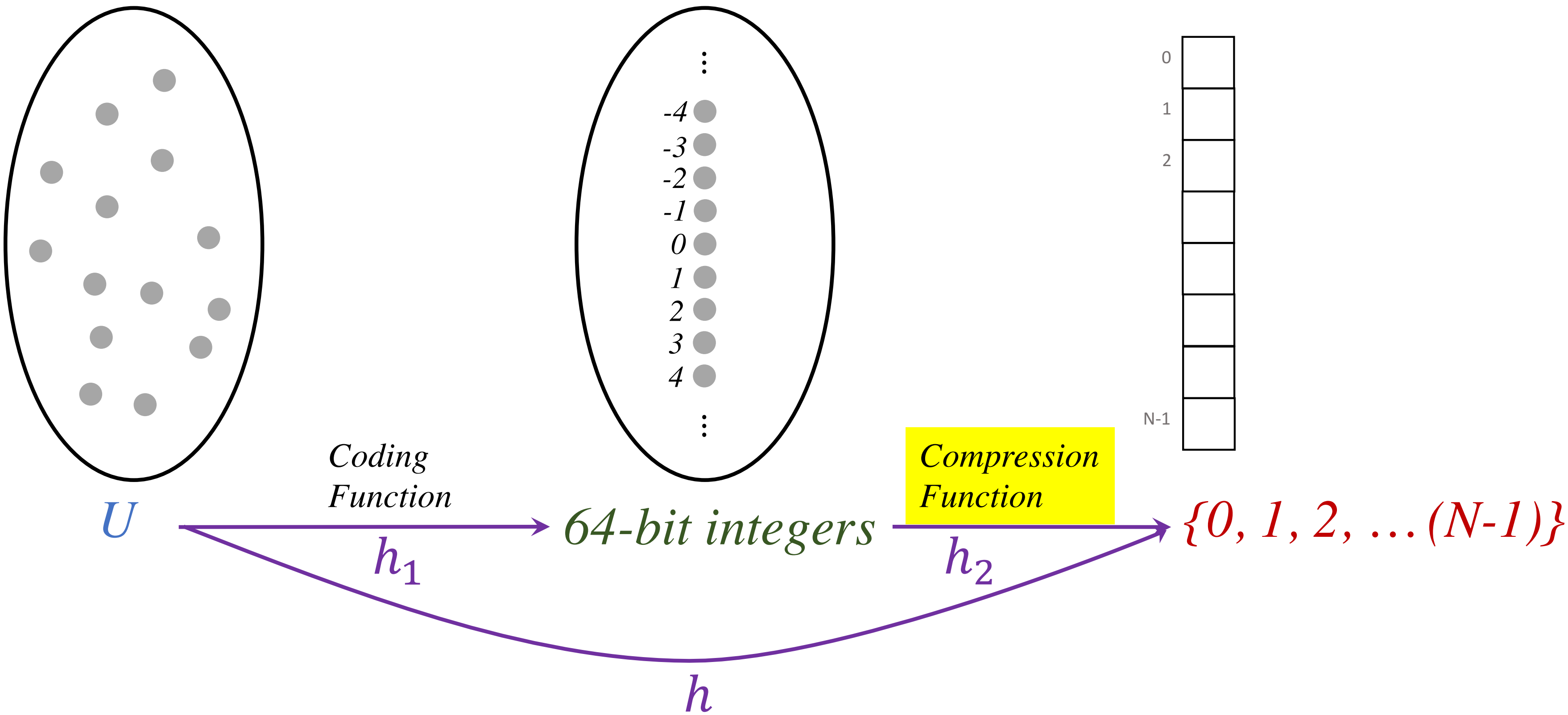
- Make sure that $(x = y) \rightarrow hash(x) = hash(y)$

Hash Functions



$$h(k) = h_2\big(h_1(k)\big)$$

Hash Functions



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h_2: $(64\text{−}bit\ integers) \rightarrow \{0, 1, \ldots, N-1\}$

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<u>Common Approaches</u>:

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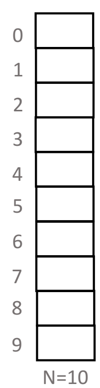

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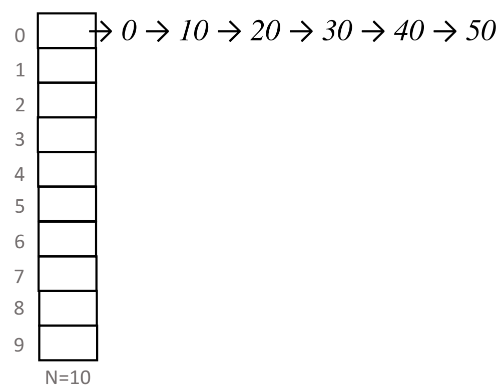

