

W-OTS# - Shorter and Faster Winternitz Signatures

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Version: 1.0

Date: 2020-07-20

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Abstract

A very simple modification to the standard W-OTS scheme is presented called W-OTS# that achieves a security enhancement similar to W-OTS+¹ but without the overhead of generating and transforming a randomization vector in every round of the chaining function. The idea proffered by W-OTS# is to simply thwart Birthday-attacks² altogether by signing an HMAC of the message-digest (keyed with cryptographically random salt) rather than the message-digest itself. The signer thwarts a birthday attack by virtue of requiring that the attacker guess the salt bits in addition to the message-digest bits during the collision scanning process. By choosing a salt length matching the message-digest length, the security of W-OTS# reduces to that of the cryptographic hash function. This essentially doubles the security level of W-OTS and facilitates the use of shorter hash functions which provide shorter and faster signatures for same security. For example, W-OTS# 128-bit signatures have commensurate security to standard W-OTS 256-bit signatures yet are roughly half the size and twice as fast. It is proposed that `Blake2b-128` and Winternitz parameter `w=4` (i.e. base-16 digits) be adopted as the default parameter set for the W-OTS# scheme.

1. Birthday Attack

A birthday attack involves an attacker forging a signature for a "malicious" message `M` by re-using a signature for an "agreed" message `m`. In this class of attack, the attacker has pre-knowledge of a message `m` that the victim is willing and intending to sign in the future.

The attacker creates variations of `m` as `{m_1..m_k}` any of which will also be deemed "valid" and signed by the victim. Whilst the victim considers each message `m_i` "identical", their hash digests are unique. This can be achieved by simply varying nonces or whitespace within `m` to create this set.

The attacker simultaneously generates variations of a "malicious" message `M` as the set `{M1..M_l}` and stops until a collision `H(m_i) = H(M_j)` is found (where `H` is the hash function used in the scheme).

Note the probability of finding such collisions is far more likely than a standard brute-force attack by virtue of the Birthday problem^{2 3}.

When a collision-pair `(m_i, M_j)` is found, the attacker asks the victim to sign valid `m_i` giving `s`
`= Sign(m_i, key) = SignDigest(H(m_i), key)`. The attacker then proceeds to forge a signature for invalid `M_i` by simply re-using `s`, as follows:

```

1: S = Sign(Mj, key)
2:   = SignDigest(H(Mj), key)
3:   = SignDigest(H(mi), key)
4:   = S

```

Unbeknownst to the victim, by signing m_i , they have also signed M_j .

2. W-OTS & W-OTS+

The Winternitz scheme is a well-documented ^{4 5} scheme whose description is beyond the scope of this document. However, of relevance is the relationship between the W-OTS "security parameter" n (the bit-length of H) and its "security level" which is generally $n/2$. This follows from the fact that if a brute-force attack on H requires 2^n hash rounds then a birthday attack requires $2^{(n/2)}$ ² hash rounds. By eliminating the birthday attack, and assuming no such other class of attacks exist for H , the security level of the scheme is restored back to that of a brute-force attack on H which is n .

W-OTS+ achieves a similar security enhancement through obfuscation of pre-images in the hashing chains, however they are performed during the chaining function which adds an overhead (significant in some implementations). W-OTS# is similar to W-OTS+ in this regard except it only obfuscates the message-digest once via an HMAC (keyed with the salt) and uses the standard W-OTS chaining function, which is faster than W-OTS+. Despite the concatenation of the salt to the signature, the overall signature size decreases by virtue of selecting a shorter hash function H .

3. W-OTS#

The W-OTS# construction is almost identical to a standard W-OTS construction for Winternitz parameter w and cryptographic hash function H . The security parameter n is inferred from the the bit-length of H .

In W-OTS, a message-digest md is computed as $md=H(\text{message})$. During signing, digits of base 2^w are read from md and signed in a Winternitz chain. In W-OTS#, the message-digest md is replaced with the "sig-mac" $smac$ defined as:

3.1 Signature Message Authentication Code (SMAC)

```

1: smac = SMAC(m, salt)
2:       = HMAC(H(m), salt)
3:       = H(salt || H(salt || H(m)))

```

The $salt$ is concatenated to the signature and used to compute $smac$ during verification.

NOTE the checksum digits are calculated and signed identically as per W-OTS but derived from $smac$ not md .

3.2 Salt

The `salt` is generated by the signer using cryptographic random number generator. The length of the `salt` is `n` bits which is the minimum value required to nullify a birthday attack (proven below). The salt is defined as:

1: `salt = {0,1}n` (i.e. `n` cryptographically random bits)

3.1.2 Proof

1. A birthday-collision is expected after $1.25 * \sqrt{U}$ hashing rounds where `U` is maximum hashing rounds ever required (non-repeating).
2. In W-OTS, $U=2^n$ where `n` is the security parameter (bits-length of `H`) and thus (1) becomes $1.25 * 2^{n/2}$.
3. In W-OTS#, adding a `d`-bit salt hardens a birthday-collision to $A = 1.25 * 2^{(n+d)/2}$ rounds. This follows from the fact that an attacker must scan for collision $(HMAC(H(m_i), salt), HMAC(H(m_j), salt))$ which involves `d` more bits (whereas in W-OTS they just scan for $(H(m_i), H(m_j))$).
4. A brute-force attack on `H` requires $B = 2^n$ hashing rounds².
5. We need to choose `d` such that $A = B$, since we only need to harden a birthday attack to match that of a brute-force attack. Hardening beyond is redundant since the security level of the scheme is only as strong as the weakest attack vector.
6. Evaluating (5) gives $d = 2 \ln(0.8) / \ln(0.2) + n = 0.2773 + n$ which is approximately `n`.
7. Thus choosing $d=n$ is sufficient to thwart birthday-attack. QED.

4. References

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