Chapter 1

A REVIEW OF THE MAJOR MULTIDIMENSIONAL SCALING MODELS FOR THE ANALYSIS OF PREFERENCE/DOMINANCE DATA IN MARKETING

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Multidimensional scaling (MDS) represents a family of various spatial geometric models for the multidimensional representation of the structure in data as well as the corresponding set of methods for fitting such spatial models. Its major uses in Marketing include positioning, market segmentation, new product design, consumer preference analysis, etc. We present several popular MDS models for the analysis of consumer preference or dominance data. The first spatial model presented is called the vector or scalar products model which represents brands by points and consumers by vectors in a T dimensional derived joint space. We describe both individual and segment level vector MDS models. The second spatial model is called the multidimensional simple unfolding or ideal point model where both brands and consumers are jointly represented by

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points in a T dimensional derived joint space. We briefly discuss two more complex variants of multidimensional unfolding called the weighted unfolding model and the general unfolding model. Here too, we describe both individual and segment level unfolding MDS models. We contrast the underlying utility assumptions implied by each of these models with illustrative figures of typical joint spaces derived from each approach. An actual commercial application of consideration to buy large Sports Utility Vehicle (SUV) vehicles is provided with the empirical results from each major type of model at the individual level is discussed.

Keywords: Multidimensional scaling; vector model; unfolding model; positioning analysis; market segmentation; clusterwise models.

1. Introduction

Using the Carroll and Arabie (1980) broad conceptualisation, we define multidimensional scaling (MDS) as a family of various geometric models for the multidimensional representation of the structure in data as well as the corresponding set of methods for fitting such spatial models. Carroll and Arabie (1980) present a taxonomy of the area of MDS based on the properties of the input measurement data (e.g., number of modes, number of ways, power of the mode, scale type, conditionality, completeness of the data, replications, etc.) and properties of the underlying multidimensional measurement model (e.g., type of geometric model, number of sets of points in the derived space, number of derived spaces, degree of constraints on model parameters, etc.). Thus, their definition extends classical MDS which typically deals only with spatial models for proximity data (e.g., similarities/dissimilarities) to various other forms of continuous and discrete representations, as well as to other data types. Our focus will be upon the two major types of models utilised for the analysis of dominance (i.e., preference, consideration to buy, choice, etc.) data as is typically collected in Marketing Research: The vector MDS model and the unfolding MDS model (Scott and DeSarbo, 2011). Readers interested in a more comprehensive discussion of this broad area of MDS are encouraged to consult the excellent book on MDS by Borg and Groenen (2005) for an in-depth treatment of these and other types of MDS approaches for the analysis of such data (e.g., correspondence analysis). For expositional purposes, we will assume that the data to be analysed is a two-way dataset of metric brand preferences where the rows of this data matrix ($\underline{\boldsymbol{P}}$) reflect a sample of consumers and the columns of the matrix represent brands in a designated product/service class. The general entry in this data matrix (P_{ij}) is the metric preference rating given for brand j by consumer i. The objective of the MDS models to be described is to estimate a spatial configuration (a joint space) of both row (consumers or derived market segments) and column (brands) objects such that their particular geometric interrelationships most parsimoniously recovers the input preference data $\underline{\boldsymbol{P}}$. We will describe both traditional individual level MDS models and more recent segment level or clusterwise MDS models for the analysis of such preference or dominance data.

2. The Vector MDS Model

2.1. *The individual level vector MDS model*

Tucker (1960) and Slater (1960) were the first to independently formulate this scalar products based model for geometrically displaying the structure in such two-way data (Carroll, 1972, 1980). Related to factor analysis, the underlying model can be mathematically represented as:

$$P_{ij} = \sum_{t=1}^{T} a_{it} b_{jt} + e_{ij}, \tag{1}$$

where:

$i =$ $1, \ldots, I$ consumers;
$j =$ $1, \ldots, J$ brands;
$t =$ $1, \ldots, T$ dimensions;
$a_{it} =$ the tth coordinate of the terminus of the preference vector for consumer i in the derived space;
$b_{jt} =$ the tth coordinate of the location of brand j in the derived space;
$e_{ij} =$ error.

