1) a] $5n^3 + 2n^2 + 3n = O(n^3)$

$f(n) = 5n^3 + 2n^2 + 3n$

$g(n) = n^3$

$f(n) \le c \cdot g(n)$, for all $n \ge n_0$

$5n^3 + 2n^2 + 3n \le c \cdot n^3$

Let $n_0 = 1$

$\therefore n \ge 1$

~~Since~~ $5n^3 + 2n^2 + 3n \le 5n^3 + 2n^3 + 3n^3 = 10n^3$

$c = 10$

~~Since~~ $5n^3 + 2n^2 + 3n \le 10n^3$

$\therefore \underline{5n^3 + 2n^2 + 3n = O(n^3)}$

b] $\sqrt{7n^2 + 2n - 8} = \theta(n)$

$f(n) = \sqrt{7n^2 + 2n - 8}$

$g(n) = n$

$c_1 = 3$

$c_2 = $

$n_0 = 4$

$c_2 g(n) \le f(n) \le c_1 g(n)$, for all $n \ge n_0$

~~Since~~ $\sqrt{7n^2} \le \sqrt{7n^2 + 2n - 8} \le \cancel{\sqrt{7n^2 + 2n - 8}} \le \sqrt{7n^2 + 2n} \le \sqrt{7n^2 + 2n^2} \le \sqrt{9n^2} = 3n$

$2n - 8 \ge 0$

$2n \ge 8$

$n \ge 4$

c] $d(n) = O(f(n))$ $\longrightarrow$ $d(n) \le c \cdot f(n)$ for all $n \ge n_0$

$e(n) = O(g(n))$ $\longrightarrow$ $e(n) \le k \cdot g(n)$ for all $m \ge m_0$

$d(n) e(n) = O(f(n) g(n))$

$d(n) e(n) \le c \cdot k \cdot f(n) \cdot g(n)$

for all $n \ge n_0$ or $n \ge m_0$

$\therefore d(n) e(n) = O(f(n) g(n))$

```
def example1 (lst):
    Θ(1) ⎡ n = len(lst)
         ⎣ total = 0
         ⎡ for j in range(n):
    Θ(n) ⎢   Θ(j) ⎡ for k in range(1+j):
         ⎢        ⎣   Θ(1)⎡ total += lst[k]
    Θ(1)[ return total
```

$$T(n) = \Theta(n^2)$$

```
def example2 (lst):
    Θ(1) ⎡ n = len(lst)
         ⎢ prefix = 0
         ⎣ total = 0
         ⎡ for j in range(n):
    Θ(n) ⎢   Θ(1) ⎡ prefix += lst[j]
         ⎣        ⎣ total += prefix
    Θ(1) [ return total
```

$$T(n) = \Theta(n)$$

```
def example3 (n):
    Θ(1) ⎡ i = 1
         ⎣ sum = 0
          ⎡ while (i < n*n):
    Θ(n²) ⎢   Θ(1) ⎡ i *= 2
          ⎣        ⎣ sum += i
    Θ(1) [ return sum
```

$$T(n) = \Theta(n^2)$$

```
def example4 (n):
    Θ(1) ⎡ i = n
         ⎣ sum = 0
         ⎡ while (i > 1):
         ⎢   Θ(i) ⎡ for j in range(i):
    Θ(n) ⎢        ⎣   Θ(1)[ sum += i+j
         ⎣   Θ(1)[ i //= 2
    Θ(1)[ return sum
```

$$T(n) = \Theta(n^2)$$