Usikkerhetsoppgaver

1-2

$$C = \frac{2\pi \ell \varepsilon}{\ln(b/a)}$$

$\Delta \varepsilon_{max} = 0.2\%$

$\Delta a_{max} = 0.7\%$

$\Delta b_{max} = 0$

$\frac{b}{a} = 2$

$$|\Delta(C/\ell)| = \left|\frac{\partial C/\ell}{\partial \varepsilon}\Delta\varepsilon\right| + \left|\frac{\partial C/\ell}{\partial b}\underbrace{\Delta b}_{=0}\right| + \left|\frac{\partial C/\ell}{\partial a}\Delta a\right|$$

$$= \left|\frac{2\pi \Delta\varepsilon}{\ln(b/a)}\right| + |2\pi\varepsilon\Delta a| \cdot \left|\frac{\partial}{\partial a}\left(\frac{1}{\ln(b/a)}\right)\right|$$

$$u = \ln\left(\frac{b}{a}\right), \quad \frac{\partial}{\partial a}\left(\frac{1}{\ln(b/a)}\right) = \frac{d}{du}\left(\frac{1}{u}\right)\frac{du}{da}$$

$$= \frac{1}{-u^2}\frac{du}{da} = \frac{1}{-u^2}\frac{d}{da}\ln\left(\frac{b}{a}\right)$$

$$= \frac{1}{-\ln(b/a)^2} \cdot \frac{\partial}{\partial v}\ln(v)\frac{dv}{da}, \quad v = \frac{b}{a}, \quad \frac{dv}{da} = -\frac{b}{a^2}$$

$$= \frac{1}{-\ln\left(\frac{b}{a}\right)^2} \cdot \frac{1}{v} \cdot \left(-\frac{b}{a^2}\right) = \frac{1}{\ln\left(\frac{b}{a}\right)^2} \cdot \frac{\cancel{a}}{\cancel{b}} \cdot \frac{\cancel{b}}{a^{\cancel{2}}} = \frac{1}{a\ln\left(\frac{b}{a}\right)^2}$$

$$|\Delta C/\ell| = \left|\frac{2\pi\Delta\varepsilon}{\ln(b/a)}\right| + \left|\frac{2\pi\varepsilon\Delta a}{a\ln(b/a)^2}\right|$$

$$\frac{\Delta C/\ell}{C/\ell} = \frac{\frac{\cancel{2\pi}\Delta\varepsilon}{\cancel{\ln(b/a)}} + \frac{\cancel{2\pi}\varepsilon\Delta a}{a\ln(b/a)^{\cancel{2}}}}{\frac{\cancel{2\pi}\varepsilon}{\cancel{\ln(b/a)}}} = \frac{\Delta\varepsilon}{\varepsilon} + \frac{\varepsilon}{\varepsilon}\cdot\frac{\Delta a}{a\ln(b/a)}$$

$$= 0.2\% + \frac{0.7\%}{\ln(2)}$$

$$= 1{,}2\%$$

1-6. $n = 8$ målinger, $\bar{x} = 44.1$, $\sigma = 2$, $p = 99\%$

Konfidensintervall: $\mu_x \in [m_x \pm t_p \frac{s_x}{\sqrt{n}}]$

$\rightarrow [\bar{x} - t_p \frac{2}{\sqrt{8}}, \bar{x} + t_p \frac{2}{\sqrt{8}}]$

$\rightarrow [44{,}1 - 3.3 \frac{2}{\sqrt{8}}, 44{,}1 + 3.3 \frac{2}{\sqrt{8}}]$

$\rightarrow [41{,}77, 46{,}43]$

1-8 $V = \pi (\frac{D}{2})^2 (X_1 - X_2)$, $X_1 \sim N(\mu = 1202, \sigma = 2)$

$X_2 \sim N(\mu = 998, \sigma = 2)$

$\mu_V = \mu_1 - \mu_2 = 204$

$\sigma_{1,2}^2 = \sigma_1^2 + \sigma_2^2 = 4 + 4 = 8 \Rightarrow \sigma_{1,2} = \sqrt{8} = 2\sqrt{2}$

$\sigma_V = \pi (\frac{5}{2})^2 \cdot \sigma_{12} = 55.5 \cdot 10^{-3}\ m^3$

$\mu_V = \pi (\frac{5}{2})^2 (1202 - 998)/1000 = 4{,}006\ m^3$

Dette er for 1 standardavvik (usikkerheten normalt), så for å få 95% konfidensintervall ser vi på ± 1.96 standardavvik, derfor setter vi $\sigma_{V,95\%} = \sigma_V \cdot 1.96 = 0.109\ m^3$.

$\rightarrow V \in [4{,}005 \pm 0.109]$

1-9 $n = 12$, $m_x = 1.700\ V$, $s_x = 0.02$, $p = 1 - 99\% = 1\%$

$\mu_V = m_x \pm t_p \frac{s_x}{\sqrt{n}} = 1.7 \pm 3 \cdot \frac{0.02}{\sqrt{12}} = 1.7 \pm 0.017$

90 ishvertfall.

$\frac{1}{\sqrt{n}}$ minker for økende n, mens t_p øker lite. Må da få $\frac{1}{2} = \frac{1}{\sqrt{n}} \Rightarrow n = 4$, altså 4 ganger flere målinger. Derfor finner ny t_p, osv, osv.

1.11 $m_{tid} = 23,3\ ms$, $s_{tid} = 0.4\ ms$

$m_{fart} = 320\ m/s$, $s_{fart} = 2\ m/s$

$$s = \frac{v \cdot t}{2}$$

$$\mu_s = \frac{m_{tid} \cdot m_{fart}}{2} = \frac{7.456\ m}{2} = 3,728$$

$$\sigma_z^2 = \sum_{i=1}^{N} \sigma_{x_i}^2 \left(\frac{\partial f}{\partial x_i}\right)^2 = s_{tid}^2 \cdot \left(\frac{v}{2}\right)^2 + s_{fart}^2 \cdot \left(\frac{t}{2}\right)^2 = 4,638 \cdot 10^{-3}$$

99% konfidensintervall:

$$\mu_s \pm \sqrt{\sigma_z^2} \cdot 2,58$$

$$\rightarrow [3.728 \pm 0.178]$$