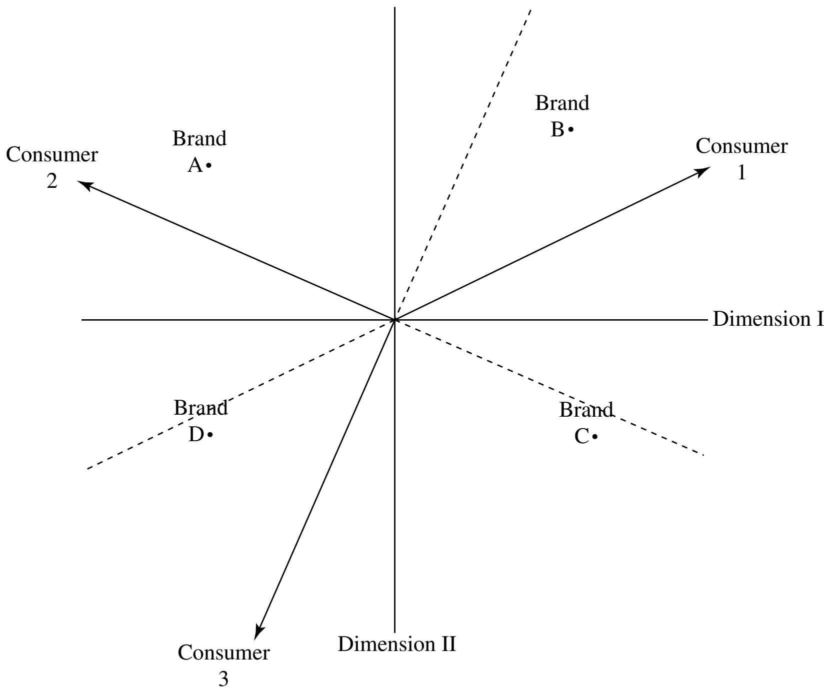

Fig. 1. The vector model.

We describe this particular geometric representation *via* Fig. 1 which illustrates the workings of the vector MDS model in Eq. (1) for the simple case of two dimensions, four brands, and three consumers. The two dimensions are labelled in the figure and represent typical scatter plot axes. The brand coordinates (b_{jt}) are plotted here for each of the four brands (A, B, C, and D) and represent the positions of the brands in this derived space. Note, the consumer locations (a_{it}) are represented in this model (labelled Consumers 1, 2, and 3) as vectors emanating thru the origin whose orientations point in the direction of increasing preference or utility for each consumer. Each of the three consumers' vectors point in different directions reflecting heterogeneity (i.e., individual differences) in their respective tastes and preferences. (Note that we draw the tails of the vectors here as dashed lines reflecting the areas of the space that are dispreferred for each consumer). The predicted cardinal preference values are given by the orthogonal projection of each of the brands onto each consumer's preference vector. Thus, Consumer 1 has the following order of predicted preference: B, C, A, D; Consumer 2: A, D, B, C; and, Consumer 3: D, C, A, B. Note, the consumer vectors are typically normalised to equal length in such joint space representations although, under certain data pre-processing conditions, the raw lengths of such vectors can be shown to be proportional to how well each consumer's preferences are recovered by the vector representation. The goal of such an analysis is to simultaneously estimate the vectors and brand coordinates in a given dimensionality that most parsimoniously captures/recovers the empirical preference data. The analysis is typically repeated for $t = 1, 2, 3, \ldots, T$ dimensions and the dimensionality is selected by inspection of a scree plot of the number of estimated dimensions *versus* a goodness of fit statistic (e.g., variance accounted-for) that measures how well the model predictions in Eq. (1) match the input preference data given the number of model parameters being estimated. Note, the cosines of the angles each consumer vector forms with the coordinate axes render information relating to the importance of these derived dimensions to that consumer. The iso-preference contours for this vector MDS model in two dimensions for a particular consumer vector (i.e., locations of equal preference) are perpendicular lines to a consumer vector at any point on that vector since brands located on such lines would project at the same point of predicted preference onto the vector. Thus, it is important to note that this vector model is not a distance based spatial model. Also, one can freely rotate the joint space of vectors and brand points and not change the model predictions (the orthogonal projections of the brand points onto the consumer vectors) or goodness-of-fit results. As noted by Carroll (1980), one of the unattractive features of this vector model is that it assumes that preference changes monotonically with respect to all dimensions. That is, since a consumer's vector points in the direction of increasing preference or utility, the more of the dimensions in that direction implies greater preference; i.e., *the more the better*. In marketing, this can create conceptual problems depending upon the nature of the underlying dimensions.

This assumption may not be realistic for many latent attributes or dimensions underlying brands in a product/service class. For example, it is not clear that consumers prefer infinite amounts of size and sportiness (assuming those were the two underlying dimensions driving their vehicle preferences) in their family Sports Utility Vehicle (SUV). In addition, it would imply that the optimal positioning of new brands would be located towards infinity in the direction of these consumer vectors which is most often not realistic. However, the vector MDS model has been shown to be very robust and estimation procedures such as MDPREF (Carroll, 1972, 1980) based on singular value decomposition principles provide globally optimum results while being able to estimate all orthogonal dimensions in one pass of the analysis.

Recently, Scott and DeSarbo (2011) have extended this individual level deterministic vector MDS model to a parametric estimation framework and have provided four variants of the individual level vector MDS model involving reparameterisation options of the consumer and/or brand coordinates. There are occasions or application where the derived dimensional coordinates regarding $\underline{A} = ((a_{it}))$ and/or $\underline{B} = ((b_{jt}))$ in Eq. (1) are either difficult to interpret or need to be related to external information (e.g., brand attributes/features, subject demographics, etc.). One can always employ property fitting methods (Borg and Groenen, 2005) where methods such as correlation or multiple regression can be employed to relate $\underline{A}$ and/or $\underline{B}$ to such external information. Unfortunately, given the rotational indeterminacy inherent in the vector model, such methods can mask tacit relationships between these estimated dimensions and such external information. As such, the model defined in (1) can be generalised to incorporate additional data in the form of individual and/or brand background variables. The coordinates for individuals (vector termini) and/or brands, as the case might be, can be reparameterised as linear functions of background variables (see Bentler and Weeks, 1978; Bloxom, 1978; de Leeuw and Heiser, 1980; and Noma and Johnson, 1977, for constraining MDS spaces). If stimulus attribute data is available, then b_{jt} can be reparameterised as:

