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Group Project

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Session 5.1.2 Random Error: 1000 trajectories of Random Error for one year

Session 5.1.2 Return: 1000 trajectories of Return for one year

Session 5.1.2 Result: 1000 trajectories for the future stock prices for one year.

Session 5.2.1: BSM Formula calculation and result

Session 5.2.2-generate: a data generator for calculating European Option

Session 5.2.2 (E1, E2, E3, E4): data collection of each case of m for European Option

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Session 5.2.3: data collection of Monte Carlo Simulation for Option 4 (European)

**Project\_task.cpp and Project\_task.exe**

Program for Monte Carlo Simulation

Vanilla Options

Research on method of pricing options

5.1 Geometric Brownian Motion Model

5.1.1 Use Market Data to Estimate Model Parameters μ and δ

1. Selected Company: Nike Inc (NYSE: NKE)

Source: Yahoo Finance (<https://finance.yahoo.com/quote/NKE/>)

2. Selected Period: 1/3/2017 - 4/27/2018

3. Display of Prices

4. Estimated μ and δ

|  |  |  |
| --- | --- | --- |
| **Log Return** | **Daily** | **Annually** |
| Mean (μ) | 0.000875 | 0.220612 |
| SD (δ) | 0.014492 | 0.230053 |

μt=ln(St-dt / St)

μannual= 250\* μdaily

More detailed data of prices in file Project.xlsm, session 5.1.1.

5.1.2 Generate Future Stock Prices Using Monte Carlo Simulation

1. S0 = 68.389999

2. Δt = 1 day

3.

(1) In file Project.xlsm, Session 5.1.2 Random Error displays 1000 trajectories of Random Error for one year.

Macro was chosen to randomly generate 1000 trajectories for Random Error.

(2) Session 5.1.2 Return displays 1000 trajectories of Return for one year.

Function to generate return:

(3) Session 5.1.2 Result displays 1000 trajectories for the future stock prices for

one year.

Function:

4.

5. The following items was displayed in Session 5.1.2 Result, Row 255 – Row 259

• Maximum and minimum prices.

• Average price for the one year period.

• Average rate of return.

• Standard deviation of the rate of return.

6.

• The simulated future price trajectories are not quite similar to the historical realization in part 5.1.1.

• The average rates of return and the standard derivations calculated from the simulated future price trajectories are close to the estimated values for μ and δ in part 5.1.1.

•Especially, the average of average return in each trajectory is equal to 0.000941, which is close to simple μ in 5.1.1 and average of standard deviation is 0.014573 which is also the same as the simple δ in 5.1.1.

The Simulation can reflect the character of a stock (such as μ and δ). And we can use Monte Carlo Simulation to estimate the possible future price of a stock.

5.2 Black—Scholes—Merton Option Pricing Model

5.2.1 Calculate Option Price Using the BSM Formula

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *European* | **S** | **K** | **Sigma** | **T** | **r** | **Div.** |
| Option 1 | 68.39 | 82.68582 | 0.230053 | 0.25 | 0.03 | 0 |
| Option 2 | 68.39 | 83.3083 | 0.230053 | 0.5 | 0.03 | 0.01 |
| Option 3 | 68.39 | 83.93546 | 0.230053 | 0.75 | 0.03 | 0.01 |
| Option 4 | 68.39 | 84.56734 | 0.230053 | 1 | 0.03 | 0 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *European* | **d1** | **d2** | **N(d1)** | **N(d2)** | **Call Price** | **Put Price** |
| Option 1 | -1.52753 | -1.64256 | 0.063315 | 0.050237 | 0.207206 | 13.88521 |
| Option 2 | -1.0702 | -1.23287 | 0.142266 | 0.108813 | 0.750985 | 14.77008 |
| Option 3 | -0.85315 | -1.05239 | 0.196787 | 0.146311 | 1.350216 | 15.53922 |
| Option 4 | -0.6775 | -0.90755 | 0.249046 | 0.182059 | 2.091063 | 15.76906 |

The calculation was handled by Excel. The function and related calculation are displayed in the Session 5.2.1.

5.2.2 Calculate Option Price Using Binomial Method

Selected Option: Option 4

1. Session 5.2.2-generate is a data generator of binomial method for calculating European Option.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **r** | **K** | **S0** | **Sigma** | **T** |
| 0.03 | 82.068 | 68.39 | 0.230053 | 1 |

The function needed in calculation:

2.

The result of European call & put option with m = 4,8,16,32

|  |  |  |
| --- | --- | --- |
| **m** | **C0** | **P0** |
| **4** | 2.633199 | 13.88572 |
| **8** | 2.482764 | 13.73529 |
| **16** | 2.634116 | 13.88664 |
| **32** | 2.580478 | 13.833 |

Session 5.2.2-E1, 5.2.2-E2, 5.2.2-E3, 5.2.2-E4 displays the process of calculating European option price for m = 4, 8, 16, 32;

The result of calculation is close to what we have got in 5.2.1. Call is cheaper than put, which satisfies the condition when K > S.

3. Session 5.2.2-American is a data generator of binomial method for calculating American Option.

The result of American call & put option with m = 4, 8, 16, 32

|  |  |  |
| --- | --- | --- |
| **m** | **C0** | **P0** |
| **4** | 2.502474 | 14.81698862 |
| **8** | 2.348852 | 14.78046273 |
| **16** | 2.495837 | 14.83035559 |
| **32** | 2.442583 | 14.80040011 |

Session 5.2.2-A1, 5.2.2-A2, 5.2.2-A3, 5.2.2-A4 separately displays the process of calculating American option price for m = 4, 8, 16, 32;

The result of calculation is close to what we have got in 5.2.1. Call is cheaper than put, which satisfies the condition when K > S.

For the price of call option, the American option is cheaper than the European option; as for the put option, at the same condition, the American option is more expensive than the European option.

All their prices are close to the answer from BSM formula.

5.2.3 Calculate Option Price Using Monte Carlo Simulation

The file Project\_task.cpp with Project\_task.exe was the program written for Monte Carlo Simulation;

Language: C++

Simulation Model: risk-neutral model

The Needed Variables:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **M (Steps)** | **N (Paths)** | **R (risk-free)** | **sigma** | **K** | **S0** |
| 250 | 100, 1000, … | 0.03 | 0.230052515 | 82.068 | 68.39 |

The result of Option 4 after Monte Carlo Simulation

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **n** | **100** | **1000** | **5000** | **10000** | **50000** | **100000** | **500000** | **1000000** |
| **Call** | 3.69062 | 2.89952 | 2.71077 | 2.66112 | 2.62298 | 2.61218 | 2.60513 | 2.59357 |
| **Put** | 13.9413 | 13.7251 | 13.5235 | 13.6584 | 13.7372 | 13.7253 | 13.7433 | 13.769 |

Basic Function:

Simulated option price is close to what we have got in 5.2.1;

The results of simulation demonstrate a trend that the price of European option is gradually closer and closer to the result from BSM formula.

Conclusion: Monte Carlo Simulation with the help of program is more efficient than conducting Binomial Method; The result of Monte Carlo Simulation can well reflect the result calculated from BSM formula.