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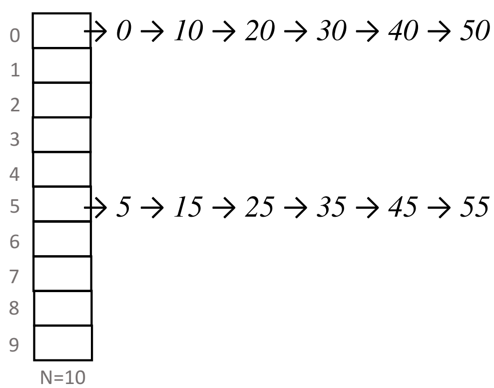

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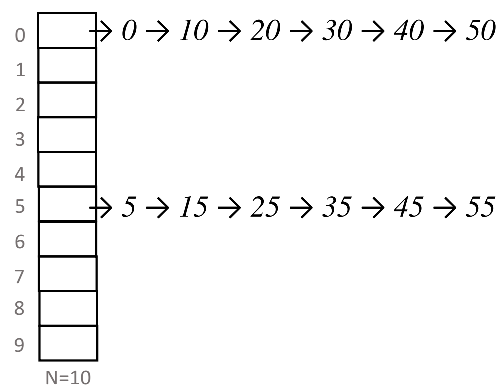

A common heuristic is to choose N to be prime.

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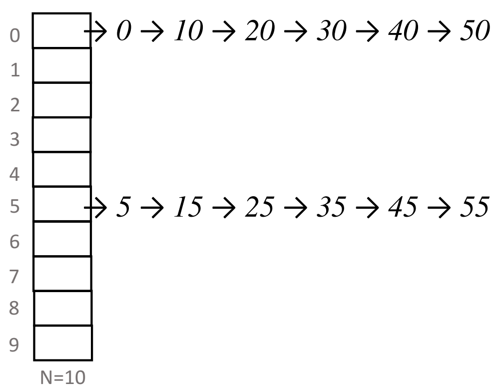

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For N=7:

Compression Functions

h_2: *(64−bit integers)* $\rightarrow \{0, 1, \ldots, N-1\}$

<u>Common Approaches</u>:

- The Division Method

  We define: $h_2(k) = k \bmod N$
  - If keys are chosen randomly → satisfies the "Uniform Hashing" property
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Example: if our keys are: {0, 5, 10, 15, 20, 30, 35, 40, 45, 50, 55}:

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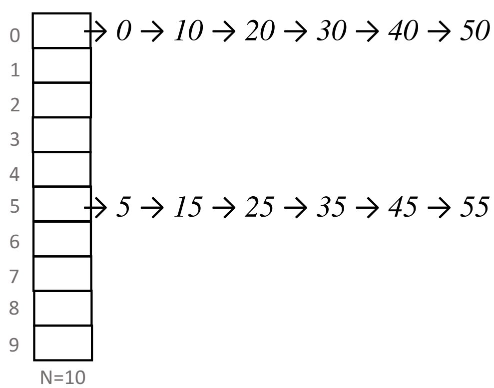

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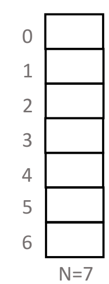

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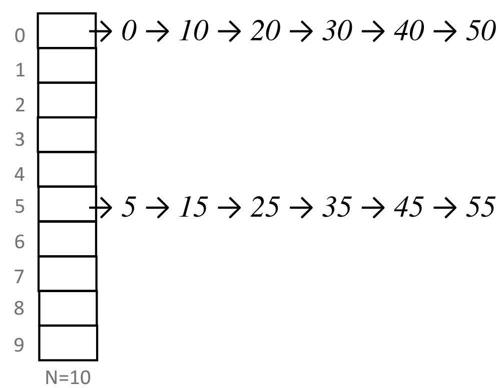

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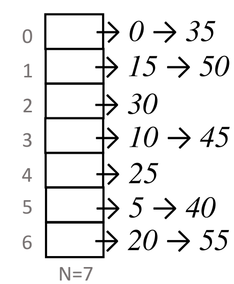

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h_2: $(64\text{−}bit\ integers) \to \{0, 1, \ldots, N-1\}$

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Example: if $|U|=60$

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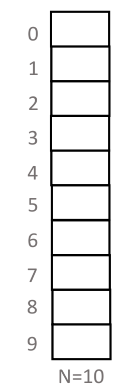

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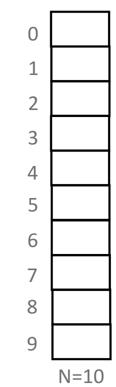

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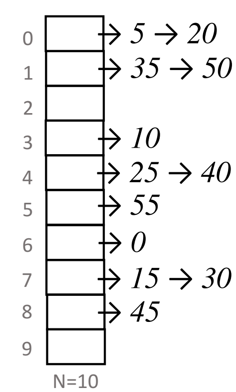

Time Analysis of $Find$

Time Analysis of *Find*

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II. Let ∝ be the load-factor of the table

Time Analysis of $Find$

Worst-Case: all keys are mapped to the same slot, it takes $\theta(n)$ to scan that chain

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That is: $\propto = \frac{n}{N}$

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Calculate the hash function and Access slot

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$\Downarrow$ Calculate the hash function and Access slot

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Implementation Points

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-
-

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-
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