$$b_{jt} = \sum_{k=1}^{K} x_{jt}\gamma_{kt}, \tag{2}$$

where x_{jk} is the value of feature k ($k = 1, \ldots, K$) for brand j and γ_{kt} is the impact of feature k on dimension t. As in CANDELINC (Carroll *et al.*, 1979), Three-Way Multivariate Conjoint Analysis (DeSarbo *et al.*, 1982), GENFOLD2 (DeSarbo and Rao, 1984, 1986), Restricted Components Analysis (Takane *et al.*, 1995), and various clusterwise or latent class MDS approaches (DeSarbo *et al.*, 2008), one can model the location of stimuli to be a direct function of their respective features. Thus, the x_{jk} are quantified features which are related to brand attributes (Lancaster, 1966, 1979). Similarly, when individual respondent background data is available, a_{it} can be reparameterised as:

$$a_{it} = \sum_{r=1}^{R} z_{ir}\alpha_{rt}, \tag{3}$$

where z_{ir} is the value of characteristic r ($r = 1, \ldots, R$) for individual i and α_{rt} is the impact of the rth individual characteristic on dimension t. When *both* stimuli and individual background data are available, both sets of coordinates can be so reparameterised. (Note that one always has the option of performing general property fitting analyses in the non-parameterised model with $\underline{A}$ or $\underline{B}$ if, for example, one did not have the full set of background variables to describe $\underline{A}$ or $\underline{B}$ completely). An option in the Scott and DeSarbo (2011) methodology also exists to estimate a stretching/shrinking parameter when a_{it} is so reparameterised to avoid potential problems with placing constraints on individual vectors as discussed in Carroll *et al.* (1979) and DeSarbo *et al.* (1982). This parameter would appear as a consumer specific multiplicative term on the right-hand side of Eq. (3). Maximum likelihood involving non-linear optimisation methods is utilised in estimating the desired set of model parameters.

2.2. *The segment level or clusterwise vector MDS model*

Recently, DeSarbo *et al*. (2008) have developed a clusterwise bilinear vector MDS model which enables one to *simultaneously* perform market segmentation and positioning analyses. This model is mathematically represented as:

$$P_{ij} = \sum_{s=1}^{S} \theta_{is} \sum_{t=1}^{T} a_{st} b_{jt} + c_i + e_{ij}, \tag{4}$$

where:

$s =$ $1, \ldots, S$ market segments (unknown);
$a_{st} =$ the tth coordinate of the terminus of the preference vector for segment is in the derived space;
$\theta_{is} =$ 0, if ith consumer is not classified in segment s,
1, otherwise;
$\theta_{is} \in$ $\{0, 1\}$;
$c_i =$ an additive constant.

This clusterwise bilinear MDS model is designed to *simultaneously* derive a single joint space where the derived segments (not individual consumers) are represented by vectors (a_{st}) and brands by coordinate points (b_{jt}), and their interrelationship in the space denotes aspects of the underlying data structure. Thus, it estimates the brand coordinates, market segment composition, and the segment vectors all at the same time. In essence, the geometry of the clusterwise vector MDS model is nearly identical to that described in Fig. 1 with the exception that individual consumer vectors are replaced by segment level vectors. Here too, the orientations of the estimated segment vectors give the direction of highest utility for each segments, and a brand has higher preference the further out in the direction of the vector (i.e., the larger the orthogonal projection of the brand onto the segment's preference vector). A major benefit of the DeSarbo *et al*. (2008) model is that it does not require distributional assumptions unlike latent class MDS models (see DeSarbo *et al*., 1991), and provide a concise spatial representation for the analysis of preference/dominance data. Here too, an option to reparameterise brand locations as a linear function of attributes via Expression (2) is available. The alternating least-squares and combinatorial optimisation estimation procedures are fast and efficient, and can accommodate large datasets. In addition, within a complete cycle of iterations, it estimates conditional globally optimum values of the parameters via analytic closed form expressions. Within each major iteration, the procedure performs four cycled steps to estimate the parameters b_{jt}, a_{st}, θ_{is}, and c_i. It monitors convergence by calculating an overall VAF (variance accounted for — akin to R^2) statistic after each cycle. Model selection (i.e., selecting R and S) is determined on the basis of scree plots of VAF versus sequential values of R and S. In addition, as with all MDS procedures, model selection is also guided by interpretation of the results and parsimony. Lastly, this model accommodates both overlapping segmentation structures as well as hard partitions.

3. The Unfolding MDS Model

3.1. *The individual level simple unfolding model*

Coombs (1950) (unidimensional unfolding) and Bennett and Hayes (1960) (multidimensional unfolding) introduced a different type of geometric MDS model for the analysis of such metric preference/dominance data called the simple unfolding model. Mathematically, the model can be represented as:

$$F(P_{ij})^{-1} = \sqrt{\sum_{t=1}^{T} (a_{it} - b_{jt})^2} + e_{ij}, \tag{5}$$

where we use the same notation as used in the vector model in Eq. (1) with some striking differences relating to the nature of how consumers' preferences are represented in this approach. In particular, in the simple unfolding model, both brands and consumers are represented as coordinate points in the derived T dimensional joint space. Here, the closer in distance between a consumer and a particular brand, the higher the predicted preference is for that particular brand. Thus, distance is *inversely* related to preference

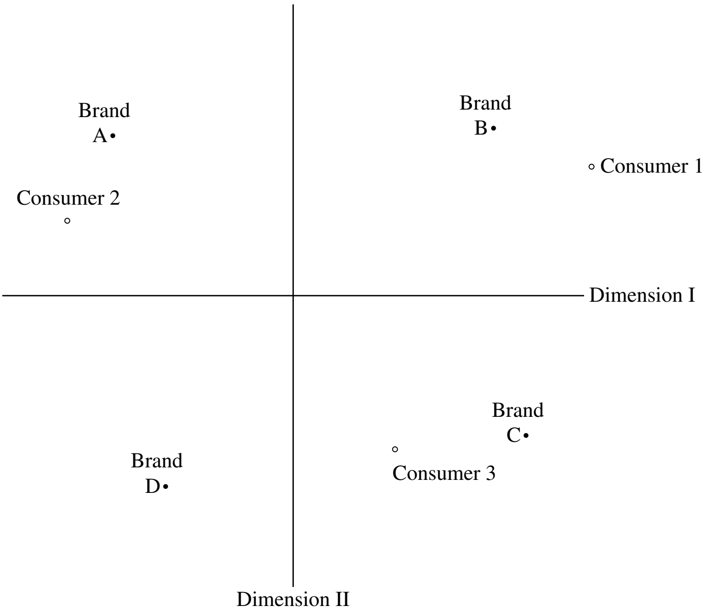

Fig. 2. The simple unfolding model.

and that is why one sees an inverse function for preference on the left hand side of Eq. (5) above. Figure 2 illustrates this model in two dimensions with three consumers and four brands. Consumer 1's order of predicted preference would be B, C, A, D; Consumer 2: A, D, B, C; and, Consumer 3: C, D, B, A. This geometric model is Euclidean distance based, and the consumer locations are often called *ideal points* as they reflect the optimal combination of the dimensions for that consumer. Thus, unlike the vector model where higher preference tends off to infinity in the direction of the estimated consumer vector, the simple unfolding model posits highest utility at the ideal point, and preference trails off uniformly in any direction from that ideal point. The farther a brand is from a consumer ideal point, the less preferred that brand is. Here, the iso-preference contours in two dimensions are concentric circles around an ideal point since all brand points of fixed radius around the particular brand point would reflect equal preference by this model. Both the brand locations and consumer ideal points are estimated to best recover the empirical preference data. As in the vector model, the analysis is conducted in $t = 1, 2, 3, \ldots, T$ dimensions, and the dimensionality is selected in terms of contrasting goodness-of-fit versus the number of dimensions estimated. Like the vector model, one can rotate the joint space configurations and not affect distances. The simple unfolding model also is indeterminate with respect to the origin of the derived joint space. Nonetheless, the simple unfolding model is more appealing to marketers given its finite preference utility assumptions and intuitive underlying Euclidean distance model. Ideal points often represent targets for marketers who try to attempt to position their brands near target segment ideal points (DeSarbo and Rao, 1986). DeSarbo and Rao (1984,1986) have implemented the linear reparameterisations of brand and/or ideal points as written in Eqs. (2) and (3) to allow for policy simulations and optimal positioning strategies. Carroll (1980) also introduced the weighted and general unfolding models which provide for additional forms of individual differences in a multidimensional unfolding context, but these more complex models involve the estimation of many additional parameters, and successful applications of these highly parameterised unfolding models in the marketing literature are lacking (we describe each in more detail below).

As first noted by Carroll (1980), while the vector and ideal point model appear to be quite different geometric representations, one can easily show that the vector model is a special case of the simple ideal point model. If one were to move an ideal point for any individual consumer further and further out along a fixed line from the origin while holding the brand locations fixed, one ends up with a vector utility model for that consumer in the limit where the ideal point tails off to infinity. The family of concentric circles surrounding that ideal point begins to flatten out and resemble the iso-preference lines of the vector model. Thus, there is additional flexibility involved in the simple unfolding model which can accommodate the vector model as a special case. However, despite its greater flexibility and intuitive appeal, the simple unfolding model suffers from frequent degenerate, uninformative solutions where the brand points and consumer ideal points are estimated to be excessively separated from each other (see Borg and Groenen (2005) for more elaborate definitions and a discussion of attempts to resolve this difficulty).

As mentioned earlier, Carroll (1980) also introduced two more general forms of multidimensional unfolding called the *weighted* and *general unfolding* MDS models. The weighted unfolding model allows each individual to have the differently weighted dimensions in addition to different ideal points. Mathematically, the weighted unfolding model can be represented as:

$$F(P_{ij})^{-1} = \sqrt{\sum_{t=1}^{T} w_{it}(a_{it} - b_{jt})^2} + e_{ij}, \tag{6}$$

where w_{it} is an estimated weighting factor which represents the importance of the tth dimension for the ith consumer. Assuming $w_{it} > 0$, this parameter allows for a stretching or shrinking of a particular dimension that is individual specific. Figure 3 illustrates the weighted unfolding model in two dimensions with three consumers and four brands. Different from the simple unfolding model, this model has ellipse shaped iso-preference contours (instead of concentric circles in the simple unfolding model) in two dimensions where the larger the weight for a particular dimension, the smaller the corresponding axis of the iso-preference ellipse. In Fig. 3, Consumer 1 equally weights the two dimensions and orders brands A, B, C, D regarding his predicted preference; Consumer 2 weights Dimension I more than Dimension II with the preference order of C, D, A, B; and Consumer 3 weights Dimension II more than Dimension I with the preference order of B, D, A, C. This geometric model is constructed on the basis of the weighted Euclidean distance defined in Eq. (6). Problems arise when the estimated w_{it} are negative with respect to respective interpretation. If for a specific consumer in two dimensions both weights are negative, then the location of that individual's ideal point becomes an *anti-ideal point* and refers to the point in the joint space of *least* preference. There, preference increases as one travels *further away* from the anti-ideal point in either direction. When the weights are mixed (i.e., in two dimensions, one

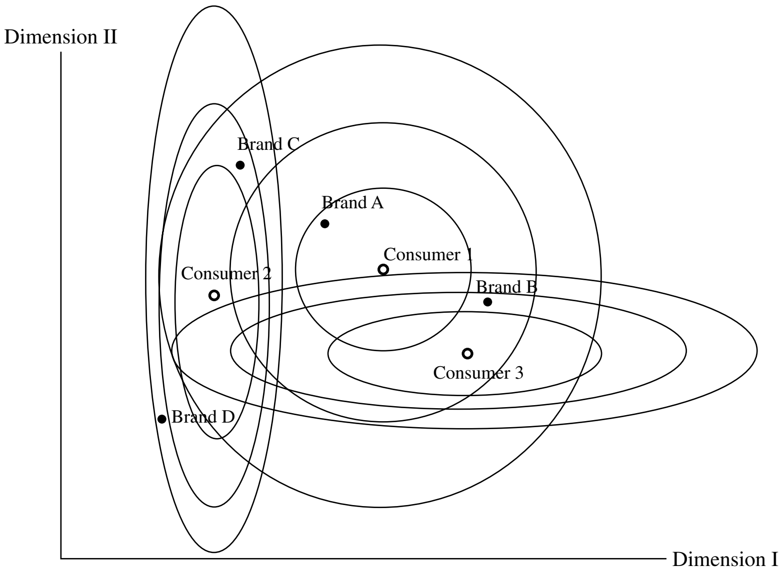

Fig. 3. The weighted unfolding model.

is positive and the other is negative), the location of the ideal point is a *saddle point* where preference increases on one direction as you go further away from the ideal point, while preference increases with the other dimension as you get closer to the ideal point. Given the dramatic increase in the number of estimated parameters and the associated difficulties involved with the interpretation of negative weights, it is no wonder why there have not been many published applications involving this particular model.

Another more complex generalisation of the simple and weighted unfolding models is the general unfolding model that can be characterised by individual specific differential rotations as well as differential weightings. The differential rotations can be conducted by allowing linear transformation on matrices $\underline{A} = ((a_{it}))$ and $\underline{B} = ((b_{it}))$. Mathematically, this general unfolding model can be represented as:

$$F(P_{ij})^{-1} = \sqrt{\sum_{t=1}^{T} w_{it}(a_{it}^{*} - b_{jt}^{*})^2 + e_{ij}}, \tag{7}$$

where $((a^*_{it})) = \underline{A}^*$ and $\underline{A}^*_i = \underline{A}_i\underline{T}_i$($\underline{A}_i$ is the ith row of the matrix $\underline{A}$ and $\underline{T}_i$ is an orthogonal transformation matrix for the ith consumer). Here, $((b^*_{jt})) = \underline{B}^*$ and $\underline{B}^*_j = \underline{B}_j\underline{T}_i$($\underline{B}_j$ is j-th row of the matrix $\underline{B}$). While the weighted unfolding model allows distinct individuals to differently weight dimensions, the general unfolding model *additionally* modifies the assumption that all individuals utilise the same set of dimensions. Here, the general unfolding model allows each individual to choose idiosyncratic sets of reference dimensions in the joint space. That is, each individual is allowed to rotate the reference frame of perceptual space and then to differentially weight these dimensions. Figure 4 illustrates (for one hypothetical consumer) that the general unfolding model also has iso-preference ellipse contours for different $\underline{T}_i$, but the ellipses need not be parallel to the dimension axes depending on $\underline{T}_i$. Because rotation alone does not make the model different from the simple unfolding model, it is the combination of differential rotations and differential weighting of dimensions that makes this a more general unfolding model.

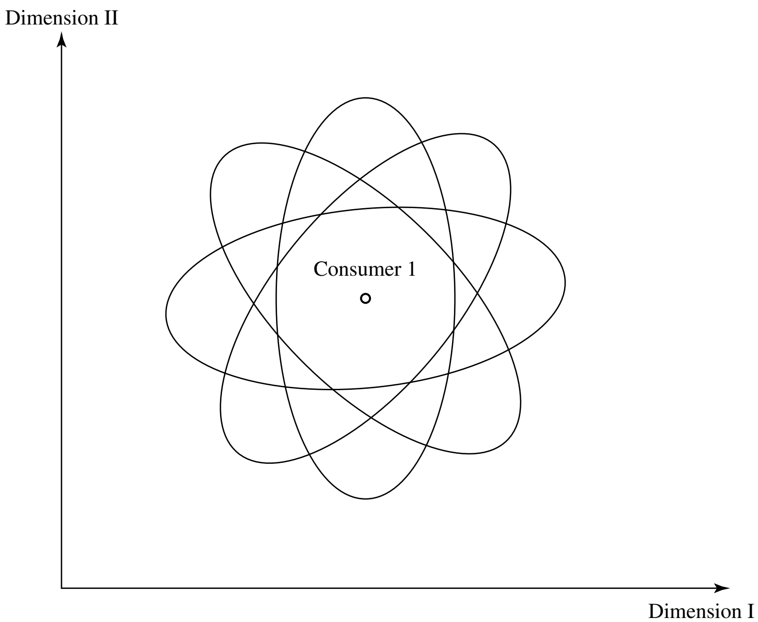

Fig. 4. Illustration of general unfolding model.

Like weighted unfolding models, there is a dramatic increase in the number of parameters involved with such a model compared to the simple unfolding model, and a paucity of any known published marketing applications exists.

3.2. *The segment level or clusterwise multidimensional unfolding model*

DeSarbo *et al.* (2008) more recently developed a two- and three-way clusterwise unfolding MDS model which simultaneously estimates a joint space of stimuli and (multiple) ideal points by derived segment, as well as the segments themselves. This model is mathematically represented as:

$$F(P_{ijr})^{-1} = \sum_{s=1}^{S} \theta_{is} \sum_{t=1}^{T} w_{rt}(a_{srt} - b_{jt})^2 + c_r + e_{ijr}, \tag{8}$$

where:

$r =$ $1, \ldots, R$ consumptive situations (e.g., time, usage occasion, goal, etc.);
$P_{ijr} =$ the preference for brand j given by consumer i in situation $r^{\cdot}$;
$a_{srt} =$ the tth coordinate of the ideal point for market segment s in situation $r^{\cdot}$;
$w_{rt} =$ weighting parameters for dimension t in situation $r^{\cdot}$;
$c_r =$ an additive constant for situation $r^{\cdot}$;
$e_{ijr} =$ error (deterministic).

This clusterwise unfolding MDS model can be used for the analysis of any type of two- or three-way metric preference/dominance data. In case of three-way data, the model shows how multiple ideal points can represent preference changes over situations, and it provides a concise spatial representation for the analysis of contextual/situational preferences. As an illustration, Fig. 5 presents a hypothetical solution for $S = 3$ segments, $J = 10$ brands, and $R = 3$ consumptive situations. The figure illustrates the spatial solution provided by this three-way clusterwise unfolding MDS model where a_{sr} labels the associated ideal point set of coordinates for the sth

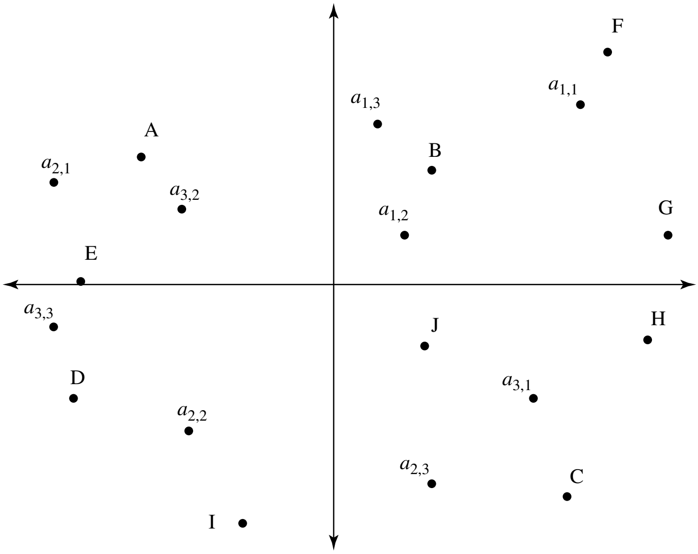

Fig. 5. Clusterwise unfolding joint space.

segment in the r-th situation, and the letters A–J label the brands. As shown in the figure, Segment 1 has three ideal points which fall into the first quadrant and the ideal points seem not to be much affected by situational factors. In contrast, Segment 2 and Segment 3 have their ideal points located in quadrants 2, 3 and 4, and the points show large ideal point movements associated with these situational factors.

This three-way clusterwise unfolding MDS model allows for a number of diverse options. First, either two-way or three-way data are accommodated by this model. Second, one can choose either stationary ideal points a_{st} or situation/context dependent ideal points a_{srt}. Third, one can estimate either non-overlapping or overlapping segments. Fourth, one can impose constraints on w_{rt} (weighting parameter) including positivity constraints or equality constraints $w_{rt} = w_r$ for all t (the equality constraint allows for a simple clusterwise unfolding model). Fifth, one can perform either internal or external analyses regarding the brand coordinates b_{jt}. Last, users can select from a variety of starting options such as given starting values, random starting values or rational starting values.

With regard to estimation procedure, the three-way clusterwise unfolding MDS model utilises an alternating least-squares iterative estimation procedure. Within each iteration, w_{rt}, c_r, a_{srt}, θ_{is} and b_{jt} are estimated by minimising the error sums of squares:

$$\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{r=1}^{R} q_{ijr}[F(P_{ijr})^{-1} - F(\hat{P}_{ijr})^{-1}]^2, \tag{9}$$

where q_{ijr} is a user specified weighting function that is adapted to prevent degenerate solutions as introduced by DeSarbo and Rao (1984) and DeSarbo and Carroll (1985). The dimensionality and number of segments are determined based on scree plots of VAF versus T and S, interpretation of the derived solutions, parsimony, etc.

4. A Marketing Application

In this section, we present a detailed Marketing application of the two individual level MDS joint space models (the vector and simple unfolding models) using customer consideration-to-buy data from a tracking study conducted in 2002 by a large U.S. automotive consumer research supplier. This particular application is modified from DeSarbo and Scott (2011). The study has been conducted semi-annually in June and December for over 20 years across several different vehicle segments. It is used to gauge automotive marketing awareness and shopping behavior among vehicle intenders. An "intender" is defined as a prospective buyer that will be "in market" or has plans to purchase a new vehicle within the next 6–12 months. The surveys used in these tracking studies were conducted among new vehicle intenders and were collected from an automotive consumer panel of more than 600,000 nationally represented households (see DeSarbo *et al*. (2008) for additional details). Using a 4-point preference response scale (4 – "Definitely Would Consider", 3 – "Probably Would Consider", 2 – "Probably Would Not Consider", and 1 – "Definitely Would Not Consider"), the respondents rated each brand in terms of their consideration-to-buy (preference) corresponding to the product segment in which he or she intends to purchase.

For this illustration, we used the large sport utility vehicle segment which included the following 16 brands: Chevy Suburban, Chevy Tahoe, Cadillac Escalade ESV, Cadillac Escalade EXT, Ford Expedition, Ford Excursion, GMC Yukon, GMC Yukon Denali, H1 Hummer, H2 Hummer, Lexus LX470, Lincoln Navigator, Mercedes G-Class, Range Rover, Toyota Land Cruiser, and Toyota Sequoia. Using this product class, we estimate both the vector model and the ideal-point (unfolding) model. The data matrix for both models consists of the same individual stated consideration-to-buy from 278 respondents on the set of 16 large SUV vehicles listed above. Thus, the input data is a two-way dataset of metric brand considerations/preferences where the rows of this data matrix are the 278 consumers and the columns represent the 16 brands.

4.1. *The vector model results*

We first consider the results of the Scott and DeSarbo (2011) stochastic vector model where the goal is to simultaneously estimate the consumer vectors and brand coordinates in a given dimensionality while best recovering the empirical consideration data. Figure 6 shows the resulting configuration in two dimensions with brand locations represented as diamond shaped points and the consumers as vectors emanating from the origin (we normalise the vectors to equal length for convenience and drop the tails of the vectors in an attempt to reduce the clutter in the figure). With respect to Dimension I (horizontal axis), we see that the GM vehicles are separated from the non-GM vehicles. We thus label this dimension as GM versus non-GM. Dimension II (vertical) represents price with the least expensive vehicles at the top of the axis and increasing in price as you travel down the vertical axis. For instance, the majority of vehicles below the horizontal axis listed for more than \$60,000, whereas the Expedition or Yukon listed in the mid \$30K range at this time. The figure also reveals that brands sharing the same manufacturer are located close together. This means that consumers give these particular brands similar consideration in their purchase plans. Note that four of the GM vehicles are closely located in the first quadrant. Similarly, the two Cadillac Escalades (also GM vehicles) are grouped

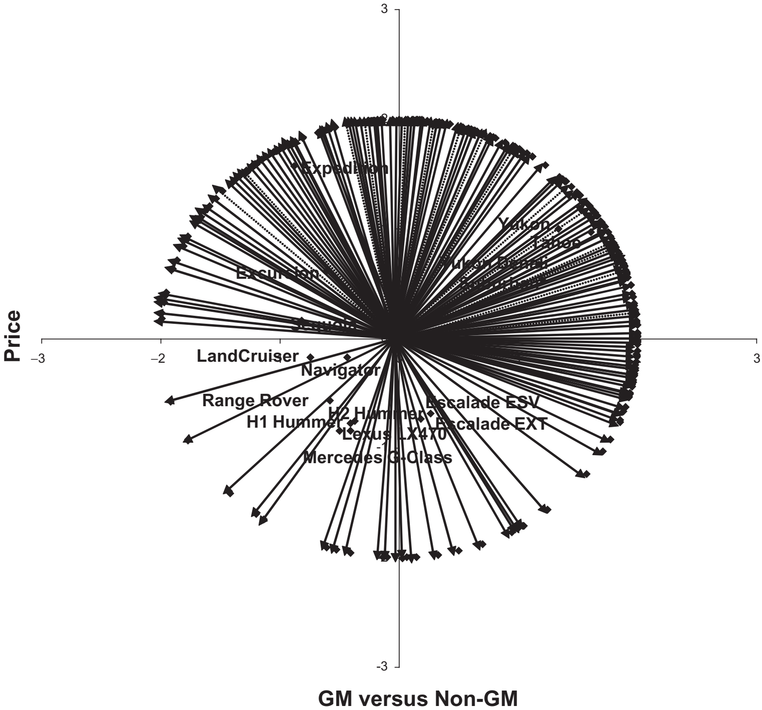

Fig. 6. Vector model — SUV application.

together, but given their premium pricing they are located with the higher priced vehicles. Other manufacturer groupings include the Ford Expedition and Ford Excursion, Toyota Land Cruiser and Toyota Sequoia, as well as the H1 and H2 Hummers. This particular joint space positioning map is problematic for this set of manufacturers in failing to make their own subset of brands sufficiently distinctive from each other and appealing to different sets of consumers in this particular product segment.

From Fig. 6, we also note a heavier concentration of vectors in the top half of the graph near the Chevy and GMC products. We may conclude that for this vehicle product segment, consideration-to-buy is greatest for the Chevy/GMC brands, followed by the Ford Expedition where we find the next largest concentration of consumer vectors. In this figure, note that a number of consumer vectors are superimposed on one another. This merely represents consumers with very similar consideration-to-buy ratings on all 16 brands. The lower part of Fig. 6 shows much less segment consideration for the large luxury sport utility vehicles like the unique Range Rover and H1 Hummer.

4.2. *The simple unfolding model results*

For the simple unfolding model representation, we first reverse scaled the same input data ($5 - P_{ij}$) given Eq. (5). Figure 7 displays the derived two-dimensional joint space spatial map for the simple unfolding or *ideal point* model obtained from the DeSarbo and Rao (1986) GENFOLD2 model. Recall that both brands and consumers are represented as points in this two-dimensional space. Here, brands are labelled and represented as diamonds for ease of identification and the 278 consumer are represented by points. The closer a consumer ideal point is to a brand, the higher the

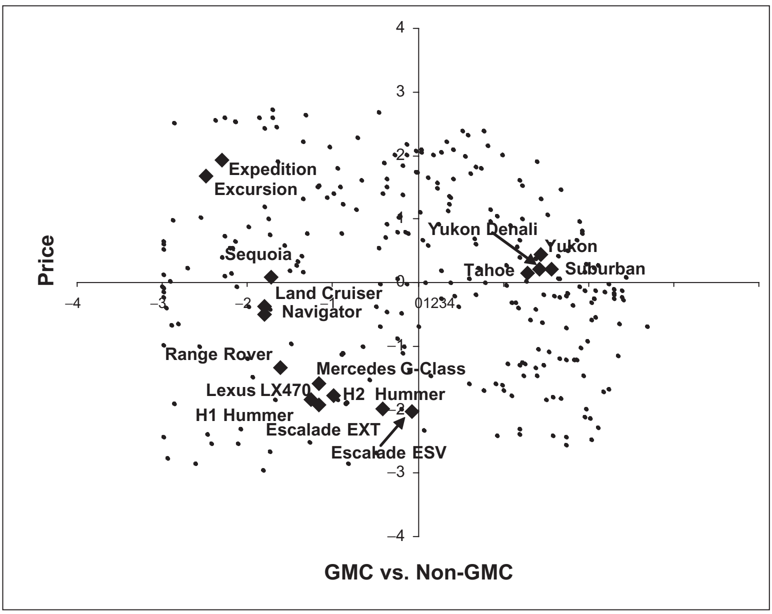

Fig. 7. Simple unfolding model — SUV application.

consideration-to-buy for that particular brand. Figure 7 shows a similar configuration of brand locations to Fig. 6 (derived from the vector model), and we label these two dimensions accordingly with Dimension I representing a GM versus non-GM factor, and Dimension II as price. Like Fig. 6, the more expensive vehicles are located lower on the vertical axis while the GM vehicles are separated from the non-GM vehicles along the horizontal axis. As indicated above, brands sharing the same manufacturer are located near one another suggesting similar consideration-to-buy and similar positioning difficulties. The ideal-point model shows a clustering of consumer points near the GM vehicles (Yukon, Yukon-Denali, Tahoe, and Suburban). There are also a good number of points in the fourth quadrant, the area between the Suburban and Tahoe and the premium priced vehicles. Because of the Euclidean distance present in the interpretation of the unfolding model, we might interpret this as potential consumer demand for different varieties of large GM SUV's given the underlying dimensions which define the space.

5. Discussion

Both Fig. 6 (vector model) and Fig. 7 (unfolding model) depict a two-dimensional joint space with 16 brands and 278 consumers. We find that the two figures are quite similar with respect to their respective brand configurations and interpretation of the underlying dimensions. Since MDS seeks to reveal the underlying structure in data, this result is not surprising given we use the same data matrix to estimate both models. Both representations reveal potential Marketing positioning problems where each manufacturer's own brands appear to compete more against their own brands than against those brands of other manufacturers.

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